Exploring Threshold concepts in basic Statistics, using the Internet

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Abstract:
Meyer and Land (2002) have introduced the notion of a threshold concept in student learning. By definition a threshold is an insight which is initially alien or counter-intuitive, is integrative in that it subsumes some previous knowledge and is transformative in that it leads to a different perspective of or within the discipline.

The notion is suggested to be applicable in many disciplines. It is phrased in terms that the threshold nature of a concept is defined mainly by the student experience rather than simply an objective analytical process.

Nonetheless past experience may lead teachers of statistics at tertiary level to surmise that threshold concepts in basic statistics will include the notion of patterns of spread or variation, randomness, sampling, the central limit theorem, and linear regression. Introductions to Bayes' theorem and interval estimation are further candidates. Hypothesis testing may present other difficulties. Some methods and results in an exploration of student perspectives will be presented.

A class of over 465 students in their second semester course of undergraduate applied statistics (STA220) participated in a survey with a short list of 4 items via internet and PC lab access to a WebCT site supporting their current course. The 4 items addressed their experience in the earlier course (STA100), and were as follows:
Explain in your own words the term random sample.
The central limit theorem tells us something about the mean of a sample. State in your own words what the theorem implies.
List three concepts you found very simple to learn about in STA100.
List the three most difficult concepts that you learnt about in STA100.

The motive for the exploration is the open question of whether or not two key concepts were clearly and articulately reported by the respondents, and whether there is initial supportive evidence for any particular concepts being experienced as threshold concepts by these students.

As teaching effort aimed at plausible threshold concepts may lead to more successful student participation and learning, the diagnostic value of an internet resource that assists in collection of data may be substantial. Text editors may assist in the analysis of typed responses. Internet connections will allow for the quicker transfer of data and for rapid interchange of improved public domain material addressing concepts that appear to have threshold qualities

INTRODUCTION

The University of Cape Town has a preparatory one-semester mathematics course focussed upon elements with applications in business, (mis)named STA101. It is generally taken in the first semester. The purpose of the course is to address the needs of some 700 school-leavers, who are not strong mathematically. The course is designed to prepare those students for some concepts in an introductory one-semester course in statistics: STA100, which is generally taken in semester 2 of the first year. The course address data summary and description, and rudiments of sampling, point and interval estimation, hypothesis testing for paired and independent samples, and linear regression.

This STA100 course has regular statistical material with an applied and business orientation. The enrolment is of the order of 1800 students. Most (about 1100) of these students have better school-leaving mathematics grades than those for STA101 students, and hence will also be taking traditional year-one calculus and mathematical courses over two semesters of the first year.

A second one-semester course STA220 geared to students in the Faculty of Commerce, is offered in their second academic year, within a three year bachelor degree BCom. This STA220 course may lead into a statistics major in Commerce. Content includes applications of multiple regression, categorical data analysis, analysis of variance, time-series, and non-parametric tests. Enrolment requires a pass in STA100 and currently varies around 465 students in number.

In Science and Business science (actuarial and finance) streams, students have a more theoretical and rigorous statistics major. There is a view within the Department of Statistical Sciences at UCT that this group needs a more extensive foundation in year one, as too large a number appear to struggle with the formal content of their second year (distributions, models, likelihood, inference, moments and generating functions).

Concern over diminishing mathematical preparedness of school-leavers despite notionally improving school exit grades, and poor pass rates in STA100, led to an initiative to explore alternative teaching methods to supplement lecture and tutorial modes. [A second initiative will extend the content of STA100 in a new course for science and actuarial majors, but is not discussed further here].

This analysis aims to identify the innovations introduced to STA100F and STA100S, the benefits, drawbacks or problems with these innovations and speculates on their effect through exposure to threshold concepts.

SOCIETAL CHALLENGES

The apparently universal challenge of creating and managing service courses appropriate to students backgrounds, needs and career objectives, operates also at UCT. In a time of real financial constraints, and competing high-cost more-favoured courses in other disciplines, the imperatives of maximising student learning and engagement have been radically highlighted. The search for the interventions that matter, i.e. that have consequential effects, receives higher priority.

