TECHNOLOGY SUPPORTED INQUIRY LEARNING IN MATHEMATICS AND STATISTICS WITH FATHOM: A PROFESSIONAL DEVELOPMENT PROJECT

by

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ABSTRACT

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Technology Supported Inquiry Learning in Mathematics and Statistics with Fathom: A Professional Development Project
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This study utilized teacher development experiment methodology to support, and deeply examine, three teachers' understandings and practices regarding content, pedagogy, and technology as they learn about and strive to integrate Fathom, data analysis software, into their curriculum and instruction. Surveys, observations, and interviews were utilized to gather data in order to research the interactions among these three factors and how they were associated with the effectiveness of the three teachers' integration of technology. Pictures of weak, developing, and exemplary facilitation of Technology-Supported Inquiry Learning, as outlined by an effective learning environment conceptual framework, resulted from this study. Exemplary practices occurred in teachers who held strong understandings and practices in all areas; content, pedagogy, and technology. This implies that professional development must be provided in such a way that teachers can learn about, practice with, and reflect on all areas simultaneously.

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CHAPTER 1

PURPOSE AND SIGNIFANCE

INTRODUCTION

The purpose of this study is to thoroughly examine teachers' understandings and practices regarding content, pedagogy, and technology as they learn about and strive to effectively integrate *Fathom*, data analysis educational software, into their curricula and instruction. The research focus of this teacher development experiment (TDE is thoroughly described in Chapter Three) is to: (1) articulate a reform-based conception of Technology-Supported Inquiry Learning (TSIL), and (2) study what teacher learning and practice looks like, via case studies of three mathematics teachers, as they strive to integrate Fathom in a TSIL classroom environment.

To further explicate the goals of this project, this chapter will: describe the National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (NCTM PSSM, 2000) and the National Research Council's National Science Education Standards' (NRC Standards, 1996) vision of mathematical literacy and scientific inquiry; introduce statistics as a bridge between mathematics and science and as a natural field of inquiry; demonstrate the links between statistical practice and how people learn; develop the role of technology in statistics and inquiry, and introduce Fathom and discuss the supports it provides for learning; summarize TSIL with Fathom; and finally, introduce the research questions of this study.

Technology-Supported Inquiry Learning is a term that has not previously been introduced. The description of TSIL, articulated below, seeks to clarify this term and demonstrate the significance of research aimed at examining changes in teachers' understandings and practices as they work within this setting. Further, the vision of TSIL expressed in this chapter will serve to frame the research questions of this study. These research questions will be presented at the end of Chapter One.

SIGNIFICANCE

The significance of this research is two-fold. First, although technology is ever-present in society and exists in many schools, there are not many examples of ways that it can be used to enhance student learning and motivation. Articulating a TSIL framework and including examples via Fathom and statistics education will benefit the education community seeking to better understand what effective technology integration looks like. Second, by working closely with three practicing teachers and developing case studies of their understandings and practices regarding technology, statistics, and teaching, this research benefits the participants and their students, and provide materials other teachers and teacher educators can utilize for their own teaching and learning.

Federal education goals highlight the importance of this chain of inquiry. The Office of Technology Assessment (OTA, 1995, p. 34) recommended the following areas for federal policy in a report commissioned by the United States Congress:

1. Federal and state leadership that articulates the value of integrated, technology based teaching and legitimizes technology as a path to achieve educational goals.

2. Increased focus on teachers, both in training and in the field, including: time and money to allow teachers to learn to use technology, support for their professional growth, respect for the complex nature of learning and the many demands facing teachers today, and research on how technology affects teaching and school change.
3. Commitment to research, development, and dissemination that will advance technology use by and for teachers.

We are living in a technologically advancing society. Mathematical, scientific, and technical knowledge play an important role in this society. This fact has been recognized by the United States government in many studies and reports (MSEB, 1989; OTA, 1995; NCMST, 2000; Bush, 2001**)**. One of the recent reports commissioned by the government on this subject, by The National Commission on Mathematics and Science Teaching for the 21st Century (NCMST, 2000), summarized the significance of technical knowledge by stating:

> At the daybreak of this new century and millennium, the Commission is convinced that the future well-being of our nation and people depends not just on how well we educate our children generally, but on how well we educate them in mathematics and science specifically (p. 5).
>
> Four important and enduring reasons underscore the need for our children to achieve competency in math and science: (1) the rapid pace of change in both the increasingly interdependent global economy and in the American workplace demands widespread math and science related knowledge and abilities; (2) our citizens need both math and science for their everyday decision making; (3) math and science are inexorably linked to the nation's security interests; and (4) the deeper, intrinsic value of math and science knowledge shape and define our common life, history, and culture. Math and science are primary sources of lifelong learning and the progress of our civilization (p. 7).

Information abounds in the technological age of today and students need to learn to understand and use the deluge of data that surrounds us. If students do not learn how to critically understand the flood of information that constantly surrounds us today, then they will not be able to effectively participate in industry, nor even in citizenship. Unfortunately, too often this has not been the case; many students are not achieving mathematical literacy and are being left behind (Bush, 2001).

Technical knowledge is often referred to as a gatekeeper to higher paying jobs and social status (Jetter, 1993). In the current "high-tech" era, which is marked by continually emerging technologies, rapid scientific development, and the ascendancy of information as "the new capital," the ability to understand mathematics, and to mathematize is an important aspect of social power (Damarin, 1995). Mathematizing is the way "in which actions or processes are transformed into conceptual mathematical objects" (Cobb et al., 1997, p. 258). Changes in the workplace increasingly demand teamwork, collaboration, and communication. Similarly, college-level mathematics courses are increasingly emphasizing the ability to convey ideas clearly, both orally and in writing. To be prepared for the future, high school students must be able to exchange mathematical ideas effectively with others (NRC, 2000).

All too often, however, education assists to reproduce stratification between those who have technical knowledge and those who do not:

> Because math holds the key to leadership in our information-based society, the widening gap between those who are math literate and those who are not coincides, to a frightening degree, with racial and economic categories. We are at risk of becoming a divided nation in which knowledge of math supports a productive, technologically powerful elite while a dependent, semiliterate majority, find economic and political power beyond reach. Unless corrected, innumeracy and illiteracy will drive America apart (MSEB, 1989, p. 74).

It is imperative that all students gain confidence and understanding in mathematics. Democracy is founded on the principle that the majority of the people are able to participate in the decision making process, with an absence of class distinctions and privileges. It is the purpose of our education system that citizens are prepared to participate in our democracy. When we fail to successfully cultivate mathematical, scientific, and technical understanding in all citizens, the education system in failing in its commitment to democracy.

One of the major reasons that schooling has not been successful in helping all students to learn, and has even served to mediate the reproduction of stratification in society (Willis, 1979), is due to the dry, objective, and shallow nature of the curricula the majority of students encounter in school (Anyon, 1980; Hiebert, 2003). These researchers have found that much school curricula has emphasized procedures over concepts (NRC, 1997; 2000) and has not accommodated a variety of perspectives and learning styles. This traditionally dry, objective, shallow curriculum has disenfranchised many students and contributed to their lack of commitment to, and belief that they are able to, gain mathematical understanding.

Instruction, too, has often not taken learners' backgrounds into consideration (Secada et al., 1995). In a recent international comparative study of mathematics and science education, Third International Mathematics and Science Study (TIMSS) (Hiebert, 2003), the instruction in United States classrooms was found to be much more teacher-centered and composed of low-order thinking tasks than those countries that demonstrated higher student achievement.

> Materials intended for our math and science students mention a staggering array of topics. This mention does not include much more than the learning of algorithms and simple facts. In mathematics, we have a highly fragmented curriculum, textbooks that are a 'mile wide and an inch deep,' and teachers who cover many topics but none extensively. We make low demands on students and have a more

> limited conception of ‘the basics’ than the international norm. We must make substantial changes if we are to compete and to produce a quantitatively and scientifically literate workforce and citizenry (Schmidt, McKnight, and Raizen, 1997, p. 7).

Without substantial changes from traditional methods of curriculum and instruction to more student and knowledge centered ones, the goals of mathematical, scientific, and technical understandings for all will not be met. Citizens of the information age must own deep mathematical and scientific knowledge structures, and have the ability to reason and solve complex problems, in order to critically participate in our society. Frankenstein (1987) points out that knowledge of basic mathematics and statistics is an important part of gaining real popular, democratic control over the economic, political, and social structure of our society. “Liberatory social change requires an understanding of the technical knowledge that is too often used to obscure economic and social realities (Frankenstein, 1987, p. 180)." As Friere (1970) has posited, “the important thing is for people to come to feel like masters of their thinking by discussing the thinking and views of the world explicitly (p. 105)”. Shallow curricula and non-engaging instructional goals and techniques do not succeed in promoting robust understandings and have contributed to the learning gap that exists between the majority of students who do not learn well and the few who succeed, and between the United States’ achievement goals and the current realities of low achievement. These current realities have led to a rash of reform recommendations.

Educational organizations recognize the necessity of changing the way mathematics and science are taught. Much of the mathematics reform literature discusses a reconceptualization of mathematics as a dynamic process, in contrast to the way it is traditionally perceived, as a bounded set of facts (NCTM, 2000; Ernest, 1994). The reform movement seeks to provide opportunities and expectations for all students to actively engage in, understand, and use mathematics (NCTM, 2000). This perspective of mathematics education is a move away from the traditional notion that mathematics is a static body of objective facts that can be transmitted by the teacher and the text to the student. The modern notion is that mathematics knowledge is socially constructed (Ernest, 1994) only through the "reflection on and synthesis of mathematical relationships within [an] activity" (Noss, 1988, p. 253). Mathematical knowledge is now thought of as being formed via an active, social process of mathematizing. "Meaningful mathematical activity is characterized by the creation and conceptual manipulation of experientially real mathematical objects" (Cobb et al., 1997, p. 260).

NCTM advocates the development of an inquiry mathematics tradition in classrooms, which "emphasizes exploration, conjecturing, proving, and problem solving on the part of the students" (Gregg, 1995, p. 443). A prominent feature of the National Science Education Standards is a focus on inquiry (NRC, 1996).

> If the core of math and science is inquiry, then too many of today's math and science classrooms come up short. Students are crippled by content limited to the What? They get only a little bit about the How? or How else? and not nearly enough about the Why? Missing almost entirely is Why should I care? It is hard to imagine that students in these classes are gaining the conceptual and problem solving skills they need to function effectively as workers and citizens in today's world - a world that increasingly depends on math and science (NCMST, 2000, p. 17).

Traditional teaching misses a tremendous opportunity to give all students the problem solving, communication, and thinking skills that they will need to be effective workers in the 21st century. Professional organizations' (NCTM, 2000; NRC, 1996) standards seek to enhance mathematics and science curricula and instruction away from the isolated, teacher-centered way it is too often taught, and involve students in actively constructing their own deep content knowledge structures. The new academic standards require students to reason, analyze, and develop the ability to solve problems and understand the processes of science and mathematics. These education reform goals aim to end exclusive aspects of mathematical knowledge by making all persons mathematically literate.

The following sections of this chapter build from these general reform goals to articulate a vision of what Technology-Supported Inquiry Learning is and how TSIL can serve to meet these goals. In order to do so, first the notion of inquiry is developed. Then, statistics is demonstrated to be a natural site of scientific inquiry. Next, the role of technology within statistics practice and statistics education is discussed. Finally, these general discussions of inquiry, statistics, and technology will serve to frame a conception of TSIL within statistics education. Fathom is introduced as a valuable software learning tool, which can aid in the process of TSIL.

MATHEMATICAL AND SCIENTIFIC INQUIRY

Traditional math and science instruction has been referred to as the banking model of education, where the teacher transfers the knowledge to the students (Freire, 1970). Reform-based notions of inquiry, on the other hand, are based on the goal of having students actively construct their knowledge. A major goal that guides this reform vision of education is the desire to exploit the natural curiosity of children so that they maintain their motivation for learning, not only during their school years but throughout life (NRC, 1997).

This goal of fostering interest in inquiry is at the heart of the NRC Standards. The Standards seek to promote curriculum, instruction, and assessment models that enable teachers to build on children's natural, human inquisitiveness. "In this way, teachers can help all their students understand science as a human endeavor; acquire the scientific knowledge and thinking skills important in everyday life and, if their students so choose, in pursuing a scientific career (NRC, 1997, p. 6)."

The term inquiry is used in two different ways in the NRC Science Standards.

> First, it refers to the abilities students should develop to be able to design and conduct scientific investigations and to the understandings they should gain about the nature of scientific inquiry. Second, it refers to the teaching and learning strategies that enable scientific concepts to be mastered through investigations. In this way, the Standards draw connections between learning science, learning to do science, and learning about science" (NRC, 1997, p. 15).

As John Dewey (1902, p. 27) said, "science is more than a body of knowledge to be learned, there is a process or method to learn as well". The two forms of inquiry that NRC posits correspond to Dewey's comments about content and process. Understanding the process of scientific inquiry is essential towards fostering confidence in students of their own scientific ability. All too often, science is thought of as something that only scientists do. By teaching science by inquiry, and relating that all persons can engage in scientific inquiry, students can come to believe that they too can think scientifically. As they engage in scientific inquiry, they are involved in the thinking-learning process and once their minds are turned on and tuned in, they have a much greater chance of gaining understandings of scientific content as well.