The increasing awareness that higher education must both deliver and be accountable, has led to more explicit criteria about desirable throughput (at a minimum, in terms of pass rates in courses). In the South African context, after decades of differential access to schooling and other advantages, the imperatives of transformation towards a just and equitable society exacerbate the challenges.

Increasingly the South African universities will be addressing the profile of the course enrolments across race and gender, to assess equity of access. In addition, the relative and absolute throughput rates within those subgroups, are partial measures of the adequacy with which current strategies cope with the consequences of historical contrasts of privilege and dispossession.

INNOVATIONS

A comprehensive examination by the teaching team of more than 12 first year texts developed a consensus for a single text as most likely to meet the needs of our students and their background. The chosen textbook was Keller and Warrack (1999).

Two determining factors in this choice of text may be of interest. The text offered the prospect of being a resource that was well-structured and therefore easy to use. It would allow teaching staff to motivate students to hold on to the text after the course, rather than simply dispose of it in the second-hand market. As importantly, the text was extensive in coverage and the syllabus of the STA220 was likely to be modified only in minor ways, if the text was adopted for that course too. For students that strategy would lead to zero additional text costs for second statistics course.

The book is a heavy volume, and not easily carried to lectures or on public transport. In South African terms the relative prices of textbooks is very high, and often students simply cannot afford the conventional bookshop prices. Special negotiations led to an arrangement that made costs manageable.

An additional CD to complement the text was purchasable with the text for a small marginal additional cost. It offered twelve-month sign-on to an internet resource

A large data base of multiple choice questions was constructed by members of the Department. Use of the software Respondus allowed this multiple choice resource to be formatted for internet use on the course website.

Experience had led to the belief that simply managing the class of 1800 students in the course has detracted from the time and energy that had been available for teaching and for support of student learning. The academic year 2002 saw the introduction of WebCT as a vehicle of teaching support that might offer considerable help to the individual student, and also simplify administration loads.

The complete STA100 course website was constructed so as to give students access through WebCT. In 2003 a similar structure was constructed for the STA220 course.

An identical copy of each of these sites can be accessed under URL = http://webedu.its.uct.ac.za with the login name = **Stats** (note Capital S) and entry password **welcome** for entry. In case of difficulties direct e-mail help is available from Greg Doyle gdoyle@its.uct.ac.za.

The website and the WebCT structure proved highly successful. A pass rate of 92% amongst students who completed all the course requirements was substantially higher that the pass rate of previous years. It was the view of the teaching staff that the exit expectations of knowledge and competence had not been reduced, so that the change was inferred to be real and a result of successful innovation and intervention.

It was clear that many students had made effective use of the resource, but there is still a need to encourage more regular use by some segments of the course enrolment. This issue has not been investigated and while it may simple arise because students are unfamiliar with WebCT, it is possible that deeper questions of computer literacy and numeracy need attention.

Not all teaching staff share equivalent experience with and enthusiasm for WebCT. If students read mixed messages other difficulties may be compounded.

Inadequate use of extra WebCT resources by the students and lecturers has reduced its impact and benefit; this has mainly been due to being unfamiliar with WebCT.

In the context of all this innovation, the teaching team believed it might be useful to explore a strategy to obviate exercise formula rote learning and uncritical application. For each test and examination situation students were permitted to construct a single A4 sheet of formulae, in their own handwriting. The sheets were submitted for checking of compliance with the no photocopy rule. The exercise of compiling a formula sheet four times in the course was presumed to be a source of organised learning. This assumption has not yet been tested, but anecdotal student response was enthusiastic.

The elements of innovation and the nature of apparent consequences, and some planned modifications, are presented in Table 1.

Table 1: Innovations, consequences and future modifications (STA100 in 2002).

	Innovations	Benefits	Drawbacks	Problems	Future Plans
1	**New Textbook**	Clear explanations, excellent and numerous examples	Expensive, heavy; Excel aspect was not adequately used	Students reluctant to either purchase the textbook or carry it with them	Introduction of Excel throughout the course will make the textbook more useful

2	**WebCT**	Available on and off Campus being an Internet based program. Easy to work with and not too difficult for students to pick up.	Students need to be taught and introduced to the package early in the program, not all students have the correct registration	Heavy usage of PC's around campus. Campus network not always reliable. A great deal of time and effort required to set-up the course.	Expand its use to encompass more of the course.
2 i	**Quizzes**	Enable students to constantly assess their progress and keep up, as a quiz is submitted every week, and is available for revision.	No identified drawbacks		Offer more revision quizzes
2 ii	**Lecture Notes**	Lecture time is not wasted watching students writing notes from the overheads, students concentrate on the explanations more, more examples can be covered and the students are able to listen and think about the material	Fewer students attend the lectures, printing costs are transferred to the students and are not collected from their fee accounts, however, students can make their own notes without printing them	Changes in lecture notes mean students must reprint the full set of notes for that topic rather than just the changes. Not all the PC labs around campus allow PDF files to be printed. Some printers alter the pdf outputs, distorting the notes.	No solutions to the examples will be placed in the lecture notes so as to encourage students to attend lectures and work through the material.
2 iii	**Tips**	Extra advice on known difficulty areas is available for all students.	No identified drawbacks	No problems	Expand tips
2 iv	**Worked Examples/ Common Problems**	Step by step worked examples available for all students enables known common problems to be highlighted	No identified drawbacks		
2 v	**Tutorial Solutions**	Weekly tutorial solutions are available half an hour after hand in. Students therefore have regular feedback	No identified drawbacks		
2 vi	**Past Papers and Tests**	Past papers and tests, and their solutions are available for view or to be downloaded. These resources can be used for revision purposes both in and out of lectures	No identified drawbacks	Past papers and tests need to be checked for errors and solutions often need to be written where not available on file	Expand the number of past papers and tests. Solutions required but not necessarily for deployment on WebCT. Changes in course material need explicit highlighting.

2vii	**Textbook Solutions**	The textbook has many questions for students to work through and all the solutions have been made available	No identified drawbacks		
2viii	**Test Solutions**	Test solutions are available straight after the test, no wastage of paper	No identified drawbacks	Test solutions need to be typed up	
2ix	**Marks**	Marks for all tests, quizzes and tutorials available for review by all students. Students are able to check that their work has been marked and any missing material can be dealt with immediately	No identified drawbacks	More reliance on vulnerable UCT systems which can crash.	Student response: The 80% MCQ is a better practice as it obviates bad marking from tutors.
2x	**Discussion**	Course information is available to all students immediately. Reduces the number of students asking the identical questions to problems and enables students to participate in the learning experience by answering other student's queries.	Requires monitoring to ensure that the discussion boards are not being misused.	Students do not take the time to read other people's responses.	
3	**Formula Sheets**	Students were required to produce their own handwritten formula sheets (1 A4 sheet), submitted and destroyed after each test. Strategy was to force concentration on organising and unifying the learning experience.	No identified drawbacks	A certain amount of monitoring required in the tests and checking afterwards for no photocopies and single A4 sheet.	
4	**Marked Written Tutorials**	Benefit similar to quizzes.			
5	**Web-based Tutorial Allocation System**	Vastly improves the administration of the course. Students are in control of the tutorial they wish to attend and can change when they wish. The tutors, reducing the admin load of the front office, enter tutorial	No identified drawbacks, though it does mean another program to explain to the students.	Teething problems with the system for the first six weeks of the 2nd semester.	System to be linked in with other admin systems

		marks. Students can check on marks.			
6	**Revision Lectures**	Consolidation for students, particularly around probability and probability distributions, marks were significantly improved and less students fell of the bandwagon!	No identified drawbacks		
7	**Revision Workshops**	Very popular with the students, especially as all those students who failed a recent test were encouraged to attend.	No identified drawbacks	Fitting in the revision workshops during the overburdened undergraduate timetable.	Possibly consider prearranging revision workshops to take place so many days after a test so students can plan to attend earlier.
8	**Computer Simulations**	Helped develop understanding of sampling distribution and inference concepts	No identified drawbacks	New concepts need to be repeated after section to help things sink in	Further development required and java applets required for student play and exploration.