Mathematical literacy contributes to the process of scientific inquiry by providing the skills through which scientific questions can be understood and evaluated. Accordingly, the NCTM PSSM highlight the fact that mathematics classrooms should include the important inquiry components of problem solving, connections, and communication. Together, these parts of the Standards (NCTM, 2000) state:

> Instructional programs from pre-kindergarten through grade 12 should enable all students to:
>
> - build new mathematical knowledge through problem solving;
> - solve problems that arise in mathematics and in other contexts;
> - apply and adapt a variety of appropriate strategies to solve problems;
> - monitor and reflect on the process of mathematical problem solving;
> - recognize and apply mathematics in contexts outside of mathematics;
> - organize and consolidate their mathematical thinking through communication;
> - communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
> - analyze and evaluate the mathematical thinking and strategies of others;
> - use the language of mathematics to express mathematical ideas precisely.

Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas. Furthermore, Paul Halmos (1980) writes, problem solving is the "heart of mathematics." Successful problem solving requires knowledge of mathematical content, knowledge of problem-solving strategies, effective self-monitoring, and a productive disposition to pose and solve problems.

To meet new challenges in work, school, and life, students may have to adapt and extend whatever mathematics they know. Doing so effectively lies at the heart of mathematical problem solving. A problem-solving disposition includes the confidence and willingness to take on new and difficult tasks. Successful problem solvers are resourceful, seeking out information to help solve problems and making effective use of what they know.

An important aspect of a problem-solving orientation toward mathematics is making and examining the conjectures that are raised by solving a problem, and posing follow-up questions (NCTM, 2000). This ability depends on being able to see connections among and between mathematics and scientific content areas. In order to be able to pose interesting and relevant questions, and gain robust understandings, one must consider how one problem solving procedure relates to another concept or procedure. An example of making connections occurs during data analysis. Examining the data graphically may lead to better understanding of the content being examined. Better understanding of the content may then, in turn, lead to different analytical ideas. In order to facilitate these advances in understanding, knowledge of both the scientific concepts being examined and the mathematical techniques for examining them are important. This idea is further developed in later sections of this dissertation.

Communication provides students opportunity to organize and reflect on their understandings and those of their peers and teachers. The process of articulating one’s own understanding has been found to be very important towards building the metacognitive ability of self-assessment (NRC, 2000). The important role that assessment and communication play in facilitating learning with understanding are further developed during the Chapter Two literature review.

There is a strong overlap between the ways that the NCTM PSSM talk about the principles of problem solving, connection, and communication and the way the NRC Science Standards talk about inquiry. In general, inquiry is the broader term and for purposes of clarity and consistency, I will refer to the composite of the skills of problem solving, connection, and communication as inquiry. Below, I have outlined the way that NRC has defined scientific inquiry. NRC (1997) emphasizes that the essential features of classroom inquiry are:

1. Learners are engaged by scientifically oriented questions.
2. Learners give priority to evidence, which allows them to develop and evaluate explanations that address scientifically oriented questions.
3. Learners formulate explanations from evidence to address scientifically oriented questions.
4. Learners evaluate their explanations in light of alternative explanations, particularly those reflecting scientific understanding.
5. Learners communicate and justify their proposed explanations.

These components of the scientific inquiry process necessarily include the mathematical principles of connection, problem solving, and communication. In order to make informed scientific decisions, learners must be able to understand how to utilize mathematical forms of analysis (connection), interpret mathematically their data and results (problem solving), and communicate their results with others (communication). Together, these skills, understandings, and processes seek to provide students with the opportunities and experiences necessary to support them in developing deep mathematical and scientific content knowledge structures and the ability to apply this knowledge to diverse tasks and settings. They form the composite of the ability referred to herein as inquiry.

The research base on learning and on effective learning environments makes a strong case for inquiry-based approaches (NRC, 2000; Minstrell, 2000; Carpenter et al., 1996; White & Frederikson, 1998; Krajcik et al., 2000). This research project seeks to demonstrate how statistics education is a natural site to apply what we already know about inquiry learning. Participating statistics teachers within this study will learn about and practice enacting inquiry pedagogy that is supported by technology. Through this process, much information about the practical aspects of teaching and learning statistics via technology-supported inquiry will be generated. Technology-Supported Inquiry Learning (TSIL) is introduced as a pedagogical practice that utilizes technology in order to assist students in understanding inquiry, both as an integral part of scientific investigation and as a method of learning content.

STATISTICS AS INQUIRY

Statistics is an important part of both the mathematics and science standards. The NCTM PSSM (2000) state that instructional programs from PreK through grade 12 should enable all students to:

1. Formulate questions that can be addressed with data and collect, organize, and display relevant data and answer them.
2. Select and use appropriate statistical methods to analyze data.
3. Develop and evaluate inferences and predictions that are based on data.
4. Understand and apply basic concepts of probability (p. 20).

The NRC Science Standards (1997) state “at each of the steps involved in inquiry, students and teachers ought to ask what counts? What data do we keep?

What data do we discard? What patterns exist in the data? Are these patterns appropriate for this inquiry? What explanations account for the patterns? Is one explanation better than another? " (p. 18). This statement demonstrates the centrality of statistics to the scientific inquiry process. Some are even so daring as to say that statistics is not just a part of inquiry but is the essence of the process itself.

> Statistics in its broadest sense is the matrix of all experimental science – and is consequently a branch of scientific method, if not Scientific Method itself; and, hence, it transcends the application of the scientific method in sundry fields of specialization. The scientist should know Statistics as he knows logic and formal language for communicating his ideas (Anderson & Loynes, 1987, p. 2).

Whether one agrees or not that statistics is the scientific method, it must be recognized that an understanding of statistics is essential towards fostering mathematical and scientific literacy in today's information age.

Statistics is concerned with collecting, analyzing, and interpreting data in the best possible way, where the meaning of best depends on the particular circumstances of the practical situation. Statistics has developed from two disciplines: the mathematical study of probability and chance events, and the scientific attempt to draw conclusions from data in the face of inevitable error and imprecision (Anderson & Loynes, 1987). Statistics largely consists of discerning, describing, predicting, and confirming patterns and relationships in data.

Statistical methods are formed and understood mathematically but the process of applying them to data consists of scientific reasoning in essence. Statistics involves both deductive and inductive thought, via its recursive relationship between trying to understand particular data by fitting it to general models. In practice, scientific progress usually involves a combination of inductive and deductive reasoning (Chatfield, 1995). For many reasons, that will be explored herein, statistics education should reflect authentic statistical practice. In so doing, students will learn much about the content and processes of mathematics and science, and their application to the real world.

In practice, the statistician is a problem solver. A statistician needs to be able to (Chatfield, 1995):

1. formulate a real problem in statistical terms;
2. give advice on efficient data collection;
3. analyze data and extract the maximum amount of information;
4. interpret and report the results (p. 1).

These abilities closely align with the scientific inquiry process introduced earlier.

Teaching statistics in a manner that is faithful to statistical practice is a natural way to foster understandings of inquiry in students. Currently, however, “a wide gap separates statistics teaching from statistics practice” (Moore, 1992, p. 1). The inductive nature of statistics is too often de-emphasized in the mathematically based, and necessarily deductive, setting of traditional statistics courses.

> When we teach statistics as if it were a branch of mathematics, students are left on their own to make the connection between theory and any practical application. Most of the students in basic statistics courses are more likely to be motivated by learning about the world, especially the world as seen through the eyes of their own interests, than by elegant mathematics. They are more likely to have the kind of “aha!” reaction that makes difficult ideas sink in and stick from an insight about a complex relationship among some real world variables than from working through the proof of a limit theorem. Students who see statistics as a tool for understanding the world find it empowering (Moore, 1992, p. 45).

In order to effectively reason about the world, one needs to be able to reach conclusions both deductively and inductively. Sometimes, one can ascertain the particulars by understanding general theorems (e.g., using knowledge of linear functions to predict certain relationships). Other times, one can only reach a generalized conclusion by studying particular instances (i.e. using information of certain relationships to gain an understanding of a step function). In practice, statistics studies the interactions between variables in order to be able to make predictions about their relationships. When statistics education ignores the contribution of inductive thought within practice, a disservice is done to the students. They are not able to see how meaning making occurs via statistics practice and they are not able to learn that they are able to participate in this process.

The aims of statistics education should include persuading students of the relevance of statistics, and to give them confidence to begin to apply what they know to real problems. Moore (1992) delineates this perspective:

> This view helps us respond to the question in the minds of many students: 'why should I bother with this?'. The answer is not 'because its required for your degree,' but rather 'because you can learn more about the world – whatever aspects of it may interest you – by using the tools, methods, and reasoning of statistics.' (p. 45).

In order to demonstrate to students how knowledge of statistics can empower them to better understand the world we live in, statistics education should involve them in authentic statistical activity. In the book *Teaching of Practical Statistics,* Anderson and Loynes (1987) delineated the abilities that are necessary for statistical practice. The abilities that these authors listed are consistent with the scientific inquiry process as well and included the abilities to effectively plan the investigative process, recognize progress during investigation, collaborate successfully with others, and to interpret and communicate findings.

Anderson and Loynes' summary is useful in showing just how many different kinds of skills are needed by a statistician, and thereby underscore the fact that traditional statistics courses that focus primary on formulas and procedures omit some of the most important aspects of the subject. Furthermore, involving students in this type of authentic statistical inquiry activity will meet many national mathematics and science standards. Students engaged in inquiry pedagogy also have opportunities to learn much about applying their knowledge to the real world and gain confidence in their knowledge and ability to do so. Finally, the process of involving students in statistical inquiry aligns with what we know about how people learn with understanding.   Chapter Two outlines the research base of this project, and introduces a practical framework that will be utilized by this researcher in order to support teachers as they learn to integrate statistics inquiry pedagogy into their practice.

THE ROLE OF TECHNOLOGY IN STATISTICS AND INQUIRY

Technology plays a central role in the process of scientific inquiry. The NRC Standards (1996) state, "Scientists rely on technology to enhance the gathering and manipulation of data. Technology used to gather data enhances accuracy and allows scientists to analyze and quantify results of investigations (p. 19)." Mathematics education researchers agree that technology can play a central role in fostering an inquiry-based approach to teaching and learning. "Technology greatly enhances investigation by providing a learning environment that encourages inquiry and extension, enables students to make mathematical connections, and creates a view of mathematics as an exploratory science” (Connell, M., 1997, p. 335).

Technology profoundly changes what can and should be done in mathematics education. In a day and age where students can use technology to actively investigate multiple representations and applications of mathematics, the traditional focus on paper and pencil symbolic manipulation does not constitute sufficient mathematics education. "Although historically there has been a strong focus on symbol manipulation in the mathematics curriculum, the advent of computers and calculators in the classroom facilitates a new approach - one where the focus is on reasoning with a variety of representations and understanding the relationships among those representations” (Dugdale et al., 1995, p. 330). Literature supports that technology can be utilized in the classroom in order to achieve more conceptual reasoning and in so doing make the subject of statistics more interesting and statistics students more knowledgeable (Anderson & Loynes, 1987; Moore, 1992; Smith, 1998; Konold, 1995, 2002).

Computers can and should play a special role in statistical practice and teaching (Anderson & Loynes, 1987; Moore, 1992; Smith, 1998; Konold, 1995, 2002). Computers can be incorporated into curriculum and instruction in ways that allow statistics classes to consider the complexities of the real world. Students know that the world is complex. When we oversimplify, we make the subject seem irrelevant. Real examples, such as scientific relationships between the orbit of the planet and the seasons, or economic relationships between manufacturing and sales, or sports data can be motivating to students. Furthermore, computers can make the classroom experience more like real-world statistical practice. Statisticians analyze data to learn something about how the world functions. To do so, they combine knowledge of statistical methods with knowledge about the subject matter giving rise to the data collection. When we work in class with real data, and utilize an inquiry teaching and learning framework, students can also participate in this process and combine the knowledge they are gaining in other disciplines with the methods taught in statistics class. This helps students to see for themselves why they should study statistics, and it also helps them to recognize the limitations of statistical analysis (Anderson & Loynes, 1987).

The ability of technology to assist in connecting the real world and in-school practices of students is a powerful contribution to student learning and motivation. Noss (1998) claims that technology can be the instrument for bridging the gap between formal symbolic and informal reasoning in mathematics. He states that previously “what we have not had at our disposal was the means for learners to engage in culturally embedded activities while simultaneously mathematizing their activities” (p. 254). Since technology exists so much in the world around us, when it is brought into the classroom in authentic ways it offers a rich opportunity for students to come to value mathematical understanding. Computers make it practical to teach with real data rather than with constructed examples. Real datasets are more interesting, partly because they come with a story. Students see courses that discuss real data as being more relevant (Smith, 1998). By understanding the context of a problem, students are able to ask relevant questions of the data. Asking new questions encourages them to become engaged in data analysis, and engagement enhances learning (Kuh, 2001).