The table records the extent of the teaching intervention as a context within which the student experience of the course material is to be studied.

INHERENT DISCIPLINE OBSTACLES TO LEARNING

Meyer and Land (2002) have introduced the notion of a threshold concept in student learning. By definition a threshold is an insight which is initially alien or **counter-intuitive**, is **integrative** in that it subsumes some previous knowledge and is **transformative** in that it leads to a different perspective of or within the discipline.

The notion of threshold is suggested to be applicable in many disciplines. An essential element of the threshold nature of a concept is defined mainly by the student **subjective** experience in acquiring it rather than simply an objective analytical process.

Nonetheless past experience may lead teachers of statistics at tertiary level to surmise that threshold concepts in basic statistics will include the notion of patterns of spread or variation, randomness, sampling, the central limit theorem, and linear regression. Introductions to Bayes' theorem and interval estimation are further candidates.

Hypothesis testing may present other difficulties than threshold challenges, due to its inherent conditionalities, and unfortunate conflations (at least in English) of the word *significance*.

If the notion of threshold is as robust as it appears, then there is a prospect of facilitating student learning by specifying and delineating the constructs that constitute thresholds, and tailoring teaching efforts to incorporate explicit strategies to assist students across them.

Ideally such initiatives require some sort of rigorous qualitative exploration with successful students to address the subjective experience. There may be value in interviewing students who fail a course, or at least the accessible subgroup of failures who repeat the course at a later stage. In fact, the later group may develop more explicit notions of what they come to experience as key threshold concepts.

Formal qualitative study was not feasible in the UCT environment. Given the apparent success of the innovations, it seemed potentially useful to explore *post hoc* whether any case could be made for the emergence of threshold concepts within the experience of that group.

Some methods and results in an exploration of student perspectives will be presented.

ASSESSING THE INNOVATIONS

A class of over 465 students in their second semester course of undergraduate applied statistics (STA220) participated in a survey via internet and PC lab access to a WebCT site supporting their current course in year 2003. Though the survey mainly addressed the demographic and academic profile of the course enrolment, it included a short set of 4 open-ended items designed to tap into the student subjective experience of the earlier material studied in the STA100 course of year 2002.

This study group may or may not constitute a sound basis for generalization. It comprises only the students who passed STA100 and were sufficiently interested or sufficiently constrained in curriculum choice, to take on a second course.

It seemed appropriate to steer student opinion as little as possible in the four items, but at the same time to establish some idea of the extent of reasonable learning in the earlier course. Accordingly, the first two items addressed student perceptions of two specified concepts, and two further items gave students an open-ended framework within which to offer particular views.

1. Explain in your own words the term random sample.
2. The central limit theorem tells us something about the mean of a sample. State in your own words what the theorem implies.
3. List three concepts you found very simple to learn about in STA100.
4. List the three most difficult concepts that you learnt about in STA100.

The motivation for the first pair of items was the question of whether or not two key concepts (random sample and CLT) were clearly and articulately reported by the respondents. These concepts are plausible candidates for threshold status.

The open format for the second pair of items seemed a plausible strategy by which to elicit some part of the subjective experience of students, and thereby markers for other threshold concepts.
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It was possible to have every one of the 465 registered students participate, but the quality of participation was not uniform. A small subset of students offered flippant or nonsensical responses to one or more of the four items.

In general there appeared to be a serious attempt to respond frankly to items. However the quality of understanding or the level of language use and of articulate expression reflected in responses to the first two items, is quite sobering. Even if the possible effect of the informality of any consequence of a poor answer and other motivational factors undermine the efficacy of the two items, the use of language is problematic.