Computers are fundamental to current statistical practice. They are tools through which we understand the world, allowing us to use increasingly realistic models and deal with growing masses of data. Computers should, therefore, play a major role in teaching statistics. “Because computers simultaneously are central to what statisticians do and are the tools that make structured inquiry possible, we cannot imagine an up-to-date introductory or applied statistics course that does not give students substantial experience with computers” (Anderson & Loynes, 1987, p. 45).

The role that computers should play in teaching statistics depends on the expectations we have of our courses, our students, and ourselves. The mathematics and science standards emphasize teaching students how to approach and make progress on problems they may later face. The successful student will know how to address and solve problems that he or she might encounter in the real world, whether or not they are isomorphic to problems seen in class. This view emphasizes collaboration and expects the teacher to establish norms and expectations for communication, and for the student to build experience as problem solvers. Both parties are active participants. Computing can be particularly helpful in achieving the goals of this view of teaching (Becker, 2000c). Moreover, courses designed with this goal are more likely to convey the philosophy and methods of modern data analysis (Anderson & Loynes, 1987, p. 46).

This dissertation introduces Technology-Supported Inquiry Learning as a pedagogical practice that utilizes technology in order to assist students in understanding inquiry, both as an integral part of scientific investigation and as a method of learning content. Towards this end, this project articulates what TSIL can look like when utilizing Fathom learning technology within statistics education. Furthermore, this dissertation project seeks to support and examine mathematics teachers as they learn to implement Fathom within their TSIL statistics curriculum and instruction. By doing so, practical understandings of the challenges associated with putting reform-based visions of TSIL into practice can be better understood by the education community.

FATHOM

One particular piece of statistics software, *Fathom* by Key Curriculum Press (Finzer, 1997), was specifically designed with teaching and learning in mind. The developers of this data analysis learning tool put much research and development effort into designing Fathom so that it allows students to interact with the software in a way that was user-friendly, but still makes the users think about what it is they want to know and how they could find out the information they want. An example of this is that students cannot blindly try to fit every possible model to a set of data, but instead must examine and understand the fit of one model at a time. An illustration of this feature, here where a student uses Fathom to simultaneously explore two different ways of modeling the consumer price index (CPI) from 1913 to 1997, is demonstrated in Figure 1 below.

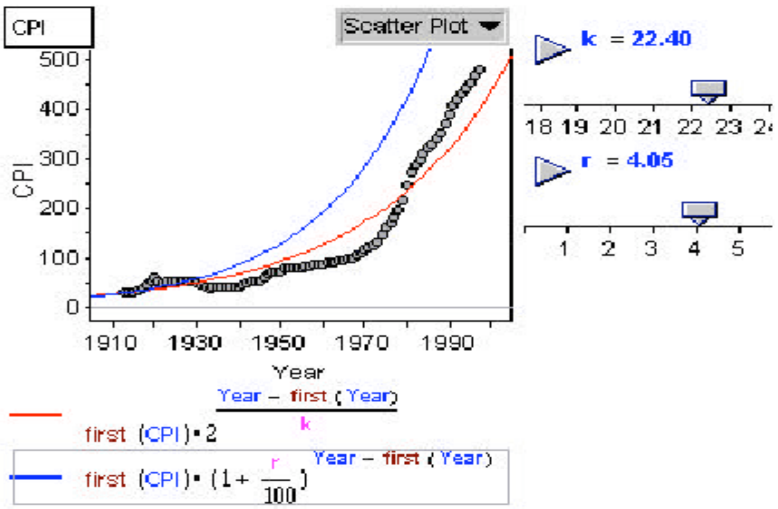


Figure 1. Here, a student uses Fathom to simultaneously explores two different ways of modeling the consumer price index (CPI) from 1913 to 1997. The top curve shows the growth as a doubling process, the bottom curve is a compounding of interest process (Finzer, 2000, p. 6).

Many features that help statisticians explore and understand data, such as the one demonstrated above, are present in Fathom but are featured in ways that are user-friendly and interactive. These features are further demonstrated below.

Many statisticians claim that graphical techniques are the principal tools for identifying patterns, structure, and regularity in data (Chatfield, 1995; Moore, 1992). Furthermore, interactive and dynamic graphics are particularly powerful in helping us to understand the world that we live in. Interactive graphics allow data analysts to interact with two-dimensional graphs. For instance, in Figure 2 below, a line that has been placed on a scatterplot can be dragged to change its slope and intercept.

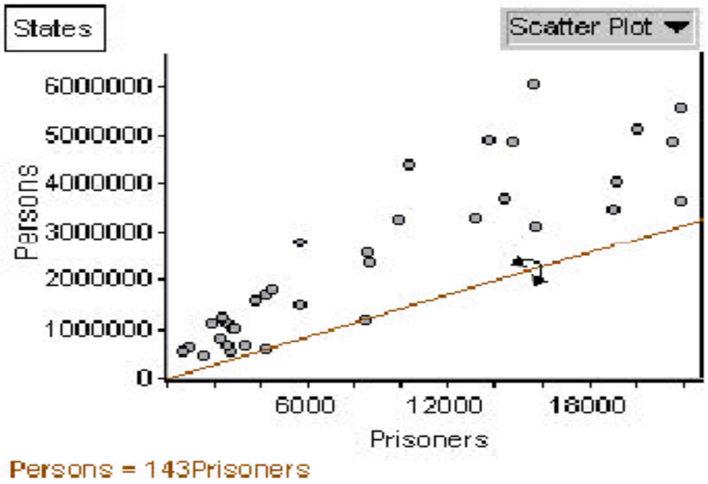
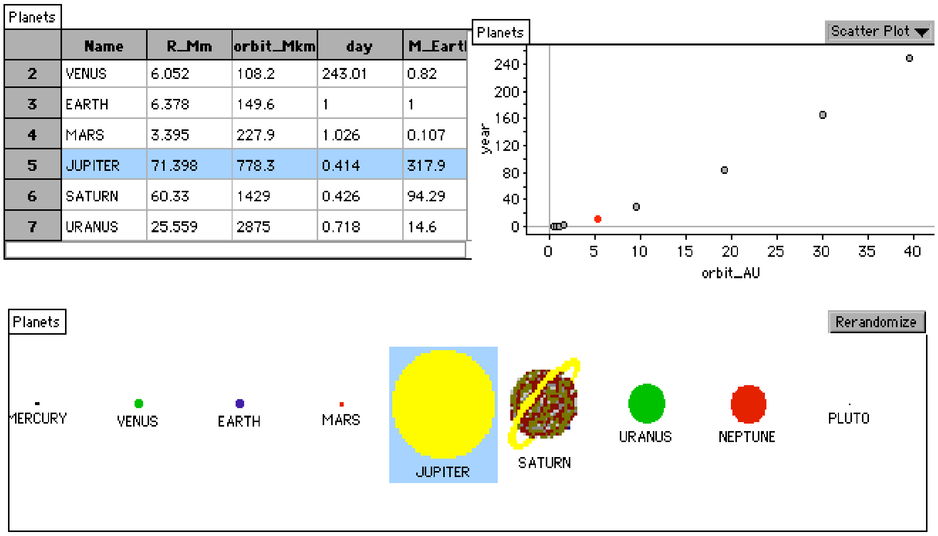


Figure 2. A moveable line, highlighted by the double arrows, is placed on a graph within Fathom, which compares the ratio of population to prisoners in the United States. The line shown goes through Washington DC and Nevada (Finzer, 2000, p. 3).

Analytic methods have dominated statistical teaching but interactive graphical methods offer an alternative approach that can complement and enhance traditional methods. These kinds of graphics allow students to actively investigate data and interactively form their own understandings as they do so. Fathom allows multiple representations of data to all appear on the computer monitor at the same time and for these multiple representations to be simultaneously and interactively investigated by the user (this technique is called brushing – see Figure 3 below).

Fathom has many dynamic features built into it, which can be utilized to demonstrate how interactive graphics can enhance understanding. Brushing exists along multiple representations. Students could utilize this feature, for example, by continually examining the affect of an outlier on a mean versus a median or mode.



Planets

	Name	R_Mm	orbit_Mkm	day	M_Eart
2	VENUS	6.052	108.2	243.01	0.82
3	EARTH	6.378	149.6	1	1
4	MARS	3.395	227.9	1.026	0.107
5	JUPITER	71.398	778.3	0.414	317.9
6	SATURN	60.33	1429	0.426	94.29
7	URANUS	25.559	2875	0.718	14.6

Figure 3. The example above highlights the brushing capabilities of Fathom within an investigation of the relationship between planets' orbit radii and length of year (Finzer, 2000). The selection of Jupiter in the collection at the bottom of the screen causes this planet to be selected in the graph and case table as well.

Sliders, as shown in Figure 4 below, allow users to manipulate components of statistical equations or graphs in small or large increments, and immediately see the results of these manipulations in their models.

In the classroom, simulation experiments can play the same role for statistics as laboratory experiments do in many sciences. Computer-generated simulations can give students more experience with randomness than they could get in years of practical data analysis (Anderson & Loynes, 1987, p. 49). Fathom has a simulation feature built into it that allows students to model and investigate how repeated samples of different sizes, and from different populations, affect statistical estimates.

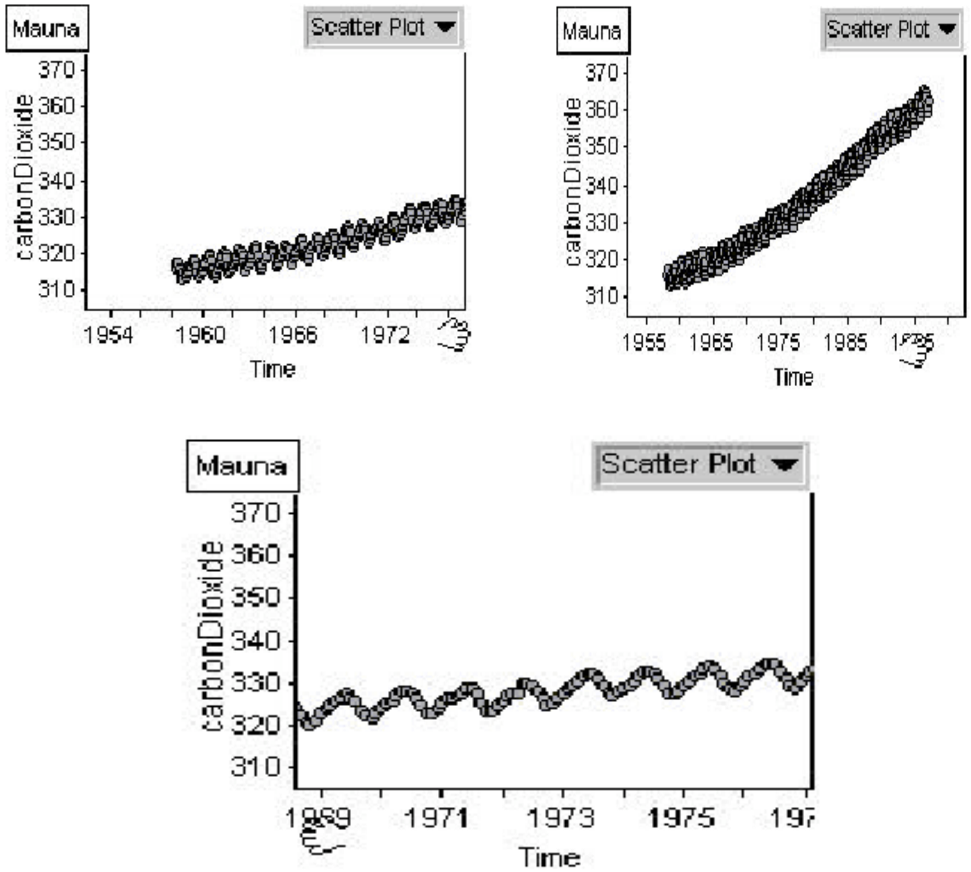
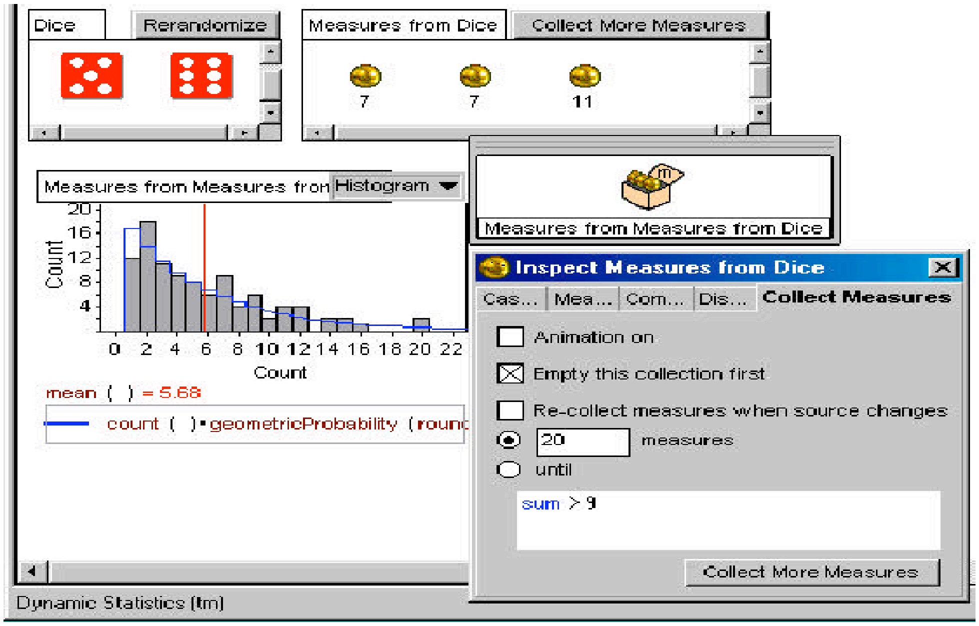


Figure 4. In the sequence of three plots shown above, which demonstrates some of the dynamic features of Fathom, the user dilates the x-axis first by dragging the upper end of the scale to the right and dragging the lower end of the scale to the left. The time series of CO_2 as measured on top of the Mauna Loa volcano gradually resolves into a seasonal saw tooth (Finzer, 2000, p. 4).

An illustration of the simulation capability of Fathom is shown in Figure 5 below. Simulations like the one shown below allow students to efficiently focus on and test probabilistic hypotheses without getting lost in the tedium of lengthy data collection.



☐ Animation on

☒ Empty this collection first

☐ Re-collect measures when source changes

● 20 measures

○ until

sum > 9

Figure 5. A Fathom simulation of how many times two dice must be rolled until the sum is greater than 9 has been run (Finzer, 2000, p. 6).

Another wonderful design feature of Fathom is that it is very easy to import real world data from the World Wide Web into the Fathom environment. As demonstrated in Figure 6 below, all that the student needs to do is grab the tag next to the url:// on a web page and drag and drop it into Fathom to import the data. “Roughly 90% of the time, it [Fathom] does 90% of the work of getting the data into a form it can be analyzed. For example, the data from this NFL site took about two minutes of work before it was ready for exploration (Finzer, 2000, p. 8).”

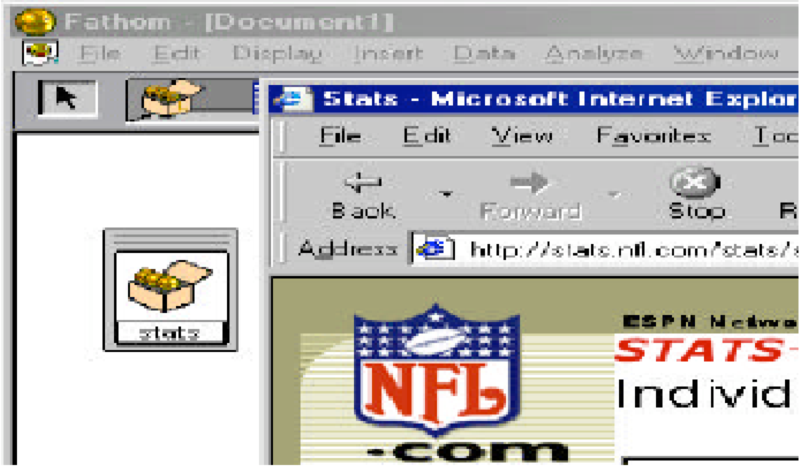


Figure 6. Importing NFL data by dragging and dropping data from the http://stats.nfl.com web page into Fathom (Finzer, 2000, p. 8).

Chatfield posits that statistical software needs to be appraised on various criteria, which include statistical, computational and commercial considerations. His guidelines reinforce the benefits of Fathom. These qualities include:

> flexible data entry and editing facilities; good facilities for exploring data via summary statistics and graphs; statistically sound procedures for fitting models, including diagnostic checking; computationally efficient programs; well-designed, clear and self-explanatory output; and adequate documentation and support. Other criteria include the cost of the package, how easy it is to learn and use, the required equipment, the needs of the target user and the possibility of extending the package (1995, p. 95).

Fathom meets all of these design requirements. It is designed to be a powerful data analysis tool and also to be user-friendly. As an example, in order to make a graph of the data, users can just grab a graph icon from the tool bar and then grab an attribute of the data and drop it on the graph. Calculating summary statistics is as simple; just grab a summary table from the tool bar and drop an attribute in to it.

The capabilities of Fathom are more than adequate for introductory to intermediate level statistics and the design of the software is excellent for those who are learning not just the technology, but forming statistical understandings as well. Interactive graphics, such as multiple representations, brushing, sliders, and simulations not only change the way statisticians explore data, but should also change the way we teach statistics (Chatfield, 1995). When Fathom technology is incorporated into statistics instruction, students are able to investigate, discuss, prove statistical relationships via interactive graphics and are more able to build conceptual understanding than when they cannot be involved with this kind of active investigation. In the process of learning Fathom, students are provided with many types of conceptual supports that assist them to engage in authentic statistical investigation and to develop deep conceptual understandings. Furthermore, when Fathom is integrated into an authentic, statistical inquiry learning environment, students are able to utilize the user-friendly, interactive features of the software towards understanding the process of scientific investigation, the role that statistics plays in this process, and their ability to do so.

SUMMARY OF TSIL WITHIN STATISTICS

A major goal of high school mathematics is to equip students with knowledge and tools that enable them to formulate, approach, and solve problems beyond those that they have studied. High school students should have significant opportunities to develop a broad repertoire of problem-solving strategies. They should have opportunities to formulate and refine problems because problems that occur in real settings do not often arrive neatly packaged. Students need experience in identifying problems and articulating them clearly enough to determine when they have arrived at solutions. The curriculum should include problems for which students know the goal to be achieved but for which they need to specify—or perhaps gather from other sources—the kinds of information needed to achieve it. Statistics education activities that follow the process of statistical practice allow students the opportunity to become problem posers and solvers.

Hunter (1981) distinguished the following stages of a statistical investigation:

1. the investigator poses the problem and describes the background to it.
2. the problem is formulated in statistical terms
3. the mathematical/statistical/computational machinery needed to produce a formal 'answer' is operated;
4. the 'answer' is interpreted and evaluated in the light of the formal questions it was meant to answer;
5. a response is communicated in the terms of the original question.

These stages of a statistical investigation align closely with national education goals (NCTM, 2000; NRC, 1996, 1997) that students gain understandings of inquiry, problem solving, connection, and communication. The NRC (1997) outlines common components shared by instructional models involving inquiry:

1. Students engage with a scientific question, event, or phenomenon. This connects with what they already know, creates dissonance with their own ideas, and/or motivates them to learn more.
2. Students analyze and interpret data, synthesize new ideas, build models, and clarify concepts and explanations with teachers and other sources of scientific knowledge.
3. Students, with their teachers, review and assess what they have learned and how they have learned it (NRC, 1997, p. 35).

Technology-Supported Inquiry Learning is viewed as a pedagogical practice that utilizes technology in order to assist students in understanding inquiry, both as an integral part of scientific investigation and as a method of learning content. Towards these ends, this project articulates what TSIL can look like when utilizing Fathom learning technology within statistics education.

The vision of TSIL with Fathom that this project posits, is one in which students first engage with a scientific question, which may be introduced by them or by their teacher, depending on the instructional goals. Students discuss this question with others in order to make sure they understand it clearly and have a pragmatic plan to gather data to answer it. Students then actively gather information towards solving their questions, utilizing Fathom, and other resources. Along the way, they articulate their thinking to others. This serves to clarify their understandings and as a way to get formative feedback on their progress. Finally, students share their findings with others, either orally or in writing, so they build their communication skills and learn how to pose, conduct, and complete a research study. Together, these stages comprise aspects of scientific inquiry. Technology can be utilized within these stages in order to support the inquiry process.

Fathom, in particular, can be utilized throughout the steps of a statistical investigation. Fathom can be used during the initial examination of the data, which includes assessing the structure and quality of the data and processing them into a suitable form for analysis and generating descriptive statistics. Fathom allows opportunity for students to investigate the merits of various procedures for definitive analysis. Finally, students can make use of Fathom graphs and descriptive statistics as they prepare, write, revise, and present their findings. The satisfaction of these demands will allow students to understand the process of scientific inquiry and learn much mathematics and statistics along the way.

CONCLUSION AND RESEARCH QUESTIONS

Technology can be utilized to assist in promoting students' deep understanding of mathematics and science content and the inquiry process. This kind of instruction and curriculum, however, requires considerable planning and organization on the part of both teachers and students. This dissertation study seeks to contribute to theoretical and practical understandings of ways that teachers can be supported as they strive to integrate Fathom into their TSIL practice. Calls for teachers to be able to integrate technology continually increase. Research that strives to support and understand teachers' efforts to do so is essential.

Educators have been targeted as a key to spreading technology into schools. Teachers are expected to utilize technological resources in instruction. NCTM (2000) states:

> technology has changed the ways in which mathematics is used and has led to the creation of both new and expanded fields of mathematical study. Thus, technology is driving change in the content of mathematics programs, in methods for mathematics instruction, and in the ways that mathematics is learned and assessed. A vital aspect of such change is a teacher's ability to use appropriate instructional technology to develop, enhance, and extend students' understanding and application of mathematics. It is essential that teachers continue to explore the impact of instructional technology and the perspectives it provides on an expanding array of mathematics concepts, skills, and applications (p. 25).

Furthermore, NCTM (2000) maintains "technological tools can and should be used [by teachers] to foster understandings and intuitions, with the goal of enriching student learning of mathematics" (p. 27).

Technology is changing the face of mathematics education and adapting to this is not always easy for teachers. "The standards suggest a classroom environment in which computers are both prominent in the experience of students and employed in order that students grow intellectually and not merely develop isolated skills" (Becker, 1994, p. 294). Technology is not often integrated into instruction in this manner, in fact "there is a large discrepancy between the level of computer use expected of teachers and its actual level" (Marcinkiewicz, 1994, p. 221). Becker (1994, 2000a) estimated that the proportion of exemplary, computer-using, mathematics teachers is less than 8%.

The National Commission on Mathematics and Science Teaching for the 21st Century (2000) urges:

> The Report's second message points in the direction of a solution: the most direct route to improving mathematics and science achievement for all students is better mathematics and science teaching (p. 7). The first step must be a preparation program that imparts a deep understanding of content, teaches prospective teachers many ways to motivate young minds, especially with the appropriate use of technology, and instills a knowledge of - and basic skills in using - effective teaching methods in the discipline (p. 22).

This study aims to articulate what effective technology integration in math and science could look like and to share practical examples of teachers' understandings and practices as they learn to implement this pedagogy. The case studies that are developed out of this teacher development experiment will examine the decisions teachers make regarding how, why, and when they choose to integrate technology. This is an area of research that is not commonly discussed and will be particularly beneficial for teachers who are trying to understand how to effectively utilize technology to enhance student learning, and for teacher educators who are trying to support teachers' developing new and improved instructional practices that incorporate technology. Therefore, the research questions of this study are:

1. What are teachers’ understandings as they learn about, practice with, and reflect upon technology-supported inquiry learning?
    a. What are the participating teachers’ understandings regarding mathematics and statistics content?
    b. What are the participating teachers’ understandings regarding pedagogy?
    c. What are the participating teachers’ understandings regarding technology?

2. What do the instructional practices look like for teachers who are trying to incorporate TSIL within their classrooms?
    a. What do teachers' practices look like as they incorporate Fathom into their teaching?
    b. What are similarities and differences regarding teachers’ practices involving TSIL with Fathom?

CHAPTER TWO

LITERATURE REVIEW AND PRACTICAL FRAMEWORK

The research questions of this dissertation project center around examining participating teachers' understandings and practices as they strive to foster technology-supported inquiry learning into their curriculum and instruction. The author of this dissertation serves the dual role of teacher educator and researcher during the course of this project. This chapter reviews prior research literature that serves a significant role in my efforts to support and examine participating teachers' technology integration efforts. The chapter is divided into two sections: a literature review section, and a practical framework section. Together, these sections will demonstrate the empirical and theoretical base that this dissertation project is built upon, and how this base will support the goal of answering the research questions of this study.

This chapter begins with a review of literature that is relevant to the purpose of this research study. Included are syntheses of prior research on inquiry education, prior statistics education research, research about professional development, and research about learning in general. This literature review serves as the background for the second section of this chapter, a discussion of how this project synthesizes the literature review into a practical framework that guides this research study.

LITERATURE REVIEW

Prior Research on Inquiry Education

Inquiry oriented, constructivist approaches to mathematics and science education have been tried many times and met with limited success (Stake & Easley, 1978; Harms & Yeager, 1980). Although educational philosophers and practitioners such as Dewey (1902) and Polya (1957, 1962) have expounded on the important components of inquiry, effectively integrating inquiry into curricula and instruction has been difficult. Dewey's (1902) problem of "determining the medium" or weaving what Ball (1997) terms a representational context in which children can do – explore, test, reason, and argue about – and, consequently, learn about particular mathematical ideas and tools is at the heart of the difficult work of teaching for understanding in mathematics. Polya (1957) states that students must be able to learn how to "understand the problem, devise a plan, carry out the plan, and look back on the solutions they have obtained" (p. 5). This is easier said than done however. Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne (1996) point out

> the teacher bears the responsibility for developing a social community of students that problematizes mathematics and shares in searching for solutions. The teacher will need to take an active role in selecting and presenting tasks. Tasks do not just appear, and it is unlikely that students spontaneously will create tasks that sustain reflective inquiry in mathematics (p. 16).