Many students seem unable to write a coherent sentence, or a logically constructed set of points. Two sets of 5 extracts from the student responses for randomness and the central limit theorem illustrate this claim.

1. It is a group of things taken out of a population in any way possible.

2. A random sample is some type of variable picked out of many at random and used as a basis on estimating/concluding what happens to the whole population.
3. A random sample is a selection of items from a population in a way that no formula has been used, on i.e. only capably done by a human
4. randomly chosen figures from a huge choice
5. It items selected without prior research

1. The theorem says that the mean of the sample is the middle
2. When the man of a sample lies close to the mean of the population the distribution is approximately normal of sufficiently large enough to be normal.
3. the limit implies that the mean is close to the central figure
4. This informs us where the mean lies. e.g. variability if the mean is close to bottom of sample or upper level of sample
5. That if we choose a large i.e. very large sample size we will get closer to resembling a normal distribution.

A separate linguistic analysis is under design so that the challenges, in the UCT environment, of communicating a real appreciation of randomness and of the behaviour of sample means as sample size increases, may be better understood.

Many of the 465 respondents claimed fewer than 3 areas of difficulties, and many reported fewer than 3 simple concepts. The text answers were screened for the presence of particular strings of letters.

The strings were chosen from a list of course concepts and augmented by an examination of the terms actually given by students. A collection of 86 strings was the subject of the search and the total counts of occurrences of these strings in the composite set of simple and difficult concepts were 686 and 726 respectively.

Some 39 of the 86 strings did not appear in a single student response: alternative hypothesis, class intervals, class limits, class width, classes, CLT, conditional independence, critical values, cumulative frequencies, cumulative probabilities, CV, difference of means, disjoint events, elementary events, equally likely, goodness-of-fit, hypergeometric, independent samples, intercept, matched samples, model, moment, ogives, one-tail, outcomes, paired samples, parameters, pmf, power, probability density functions, probability formula, regression coefficient, SD, slope, standardized, strays, t-distribution, test-statistics, variance

Table 2 indicates that some 26 concepts dominating student responses of each type (simple and difficult).

What may be of interest is the set of concepts with frequent occurrence in both types. These concepts perhaps include thresholds that give rise to an apparent divergence of opinion amongst respondents. In decreasing frequency of total occurrence in student responses, possible thresholds are SS, probability, chi-square, t-test, Poisson, F-test, binomial, normal, regression, exponential, z-test, chi-test,

Mean, median and uniform were fairly often specified as simple, and virtually never as difficult. The terms continuous and exponential, when specified, were fairly uniformly described as difficult.

As teaching effort aimed at plausible threshold concepts may lead to more successful student participation and learning, the diagnostic value of an internet resource that assists in collection of data may be substantial.

Text editors may assist in the analysis of typed responses. Internet connections will allow for the quicker transfer of data and for rapid interchange of improved public domain material addressing concepts that appear to have threshold qualities.

Table 2: Frequencies of concepts in 465 student responses.

Simple	Count		Difficult	Count
probability	92		chi-square	110
median	79		SS	106
SS	60		probability	72
mode	53		Poisson	70
normal	51		F-test	58
binomial	45		t-test	54
histograms	39		exponential	49
t-test	36		regression	30
chi-square	35		chi-test	29
z-test	28		z-test	23
means	25		binomial	21
regression	24		continuous	16
Poisson	20		means	12
F-test	13		estimation	10
linear regression	11		comparison	9
Correlation	9		Correlation	7
chi-test	8		normal	7
standard deviation	8		discrete	6
discrete	7		linear regression	6
theorem	5		theorem	6
uniform	5		central limit theorem	4
correlation coefficient	4		estimator	2
confidence intervals	3		events	2
distribution functions	3		histograms	2
estimation	3		samples	2
exponential	3		standard deviation	2
14 **Other** concepts	17		10 **Other** concepts	11
Response count	**686**		**Response count**	**726**

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