White and Frederiksen's (1998) recent research and practice on scientific inquiry synthesized recent advances in cognitive science (particularly the work on metacognition) with advances in educational technology (particularly the creation of computer simulation and modeling tools). This set the stage for them to be able to develop more effective approaches to the teaching of scientific inquiry. They hypothesized that scientific inquiry could be made accessible to a wide range of students by recognizing the importance of metacognition and creating an instructional approach that develops students' metacognitive knowledge and skills through a process of scaffolded inquiry, reflection, and generalization. Their resulting ThinkerTools Inquiry Curriculum centers around a metacognitive model of research, called the Inquiry Cycle, and a metacognitive process, called Reflective Assessment, in which students reflect on their own and each other's inquiry.

White and Frederikson's approach to inquiry education is to create climates for students and for teachers in which ideas can be freely expressed and explored (Frederikson et al., 1998). They explain that in order to do so, they

> carefully scaffold the creation of a classroom research community through providing detailed and explicit lesson plans, computer simulations and activities, and student research materials and then rely on the same process of implementation coupled with reflection to allow both teachers and students to develop a mature understanding of scientific inquiry (1998, p. 10).

Students in their research sites go through an inquiry cycle for each research topic in the curriculum and, by the end of the curriculum, develop the skills to do independent research on topics of their own choosing. These researchers found that successful students were able to acquire the metacognitive expertise of being able to monitor and reflect on their research. Students learned to use criteria, such as 'reasoning carefully' to evaluate their progress while involved in the inquiry cycle. As they continually engaged in this reflective self-assessment process (scaffolds), their knowledge of inquiry developed and improved (White and Frederikson, 1998).

As Ball (1997) points out in discussing this type of instruction, “all this sounds both sensible and elegant – achieving it, however is difficult (p. 160)”. Julyann & Duckworth (1996) reiterate

> perhaps first and foremost, the phenomenon students are asked to think about needs to be interesting, worthy of engaging their time and attention. In addition, it should offer a variety of avenues for exploration, various routes of approach. Once these parameters are established, the teacher needs to listen carefully to students’ interpretations of the data, paying particular attention to any individual’s conundrums, puzzlements, confusions. And the teacher equally needs to pay attention to differences of opinion within the class, giving equal respect to each one, for as long as any student still takes it seriously. By focusing on puzzlements and contradictions, the teacher establishes the notion that ideas are complicated and worthy of time and consideration and that each student is capable of formulating interesting ideas. Further, the teacher acknowledges that ‘not knowing’ is a state that is important to live with – the state that most of us are in most of the time (p. 4).

Edelson, Gordin, and Pea (1999) found that there are five significant challenges to the successful implementation of inquiry-based learning. The five challenges are “motivation, accessibility of investigation techniques, background knowledge, management of extended activities, and the practical constraints of the learning context" (p. 7). Inquiry instruction is different than the way most teachers learned and is difficult for them to implement effectively. This chapter provides a practical framework for how this dissertation project will support participating teachers’ learning. Research by White and Frederikson (1998) and Edelson et al. (1999) will provide helpful models for teachers to read, think about, and discuss.

Prior Statistics Education Research

Not until recent decades has there been much research in the area of probability and statistics education. Shaughnessy (1992) synthesized research in these areas for the Handbook of Research in Mathematics Education and stated

> it is not surprising that there has not been much involvement by mathematics educators in research on the teaching and learning of stochastics. Much of what is researched in mathematics education is driven by what is taught in schools. Since very little probability or statistics has been systematically taught in our schools in the past, there has been little impetus to carry out research on the problems that students have in learning it (p. 465).

Much of the research conducted before the last decade was by psychologists. Shaughnessy (1992) states that these psychologists were mainly "observers and describers of what happens when subjects wrestle with cognitive judgmental tasks" (p. 469).

Kahneman, Slovic, & Tversky (1982) synthesized much of their and other's research on how people think about probability and judgment under uncertainty. This body of work "investigates primitive conceptions or intuitions of probability and statistics, misconceptions, fallacies in thinking, judgmental biases, and so forth" (Shaughnessy, 1992, p. 470). Tversky and Kahneman (1982) report "people rely on a limited number of heuristic probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors" (p. 3). For example, these researchers demonstrated that people are insensitive to the role prior probabilities, sample size, chance, predictability, validity, and regression effects in reasoning about stochastic events. While these psychologists have not been concerned with seeing if these misconceptions can be diminished under instruction, mathematics educators have discovered that they must become familiar with students' preexisting stochastic conceptions before they try to teach the mathematics concepts of probability and statistics (Toughness, 1992). By eliciting students' existing stochastic conceptions, and then structuring learning activities that challenge them, it is believed that constructivist teaching can help to correct the types of misconceptions Tversky and Kahnemann have demonstrated via their research.

Clifford Konold (1995, 2002) and his research group (Pollatsek, Lima, & Well, 1981) have been working for over 20 years to investigate student understandings of various probabilistic and statistical concepts and ways that curricula and instruction may affect them. Three of their major findings from their research are

> students come into our courses with some strongly-held yet basically incorrect intuitions, these intuitions prove extremely difficult to alter, and altering them is complicated by the fact that a student can hold multiple and contradictory beliefs about a particular situation (Konold, 1995, p. 2).

Konold's findings imply that correcting students' stochastic misconceptions is no easy task. Many times, students can understand classic notions of probability and statistics yet not reconcile proper notions with their own existing beliefs. More research and practice in the area of statistics education is necessary in order to find ways to help students learn with understanding.

Research findings by Konold and others have led many to question the way that statistics is taught and search for curricular and instructional means to improve statistics education. In the 1990's, the National Science Foundation funded many projects whose goals were to produce and offer materials and ways of thinking about statistics teaching that would make statistics education more effective. Of these NSF funded projects, "nearly all involve statistical laboratories. The prominence of the lab approach accords with the movement of statistics back towards its roots in science, and with research in education that demonstrates the importance of active learning (Cobb, 1993, p. 3)".

George Cobb (1993) reviewed the progress of over a dozen NSF grants focusing on statistics education. He summarized that "many of these projects were successful in developing projects and activities that assist in allowing statistics education to be "more data, less lecturing (p. 1)". All of these projects, however, took place at universities. While they were successful in allowing for a more laboratory-based approach to statistics education in the university, and in providing more materials and ideas at that level, they did not necessarily allow K-12 teachers to integrate more student-centered statistical learning activity into their instruction. A gap still exists in the ability of K-12 teachers to effectively enact reform-based statistics education (Konold, 2002; Shaughnessy, 2002).

The difficulty teachers have moving their instructional practices towards more student-centered activity is not isolated to statistics education. Shaughnessy (1992) explains this notion:

> the impediments to effective teaching of probability and statistics in our schools are the same ones that hinder effective implementation of problem solving in our schools. The teaching and learning of stochastics involves building models of physical phenomena, development and use of strategies, and comparison and evaluation of several different approaches to problems in order to monitor possible misconceptions or misrepresentations. In these respects, teaching stochastics is teaching problem solving, albeit in a particular content

> domain. In addition, teachers' backgrounds are weak or nonexistent in stochastics and in problem solving (p. 467).

Comparing statistics instruction to that of problem solving allows a more comprehensive investigation of factors that may inhibit or enhance effective education. Instead of relegating the discussion of improving statistics instruction to only those who teach statistics, conversation can be opened to include others lessons that have been learned when implementing constructivist pedagogy. Specifically, TSIL pedagogy offers exciting possibilities towards helping to assist students in alleviating misconceptions via asking students to conjecture, test, and reflect on their beliefs, and what they discovered about their conceptions by examining them. Furthermore, prior research has highlighted the importance of studying teachers' understandings and decisions related to their implementation of inquiry pedagogy.

Lester and Charles (1992) state, "research on problem solving has provided little specific information about problem-solving instruction" (p. 1). In particular, they posit that "relatively little attention has been given to the role of the teacher in instruction" (p. 1). They go on to discuss that "the various decisions [teachers] made before, during, and as a result of instruction … has been largely ignored as a factor of importance in problem-solving instruction research" (p. 6). This dissertation aims to directly support teachers in learning about, enacting, and reflecting upon inquiry pedagogy in statistics. This will be done by leading participants through cycles of thinking and action about TSIL, as guided by the practical framework model introduced later in this chapter. As a result, this research will help teachers to reflect upon their decision making process, and will provide much needed attention to this area of mathematics education.

Since Shaughnessy's review of statistics education research (1992), there have been examples of constructivist instructional practices that have successfully helped students to enhance their attitudes about statistics and to build strong conceptual understandings. Smith (1998) found that students' enthusiasm and examination scores improved dramatically as a result of his move to more student-centered activity. Mickelson (1997) and Derry, Levin, Osana, Jones, and Peterson (2000) also found that many students improved their attitudes towards and understanding of statistics through the use of constructivist methods. For example, Derry et al. (2000) focused on "anchoring instruction around small-group collaborative activities that simulated complex real-life problem solving (p. 747)". Her research group found that students who were engaged in this instruction "made meaningful gains in their ability to reason statistically (p. 747)". These findings are valuable towards this dissertation project in that they provide evidence that reform based pedagogy can improve students' statistical understandings. However Derry et al. (2000) also found that

> some students and the instructors of the course felt overwhelmed by the amount of work involved. The commitment to developing and assessing students' higher order reasoning abilities in authentic problem solving contexts is a much more demanding, expensive, and time-consuming task than is traditional teaching. Also, both social loafing and peer mentorship were problems that emerged in this course structure (p. 766).

This caveat further points to the need for professional development and support for teachers as they move to implement reform based pedagogy. This dissertation project provides technical, financial, and cognitive support for participants in anticipation of assisting these teachers as they make the difficult transition to TSIL instruction. Furthermore, by sharing the ways that participants are supported through cycles of reflection and action, and their subsequent understandings and practice, the research community will gain greater understandings of the challenges and successes associated with reform based pedagogy.

Research about Professional Development

“Despite infusions of funding and enthusiastic endorsements by the federal government, technology is not widely used by classroom teachers, and there is remarkably little professional development to help them [teachers] become proficient (Blumenfeld et al., 2000, p. 151).” Nonetheless, the small amount of professional development that has occurred regarding technology has provided some important lessons for teacher educators to consider when supporting teacher learning.

Norton and Wiburg (1998) point out that the introduction of technology can assist in changing the fundamental nature of the classroom from a teacher-directed to a learner-centered environment. For this to happen, the responsibility for learning must shift toward the learner, with teachers exerting less direct control. Technology provides access to more and different kinds of information than was previously accessible in the classroom. Much of this information is available through the use of technology and does not come directly from the teacher. In order to allow students to access this information, teachers are required to relinquish some control in the classroom. The teacher becomes a facilitator of learning, coach, or mentor, rather than a transmitter of information. In discussing how inquiry supported by technology can allow students to investigate conjectures and reach conclusions based on their own experiences, Kaput (1986) argues that "authority no longer is the exclusive purview of teacher and text, but is provided by proof, convincing proof (p. 7)." Research by Norton & Wiburg, and Kaput, highlights the fact that in order for effective TSIL activity to take place in the classroom teachers must provide students with opportunities to explore, conjecture and learn on their own.

The recommended changes in classroom authority and responsibility are consistent with the goals of education reform and TSIL. In fact, educators who hold a constructivist view of instruction may best be able to teach in the ways recommended for computer-based instruction and education reform in general (Becker, 2000c). The Office of Technology Assessment (1995) concluded, "teachers who use technology to support more student-centered approaches to instruction are among the most enthusiastic technology users, since technology is particularly helpful in supporting this kind of teaching" (p. 49). However, Becker (2000c) later found that "having a compatible teaching philosophy makes frequent use of computers more likely, but by itself is insufficient to make frequent computer use a modal teaching practice" (p. 20). This evidence suggests that in order for educators to become exemplary computer-using teachers not only do they need training and support in technology but also in examining their conceptions of teaching and learning, and how to utilize technology within an inquiry framework.

In addition to providing cognitive support for teachers trying to learn to effectively integrate technology into their curriculum and instruction, technical and financial support have also emerged from a review of the literature as important considerations. We are learning that "to achieve sustained use of technology, teachers need hands on learning, time to experiment, easy access to equipment, and ready access to support personnel who can help them understand how to use technology well in their teaching practice and curriculum (OTA, 1995, p. 140)."

Some research projects have been successful in providing professional development that has motivated teachers to integrate technology in ways that support the content and processes of student learning (Wetzel, 1998; Blumenfeld et al., 1991, 2000). These studies have cited the value of having teachers learn together in teams; giving opportunities for as much hands-on experience as possible; and of providing strong ongoing technical support. For example, Blumenfeld et al. (2000) emphasize the importance of collaboration among teachers and researchers and the need for cycles of practice and reflection as a route to successful changes in practice. Moreover, they suggest that change takes considerable time and support and requires a great deal of effort. Becker (1994) found

> extensive time to work with technologies and to reflect on teaching and learning; technological, financial, and cognitive support; and continual effort by teachers and researchers, systematically surface as important factors to attend to in teaching with technology professional development initiatives (p. 24).

Richardson (1992) points out that a new generation of staff development programs is evolving which “attempts to introduce new ways of thinking and practices within a context that attends to what we know about how and why teachers change their practices (p. 286)”. She goes on to state:

> This new form of staff development is generally cognitively framed in ways of helping teachers explore their beliefs and knowledge, reconstruct their premises related to teaching and learning, and alter their practices. In order for their teachers to participate in this reconstructive process, they must acknowledge the power of their own practical reasoning and expertise, and share in the ownership of the new content that helps them reconstruct their practical knowledge (p. 287).

This statement points out the importance that professional development initiatives attend to teacher beliefs and provide opportunities for teachers to reflect on practice. The kinds of changes from traditional methods that TSIL instruction implies do not come easily for teachers. Teachers wishing to implement this pedagogy face challenges learning about technology and understanding how they can utilize it to support students' learning. Effective reform-based teaching necessitates that teachers allow time and space for students to investigate, conjecture, and problem solve. This method of instruction is often much different than the way that teachers learned to teach. Prior research, outlined above, highlights that teachers must take ownership of their professional responsibility to teach in a way consistent with reform recommendations, and must be supported by others as they learn to do so. One particularly exciting professional development model, CERA, has proven to be particularly valuable towards supporting teachers in understanding and implementing reform based pedagogy supported by technology (Blumenfeld et al., 2000). CERA will be introduced later in this chapter and is utilized in the practical framework that will guide this study.

Research on Learning

Recently, prior research on cognition and learning has been synthesized in such a way as to focus on four components of effective learning environments (NRC, 2000). These four aspects, which are essential towards the establishment of effective learning environments are, learner, knowledge, assessment, and community. These components are fundamental towards the successful development of any learning environment, whether it is student learning or teacher learning. Below, each aspect will be introduced. The connection between the components of effective learning environment and this dissertation project will also be discussed. Explanation of how this dissertation aims to honor the tenets of effective learning environments by supporting student learning via the TSIL model (Table 1), and teacher learning via the CERA model (Table 2) are summarized at the end of this chapter.

Learner-centered Environments

Learner centered environments pay careful attention to the knowledge, skills, attitudes, and beliefs that learners bring to the educational setting. Accomplished teachers respect and understand their students' prior experiences and understandings and use these as a foundation on which to build new understandings (Duckworth, 1987; NRC, 2000). John Dewey (1902) urged educators long ago to

> abandon the notion of subject-matter as something fixed and ready-made in itself, outside the child's experience; cease thinking of the child's experience as also something hard and fast; see it as something fluent, embryonic, vital; and realize that the child and the curriculum are simply two limits which define a single process ... it is continuous reconstruction, moving from the child's present experience out into that represented by the organized bodies of truth that we call studies (p. 37).

Theodore Sizer (1992) states "it is a truism that we learn well only when we are engaged. That is, if we do not pay attention, we will not 'get it'. Our attention is caught by things that interest us, that so intrigue us, that we are compelled to find out more about them, that we believe we had better attend to or we might miss something" (p. 85). For a century, researchers have been advising that for effective learning to occur, education needs to take into account the fact that all we come to "know" is filtered through our own identities, experiences, and perspectives (Greeno et al., 1998; John Dewey, 1902).

Students build new knowledge and understanding on what they already know and believe. Students formulate new knowledge by modifying and refining their current concepts and by adding new concepts to what they already know (Lave, 1988). The research on conceptual change indicates that in order for students to change their ideas, they need to be provided with opportunities to explore and discover that their ideas may not sufficiently describe or explain an event or observation (NRC, 1997).

Research on inquiry demonstrates that learners come to change their ideas when they discover alternatives that seem plausible and appear to be more useful (Saxe, 1990). TSIL with Fathom activities give students opportunity to pursue interesting problems, and to make conjectures, gain and weigh evidence about, and consider multiple solutions to these problems. In this way, the TSIL process respects the tenets of learner-centered environments by allowing students to build off of their prior experiences and understanding and to actively be involved in constructing their own knowledge.

The professional development process that will be utilized during this project also respects learner-centered aspects of learning environments. In particular, the cognitive, technical, and financial support that will be provided honor kinds of assistance that research highlights is important for teachers struggling to learn to integrate technology. Furthermore, stages of collaboration and reflection during this project will be provided, in order to be able to elicit and work with teachers' emerging conceptions of TSIL integration. Fenstermacher's (1994) practical argument methodology, as outlined in the methodology section of this dissertation will be utilized to guide teacher/researcher interactions during these stages and to attend to and incorporate teachers' knowledge and beliefs into their enactment and assessment of TSIL with Fathom.

Knowledge-centered Environments

Knowledge centered environments help students develop well-organized bodies of knowledge and organize that knowledge so that it supports planning and strategic thinking. In these kinds of environments, students "learn their way around" a discipline and are able to make connections among ideas (NRC, 2000). In these kinds of learning environments, teachers help students think about the general principles or "big ideas" in a subject. When they learn new knowledge, students also learn where it applies and how. They have opportunities to practice using it in novel situations. These learning environments promote the sort of problem-solving behavior observed in experts. Research on people who have expertise in a field demonstrates that they (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that allow for retrieval and application. For their knowledge to be useable in these ways, it must be connected and organized through important concepts. Experts must know the contexts in which knowledge is applicable and must be able to transfer that knowledge from one context to another (NRC, 2000).

Students should be able to use what they learn, understand major concepts, build a strong base of supporting factual information, and know how to apply their knowledge effectively. They should be able to describe a problem in detail before attempting a solution, determine what relevant information should enter the analysis of a problem, and decide which procedures can be used to generate descriptions and analyses of the problem (NRC, 2000). Through scientific inquiry, students can gain new data to change their ideas or deepen their understanding of important scientific principles. They can also develop important abilities such as reasoning, careful observing, and logical analysis (Minstrell, 2000).

It is not just the understanding of the content that is the goal of TSIL, but also that students understand the scientific inquiry process, and develop the confidence and interest so that they can successfully engage in this process. Research on inquiry learning suggests that this pedagogy can help students learn with understanding, and enjoy the learning process. Elaine Duckworth quotes David Hawkins as saying of curriculum development "you don't want to cover a subject; you want to uncover it" and goes on to say herself that "wonderful ideas are built on wonderful ideas" (1987, p. 7). Vivian Paley says, "the key is curiosity, and it is curiosity, not answers, that we model" (1979, p. 127). This project strives to support students in valuing the role of statistics in the world around us, and building conceptual understandings of stochastics in the process.

A major goal of TSIL with Fathom will be to respect knowledge-centered aspects of effective learning environments by helping students develop deep conceptual statistical understandings and the empowerment to engage in statistical investigation themselves. Chapter One outlined the ways that Fathom technology can support statistics learning. Within a TSIL framework, students should be able to take advantage of the multiple representations and interactive graphics within the Fathom environment to gain deeper understandings of statistics concepts and procedures. The goal of this activity is for students to be confident and able to wade through the abundance of information that exists in this day and age, and to be able to represent, understand, and communicate the important aspects of this data. Furthermore, another goal of this project is that teachers will come to gain knowledge and understanding of how to integrate TSIL into their curriculum and instruction. Teachers will have multiple avenues of support during this research project to aid their learning and practice involving TSIL. These supports will be further discussed in the practical framework and methods sections of this dissertation.

Assessment-centered Environments

Assessment centered environments help students learn to monitor and regulate their own learning. Students learn to question "why it is they believe what they believe, and whether there is sufficient evidence for their beliefs" (White and Frederiksen, 1997, p. 98). These environments provide students with opportunities for feedback and revision. Assessment centered environments also help teachers shape classroom activities, diagnose students' ideas and products, and guide teachers' decisions (NCTM, 2000; NRC, 1997; Black & William, 1998). As Black and William (1998) note from their extensive review of the research on classroom assessment, "there is a body of firm evidence that formative assessment is an essential component of classroom work, and that its development can raise standards of achievement (p. 9)."

A particularly important form of assessment is students' self-assessment. Effective learning requires that students take control of their own learning. Students need to learn to recognize when they understand and when they need more information. Good learners articulate their own ideas, compare and contrast them with those of others, and provide reasons why they accept one point of view rather than another (NRC, 2000). They are "metacognitive", that is, they are aware and capable of monitoring and regulating their thoughts and their knowledge (White and Frederiksen, 1998). Research underscores the value of student self-assessment in developing their understanding of science concepts, as well as their abilities to reason and think critically (Black and William, 1998). As Black and William note, it is only when students are trained in, and given opportunities for self-assessment, that they "can understand the main purposes of their learning and thereby grasp what they need to do to achieve (1998, p. 143)." Engaging students in assessment of their own thinking and performance allows them to be more self-directive in planning, pursuing, monitoring, and correcting the course of their own learning. Self-assessment nurtures discovery, teamwork, communication, and conceptual connections (NRC, 1997, p. 80).

When facilitating inquiry-based teaching, involving students in assessment both reduces the burden on teachers and lets students know what's expected of them. Unless students can see the criteria by which they will be judged and examples of successful performance, assessment becomes a game of guessing what's in the teacher's head (NRC, 1997, p. 80). Involving students in all stages of inquiry and letting them know what is expected, they can come to make the traditionally neglected connection between what they have just learned and why they learned it.

At the broadest level, assessment of inquiry “measures the capacity of students to evaluate the kinds of questions that scientists investigate, understand the purposes of investigation, and assess the qualities of data, explanations, and arguments (NRC, 1997, p. 76). Assessment of TSIL activity should be both formative and summative and assist both the students and the teacher in determining whether students can generate and/or clarify questions; develop possible explanations; design and conduct investigations; and use data as evidence to support or reject their own explanations. Rubrics that clearly state the objectives that teachers hold for student TSIL activity and the way that these objectives are assessed should be provided to students and discussed with them before, during, and after they work on activities.

Similar to student TSIL activity, assessment of teachers’ integration efforts should be formative and summative. During the course of this project, teachers will have the opportunity to undergo cycles of action and reflection. By doing so, teachers will be able to receive feedback about their integration efforts in a formative manner. Teacher interviews and observations will also serve to provide summative evidence regarding the impact of this professional development project. More details on how these cycles of action and reflection will be provided during the course of this study are outlined in the methodology section of the dissertation.

Community-centered Environments

Community-centered environments require students to articulate their ideas, challenge those of others, and negotiate deeper meaning along with other learners. Such environments encourage people to learn from one another. They value the search for understanding and acknowledge that mistakes are a necessary ingredient if learning is to occur. Furthermore, such environments are open to new ideas and ways of thinking, as the community members are both encouraged and expected to provide each other with feedback and work to incorporate new ideas into their thinking (NRC, 2000). These researchers share

> studies of effective environments for learning science "emphasize the importance of class discussions for developing a language for talking about scientific ideas, for making students' thinking explicit to the teacher and the rest of the class, and for learning to develop a line of argumentation that uses what one has learned to solve problems and explain phenomena and observations (p. 171).

Research indicates that learners benefit from opportunities to articulate their ideas to others, challenge each other's ideas, and, in doing so, reconstruct their ideas (CGTV, 1992)

Other research points out the fact that communication is often motivational for learners. The NCTM Standards (2000) share an example where students kept working at a complex problem, in part, because it was a collaborative effort and they were discussing their work. The Cognition and Technology Group at Vanderbilt (1997) share that students are motivated to work for days and weeks at a time to solve Adventures of Jasper Woodbury's challenges when they know they will be sharing their results with authentic audiences of peers, parents, and teachers.

Inquiry research demonstrates that when ideas are exchanged and subjected to thoughtful critiques, they are often refined and improved (Brown & Campione, 1996). Furthermore, NCTM (2000) states “as students develop clearer and more-coherent communication (using verbal explanations and appropriate mathematical notation and representations), they will become better mathematical thinkers” (p. 33). Chatfield (1995) points out that communication is an important component of statistical investigation and should be incorporated into statistics education.

This study seeks to respect community-centered tenets of effective learning environments via its content and processes. TSIL with Fathom activity should allow students to collaboratively investigate statistical phenomena. Students will utilize Fathom in order to learn statistics and come to understand ways that they can use statistics to better understand the world around them. When they enter into the world of statistics by investigating content that is meaningful to themselves, and collaborate with others to gain understandings they are building learning communities within their classroom and participating in a scientific community of meaning making. When students communicate the results of their investigations with authentic audiences of peers and teachers, they further build community. In so doing, they are also able to facilitate the classes’ understandings of the relationship between statistics and the world, and also appreciate the fact that they are able to participate in the process of scientific inquiry.

The teachers involved in this research project will also be involved in community-centered aspects of learning. Together, they will share their thoughts and efforts as they learn about and enact TSIL pedagogy. Throughout their learning process, teachers will collaborate and communicate with the researcher and with other participants. Together, and in subgroups, they will engage in reflective assessment regarding their integration efforts. The process of communicating their thoughts and actions regarding TSIL gives teachers opportunity to take advantage of the fact that they are part of a learning community to critically reflect on, and make adjustments to their instructional practices. In the above-mentioned ways, both the teachers in this project, and their students, will benefit by the attention of a community of learners.

Effective Learning Environment Summary

There are interesting parallels between research on effective learning and learning environments and the process of scientific inquiry itself (Duschl, 1992).

> Both learner and scientist actively construct knowledge through confrontation with a new question, problem, or phenomenon, gathering information, and creating explanations. Throughout the process of inquiry, both constantly evaluate and reevaluate the nature and strength of evidence and share and then critique their explanations and those of others. A classroom in which students use scientific inquiry to learn is one that resembles those that research has found the most effective for learning for understanding. This consequence strengthens the argument for inquiry-based teaching (NRC, 1997, p. 124).

A pattern of general support for inquiry-based teaching continues to emerge from the research. The research literature reviewed above demonstrated that TSIL instruction honors the tenets of effective learning environments by attending to the learner, knowledge, assessment, and community centered aspects of effective learning environments. The alignment of TSIL with the tenets of effective learning environments is outlined in Table 1 on page 61. This dissertation project seeks to support teachers as they learn about and integrate inquiry-based learning activity that is supported by technology into their curriculum and instruction. This vision of TSIL will be used as a gauge to measure the progress that teachers make integrating technology into their practice during the course of this study.

PRACTICAL FRAMEWORK

The Highly Interactive-Computing in Education (HI-CE) group at the University of Michigan (Phyllis Blumenfeld, Ron Marx, Joe Kracjik, Barry Fishman, Nancy Songer, and Stephen Best) have been very active in the area of supporting science teachers' learning of technology-supported reform based pedagogy. They have articulated a pedagogical method that they refer to as Project Based Science (PBS). Their PBS curriculum is consistent with the National Research Council's emphasis on scientific inquiry. Like the vision of TSIL articulated in Chapter One, the PBS learning cycle emphasizes that curricula should provide driving questions that will anchor student investigation, data collection and analysis, provide opportunities for collaboration and communication, and be supported by technology during all phases (Krajcik, J. et al., 2001). During their decade plus activity of supporting teacher learning and practice, the HI-CE group has "worked with teachers to develop project-based science curriculum and pedagogy and learner-centered technologies to support inquiry (Blumenfeld et al., 2000, p. 149). During this time, their research group has gained many lessons about teacher learning and professional development.

Due to their continual efforts in helping teachers appropriate and learn how to use technological innovations to support reform-based instruction that utilizes technology, the HI-CE group has articulated a framework for professional development that they call CERA: "Collaborative construction of understanding; Enactment of new practices in classrooms; Reflections on practice; and Adaptation of material and practices" (Blumenfeld et al., 2000, p. 151). They go on to articulate "the implicit goal for the design of our professional development activities is to provide opportunities for teachers to enhance their knowledge, beliefs, attitudes about content, teaching, and technology use (p. 151)". This professional development project seeks to support and document changes in teachers' understandings and practices regarding technology integration over the course of this research project and the CERA model will serve as a "practical framework" in order to do so.

Margaret Eisenhart (1991) has defined a practical framework model in the following way:

> A practical framework guides research by using 'what works' in the experience or exercise of doing something by those directly involved in it, e.g., in the case of educational research: by using 'what works' in teaching as a 'kernel' idea or action that, if extended to other teachers could help to alleviate some educational problem. The study is structured to determine key features of the practice, and whether, or in what circumstances, a practice works as expected or envisioned. This kind of framework is not informed by formal theory but by accumulated practical knowledge (ideas) of practitioners … and the findings of previous research. Research hypothesis or questions are derived from this knowledge base, and research results are used to support, extend, or revise the practice. In selecting practice as the basis for a research framework, the researcher is deciding to follow conventional wisdom as understood by people who are stakeholders in the practice (p. 207-208).

The HI-CE group has found that the CERA model of professional development serves as an effective framework to support teacher learning. The CERA model includes cycles of Collaborative construction of understanding; Enactment of new practices in classrooms; Reflections on practice; and Adaptation of material and practices. I plan to utilize the CERA model as a practical framework to guide the interactions between the participating teachers and myself, the teacher educator/researcher, as I support teachers' learning and enactment of TSIL. The CERA cycle of collaboration, enactment, reflection, and adaptation honors the tenets of effective learning environments that were described in the literature review section of this chapter. The relationship between the CERA model and the tenets of effective learning environments are outlined in Table 2 on page 62.

The methods that I use as I interact with these teachers, and the practical framework that guides these interactions, will allow me answer the research questions of this study. The first research question of this study focuses on teachers' understandings about content, pedagogy, and technology over the course of this project. By engaging in cycles of collaboration and reflection with teachers there will be multiple opportunities to survey and interview teachers about their understandings of these three areas. Specific methods for gathering and analyzing this data are discussed in Chapter Three. Similarly, the opportunities for me to observe teachers' enactment and adaptation of TSIL pedagogy, which the CERA model provides, will allow me to answer the second research question of this study, regarding teachers' instructional practices during the course of this study. Again, details of the methods that will be used to collect and analyze data that will help to answer this research question will be outlined in Chapter Three.

Answering the research questions of this study, as guided by the CERA practical framework articulated above, allows me to contribute to theory and practice and the interrelationship between the two. This will be accomplished via the opportunity that this dissertation project provides to build from knowledge accumulated through research and practice and examine how this knowledge supports participating teachers’ understandings and practices related to technology integration.

Effective Learning Environments	**Technology Supported Inquiry Learning via Fathom**
Learner centered	TSIL with Fathom activities give students opportunity to pursue interesting problems, and to make conjectures, gain and weigh evidence about, and consider multiple solutions to these problems. In this way, the TSIL process respects the tenets of learner-centered environments by allowing students to build off of their prior experiences and understanding and to actively be involved in constructing their own knowledge. TSIL statistical activities utilize authentic data in order to connect school to the real world that surrounds the learner.
Knowledge centered	A major goal of TSIL with Fathom will be to respect knowledge-centered aspects of effective learning environments by helping students develop deep conceptual statistical understandings and the empowerment to engage in statistical investigation themselves. Within a TSIL framework, students should be able to take advantage of the multiple representations and interactive graphics within the Fathom environment to gain deeper understandings of statistics concepts and procedures. The goal of this activity is for students to be confident and able to wade through the abundance of information that exists in this day and age, and to be able to represent, understand, and communicate the important aspects of this data.
Assessment centered	At the broadest level, assessment of inquiry "measures the capacity of students to evaluate the kinds of questions that scientists investigate, understand the purposes of investigation, and assess the qualities of data, explanations, and arguments (NRC, 1997, p. 76). Assessment of TSIL activity should be both formative and summative and assist both the students and the teacher in determining whether students can generate and/or clarify questions; develop possible explanations; design and conduct investigations; and use data as evidence to support or reject their own explanations. Rubrics that clearly state the objectives that teachers hold for student TSIL activity and the way that these objectives are assessed should be provided to students and discussed with them before, during, and after they work on activities.
Community centered	TSIL with Fathom activity should allow students to collaboratively investigate statistical phenomena. Students will utilize Fathom in order to learn statistics and come to understand ways that they can use statistics to better understand the world around them. When they enter into the world of statistics by investigating content that is meaningful to themselves, and collaborate with others to gain understandings they are building learning communities within their classroom and participating in scientific community of meaning making. When students communicate the results of their investigations with authentic audiences of peers and teachers, they further build community. In so doing, they are also able to facilitate the classes' understandings of the relationship between statistics and the world, and also appreciate the fact that they are able to participate in the process of scientific inquiry.

Table 1. This table maps the relationship between TSIL with Fathom and research on effective learning environments. TSIL is introduced in this dissertation project as vision of how teachers can incorporate Fathom technology into reform based inquiry statistics instruction in order to foster student understanding and motivation. This mapping is utilized during this professional development project to guide and analyze the interactions between the participating teachers and their students.

Effective Learning Environments	**The CERA practical framework model of Collaboration, Enactment, Reflection, and Adaptation.**
Learner centered	The CERA model supports teachers to actively participate in their own professional development. They are not being lectured to about how they should teach. Instead, they participate in cycles of action and reflection, are able to critically assess their own learning and teaching, and are supported as they do so. In this way, the CERA process respects the tenets of learner-centered environments by allowing teachers to build off of their prior experiences and understanding and to actively be involved in constructing their own knowledge. This project provides the technical and financial assistance that has been proven beneficial towards teacher growth, and the CERA model provides a framework through which the teacher educator can provide sustained cognitive support.
Knowledge centered	A major goal of this project is that teachers will come to gain knowledge and understanding of how to integrate TSIL with Fathom into their curriculum and instruction. The CERA model has been demonstrated by the PBS group to be effective in supporting teachers to learn to implement reform based pedagogy. The CERA model provides teachers with sustained opportunity to think about, practice with, and reflect upon their understandings and practices regarding TSIL. The learning goal of this professional development project is that teachers will come to understand and utilize TSIL pedagogy and CERA provides a practical framework via which the teacher educator can support and evaluate teachers' learning.
Assessment centered	Assessment of teachers' TSIL integration efforts should be formative and summative. The CERA model of professional development provides the teacher educator/researcher a framework through which to have regular and sustained interaction with participating teachers during their learning and practice of TSIL. The CERA framework will facilitate teachers in eliciting their emerging conceptions of TSIL. Via regular cycles of collaboration, enactment, and reflection, and adaptation, teachers will be able to participate in the assessment process and receive feedback about their integration efforts in a formative manner. Teacher interviews and observations will also serve to provide summative evidence regarding the impact of this professional development project.
Community centered	The teachers involved in this research project will be involved in community-centered aspects of learning. The CERA model provides a framework through which teachers will collaboratively investigate and reflect upon TSIL. Throughout their learning process, teachers will collaborate and communicate with the researcher, their teammates, and with other participants as they learn about and enact TSIL pedagogy. Together, and in subgroups, they will engage in reflective assessment regarding their integration efforts. The process of communicating their thoughts and actions regarding TSIL, which the CERA model provides, allows teachers to take advantage of the fact that they are part of a learning community in order to critically reflect on, and make adaptations to, their instructional practices.

Table 2. This table maps the relationship between CERA and research on effective learning environments. CERA is utilized in this dissertation project as a practical framework to guide the interactions between the participating teachers and myself, the teacher educator/researcher as I support teachers' learning and enactment of TSIL.

CHAPTER THREE

METHODOLOGY

The key to long-term improvement [in teaching] is to figure out how to generate, accumulate, and share professional knowledge (NCMST, 2000, p. 17).

INTRODUCTION

This study is structured as a teacher development experiment (TDE). A teacher development experiment (Simon, 2000) research design utilizes a mix of qualitative and quantitative methods of data collection - observations, interviews, and surveys, and of data analysis – descriptive statistics, hypothesis tests, and comparative analysis (Miles & Huberman, 1984). The TDE methodology is an adaptation of the constructivist teaching experiment (Steffe & Thompson, 2000) that is used to collect and coordinate individual and group data on teacher development (Simon, 2000). Steffe & Thompson (2000) posit that "teaching experiments allow the teacher-researcher to generate situations of learning systematically and are directed toward understanding the progress students make over extended periods (p. 274)". The TDE provides a methodology for researchers who seek to implement and study teacher professional development simultaneously. This methodology has been fully articulated by Martin Simon (2000), and is well suited for this project since the goals of this methodology fit the goals of this dissertation. The goals of teacher development experiments include not only providing opportunities for teachers' development, but also closing the gap between research and practice by investigating the solution of practical problems in schools (Simon, 2000). The goal of this dissertation research is to support practicing teachers' capacity to integrate technology into their curriculum and instruction, and to contribute understandings that develop out of this endeavor to the education research community.

The CERA practical framework, described in Chapter Two, provides a model for professional development that allows for regular and sustained contact between researcher and teachers. The teacher development experiment supplies a methodology that can be utilized to fulfill the vision of supporting and researching teacher professional development. Together, the CERA practical framework and the teacher development methodology, described herein, will allow me to interact with the participants and be able to gather data necessary for answering the research questions of this study. The TDE methodology will therefore be articulated in the following sections of this chapter, and will be utilized in this dissertation project. In order, the following sections introduce the goals, methods, and participants and roles of teacher development experiments. Next, the professional support that this project will provide for participants, and an outline of the summer professional development are described. Finally, methods of data collection and data analysis are discussed.

TEACHER DEVELOPMENT EXPERIMENTS

Goals of TDE

The TDE methodology is an adaptation of the constructivist teaching experiment (Steffe & Thompson, 2000), and is used to collect and coordinate individual and group data on teacher development (Simon, 2000). Steffe &

Thompson (2000) state “teaching experiments are aimed at understanding students’ current knowledge, allowing the teacher-researcher to generate situations of learning systematically and to test conjectures and local hypotheses about the mathematical learning of the students, and are directed toward understanding the progress students make over extended periods” (p. 274). In a teaching experiment, it is the researcher who is striving to learn what change they can bring forth in their students and how to explain such change. It is the same in the teacher development experiment except the students that are being instructed are, themselves, teachers. Thus, the teacher development experiment methodology provides a framework for researchers to work at the edge of their evolving knowledge and to implement and study teacher professional development simultaneously.

> The comprehensive nature of the TDE study derives from its inclusion of three pairs of conditions: (1) study of groups of teachers and of individual teachers, (2) study of how teachers develop during PD and in their own classrooms, and (3) study of teachers’ mathematical development and their pedagogical development (Simon, 2000, p. 339).

The potential contributions of this type of interpretive research concern context and meaning. Teachers reorganize their beliefs and instructional practices as they attempt to make sense of classroom events and incidents (Ball, 1993; Putnam & Borko, 1998). Hence, teachers’ learning, as it occurs in a social context, becomes a direct focus of investigation in a teaching development experiment. By studying a group of teachers’ development, it is possible to compare, contrast, and relate different enactments of innovations (Simon, 2000). Analyses of teachers’ development that emphasizes the contexts and meanings associated with their actions can make important contributions to education research by contributing sophisticated understandings of teachers’ efforts to learn and implement reform pedagogy (Schoenfeld, 1999; Lampert, 1998).

> In experiments conducted in collaboration with teachers, the researcher typically learns something about the process by which teachers reorganize their instructional practices. The context of the teaching experiment is closer to that in which reform must take place eventually. Consequently, researchers who collaborate with teachers accept what can be called realistic constraints as they explore what might be possible in students’ mathematics education (Cobb, 2000, p. 330).

Research that focuses on the social contexts (i.e. communities of learning and practice) in which teachers formulate and verify their practical knowledge brings the worlds of teaching and research closer together. The purpose of such research “is not to determine whether general propositions about learning or teaching are true or false but to further our understanding of the character of these particular kinds of human activity (Lampert, 1998, p. 57)”. Finally, this approach to research “encourages the professional growth of teachers involved in the research, particularly their ability to cope in a conscious and reflective way with the dilemmas encountered in attempting to reform mathematics curriculum” (Romagnano, 1995, p. 13). This dissertation project seeks to support and research participating teachers’ learning regarding reform-based statistics instruction utilizing technology, and contribute to theoretical and practical understandings of professional development in the process. The teacher development experiment methodology will be utilized in this project in order to do so, and this methodology will be further articulated below.

Methods of TDE

Lesh and Kelly (2000) state that in order for researchers to reach their goals of being teachers as well as investigators, they should “bring together a diverse group of teachers then engage them in a series of activities in which they must continually articulate, examine, compare, test, refine, and reach consensus about such things as the nature of excellent problem solving activities for their students” (p. 221). These authors discuss that by being involved in a learning environment in which formative feedback and consensus building are used to optimize the chances of improvement, “teachers are able to develop in directions that they themselves are able to judge to be continually better (without basing their judgments on preconceived notions of best)” (Lesh & Kelly, 2000, p. 222). Simon (2000) expands on Lesh and Kelly’s description by discussing when and where mathematics teacher development can be supported and studied;

> because the development of math teachers involves pedagogical development, as well as mathematical development, and because that development happens not only in mathematics classes for teachers, but also in teachers’ own classrooms, and professional collaborations, TDE researchers participate in teacher development at these diverse sites (p. 345).

The CERA framework provides a model through which I will be able to observe, discuss, and interview participating teachers in the diverse settings of professional collaborations and classroom practice. The regular opportunity to be involved with teachers’ collaboration, enactment, reflection, and adaptation that CERA provides, will allow me access to gather the kinds of rich data necessary for answering the research questions of this study. The particular methods of data collection and data analysis that I will utilize during this TDE in order to gather evidence towards addressing changes in teachers' understandings and practices regarding TSIL will be detailed later in this chapter. Next, however, it is necessary to further articulate the roles of the participants in the TDE.

Participants/Roles in TDE

The TDE can allow researchers to generate powerful schemes for thinking about the development of teachers in the context of teacher education opportunities. The TDE takes as its object of study,

> a teaching-learning complex that encompasses three levels of participants: the researcher/teacher educator, the teacher, and the teachers' students, and two levels of curricula: the teacher education curricula and the mathematics students' curricula. By focusing on different aspects of this complex (indivisible) whole, one generates schemes about development (potentially at three levels), schemes about teaching (at two levels), and schemes about curricula (at two levels) (Simon, 2000, p. 338).

This project matches the multi-level focus that the TDE provides for. There are three levels of participants in this dissertation project, the researcher/teacher educator, the participating teachers, and students of the participating teachers. The CERA model, as outlined in Chapter Two, guides the interactions between the researcher/teacher educator and the participating teachers. The interactions between the participating teachers and their students are guided by the TSIL vision of pedagogy, as outlined in Chapter One. All levels of interactions are informed by what is known about effective learning environments (mapping in Tables 1 and 2, Chapter Two).

Six mathematics teachers participated in this project. I interacted with them during Summer 2002 as they learned Fathom and thought about how to integrate it, and during Fall 2002 as they integrated Fathom into their practice. Areas of contrast regarding participants in this study included such things as the grade level taught, years of experience teaching, content knowledge, technology skill, and philosophy of education, in general, and involving technology, in particular. These teachers' understandings and practices regarding content, pedagogy, and technology, which were documented during Summer and Fall 2002, were compared and contrasted with pre-intervention data gathered Spring 2002. In-depth case studies of three of these teachers' understandings and practices integrating TSIL with Fathom are introduced and discussed in Chapters Four and Five.

Professional development activities for this TDE built on prior research and practice. Romagnano (1995) urges that mathematics teachers be immersed in inquiry mathematics processes of doing real mathematics. They should be asked to reflect on their actions, roles, and responsibilities as students and as teachers as they do mathematics. Teacher educators who orchestrate this community must make explicit to their students the thoughts and decisions that guide their actions as they work (Romagnano, 1995). In order to foster these kinds of activities for teachers, Steffe & Thompson (2000) strongly recommend that "any researcher who has not conducted a teaching experiment independently, but who wishes to do so, first engage in exploratory teaching in order to become thoroughly acquainted with students' ways and means of operating with respect to whatever particular domain of mathematical concepts and operations are of interest" (p. 354). Towards these ends, I taught a semester of middle school prior to this inservice and implemented TSIL within my own curriculum and instruction, in preparation for the upcoming teacher development experiment. I learned a lot about the value of building a healthy learning community and of having students involved in all stages of the assessment process. I was able to share ways that I have worked to build community, involve students in assessment, and provide examples of my students' work during this professional development project.

During the course of this dissertation study, my role alternated between that of a teacher educator, and that of a researcher. Ulichny and Schoener (1996) offer a valuable conceptualization of the relationship between the researcher/teacher educator (herein referred to as researcher) and the teacher as situated along two dimensions, one of relationship and one of action. They posit that relationship can run along a continuum from relative strangers to mutual friends. In terms of action, they viewed the researcher as positioned along a continuum from distant observer to full participant in the work under study. The teacher, on the other hand, though a full participant in the classroom work being studied, may only be a very distant observer of the research project, the data analysis and interpretation, and the communication of the results. While pros and cons can be argued for and against various dimensions of relationship and action between the researcher and the participants, what is most important is to be able to delineate what the stances are within a particular research project, how these choices are made and justified, and how they will be maintained. For these reasons, the next paragraphs will document the stances that I strived to develop regarding relationship and action during this teacher development experiment.

Simon (2000) recommends that TDE researchers should take on the role of clinical supervisor in order to foster and study teachers' development in their own classrooms. M. Simon and R. Tzur (1999) found that the classroom consultant role was a key component in supporting the development of mathematics teachers. The researcher's role as a clinical supervisor (based on Simon, 2000) involves regular observations of the teacher during the teacher's mathematics class and regular meetings with the teacher following these classes. Conversations may focus on the lesson; what came before it or what will follow it; the teachers' thinking prior to, during, or after the lesson (including the teacher's evaluation of the lesson); the mathematics involved; and the activity of individual students. These conversations are opportunities to unearth the thoughts, decisions, and issues that drive observed actions (Romagnano, 1995). Without querying teachers' about their thinking, the decision-making processes that they go through will not be made known and available. If these understandings are not made explicit, the participants themselves, and those who will read the case studies of these teachers, will not be able to critically learn from their actions.

In this study, my stance during post-observation conversations with teachers took the form of a "critical friend", as guided by the notion of Fenstermacher's (1994) notion of practical inquiry. The concept of practical inquiry is designed to allow teachers to examine their explanations for their practices. The goal of this process is to support teacher learning by assisting them in considering the relationship between their beliefs and teaching practices. Practical inquiry can be viewed as a teacher questioning and reflecting about his/her practice with a specific focus (Fenstermacher, 1994; Richardson, 1992). Practical inquiry has been successfully used in previous professional development initiatives (Richardson, 1992; Franke et al., 1998) as a way to help facilitate positive teacher growth. Franke et al. (1998) summarized the connection between practical inquiry and teacher growth:

> A teacher who searches for successful practices can be seen as engaged in practical inquiry at a level of experimentation about what works. A teacher who examines his or her practices in relation to his or her own thinking and the thinking of his or her students engages in a different level of practical inquiry, where the focus is on detailed analysis. As teachers engage in this detailed analysis, they come to understand principled ideas that can then drive their practice and their continued practical inquiry. We view the first level of practical inquiry as leading to self-sustained change but the second level of practical inquiry as necessary for generative change (p. 68).

The researcher also serves as a resource for the teacher by providing references for textual and instructional materials, ideas for lessons, and insights into aspects of the teaching-learning process. Another way that researchers are different from teachers is that the researchers need to play some metacognitive roles that the teachers do not need to play. For example, the researchers need to ensure that sessions are planned that support and challenge teacher development. Also some additional clerical services need to be performed to ensure that records are maintained in a form that is accessible and useful (Lesh & Kelly, 2000). Further, the researcher acts as a support person for the teacher and as a confidante for the teacher's emotional experiences that accompany engagement in radical professional change. "Each aspect of the researcher's classroom supervision role contributes to their ability to understand the social, affective, and cognitive components of teacher's development (Simon, 2000, p. 348)". "Both the researcher and the teacher are learning in the context of the TDE; the teacher is learning about teaching, and the researcher is learning about the teacher's development. Each is key to the other's learning (Simon, 2000, p. 338)".

In this study, I assumed the role as a teacher educator who organized and conducted a summer professional development program focused on TSIL with Fathom, a clinical supervisor who supported teachers' efforts to understand and integrate TSIL with Fathom into their curriculum and instruction. During this whole process, I assumed the role of a researcher by gathering, organizing, and analyzing data regarding changes in teachers' thinking and action over time about TSIL with Fathom. The ways that I gathered, organized, and analyzed data are described in the data collection and data analysis sections of this chapter.

At times, such as during the summer professional development sessions, I had on the hat of the teacher educator. While in teachers' classrooms observing their TSIL practices and interviewing them about their understandings, I had on the researcher hat. This dual role, undoubtedly, had an effect on this project. Many of these teachers, and their students, would have gained tremendously if I had interfered with their lessons to provide assistance, input, or advice. However, doing so would have skewed my data and I would not have been able to as clearly see and discuss the affect of teachers' understandings regarding content, pedagogy, and technology on their practice. Due to the nature of this study, these teachers were not able to continue receiving professional development from me during the course of this project. Education research points to the fact that teacher education professional development must be sustained, ongoing work (Putman and Borko, 1998; NRC, 2001b). These teachers would have benefited more from sustained professional development and this is a major recommendation from this study.

Although I was not able to take advantage of opportunities to provide professional development while I was observing and interviewing teachers during this project, the process of asking them questions about their practice provided them with extra opportunity to reflect on their decision making and think consciously about their actions. I also shared my findings and conclusions with them this summer and hope that this helps them continue to learn and grow and refine their practice. Results and conclusions from this project are shared in Chapters Four and Five.

PROFESSIONAL SUPPORT

Cognitive, technical, and financial support has been found to be essential components of teacher technology professional development. I was able to obtain and provide valuable financial and technical support for the teachers who participated in this project. Key Curriculum Press provided all participants with complimentary copies of Fathom. The University of Colorado College of Education's Educational Technology Advisory Committee, the Institute of Cognitive Sciences at the University of Colorado, and the United States Department of Education funded MathStar project have all provided financial support for this project. These financial supports allowed this project to pay teachers' stipends during Summer and Fall 2002 for their extra time related to this project. These funds also supported travel costs related to data collection and contributed to data analysis costs such as transcription. Funding also was used to support teacher travel and per diem expenses during Summer 2002. Participants received further technical support from collaboration with one another, other Fathom users via the Fathom listserv, and myself as I interacted with them during the duration of this project.

Cognitive support was also provided to teachers from each other, the listserv, and from myself.

During the teacher development experiment, participating teachers engaged in a process of inquiry consisting of an ongoing cycle of collaboration, enactment, reflection, and adaptation around their practice (CERA) and I engaged in a equivalent cycle regarding the impact of the professional development and the participants' development. "This sort of rapprochement of teaching and research is consistent with Cobb's view of the changing relationship between theory and practice. He wrote 'theory is seen to emerge from practice and to feed back to guide it' (Simon, 2000, p. 334)". The CERA framework, via regular cycles of reflection and action among participants, and with the teacher educator, provided a model of cognitive support for teachers' development. It too, served as a framework that provided systematic opportunity for me, as a researcher, to reflect on teachers' learning, as I simultaneously supported and studied teacher growth.

SUMMER PROFESSIONAL DEVELOPMENT

I facilitated six days of summer professional development for participating teachers to learn about Fathom and TSIL. All six teachers participated in three days of professional development, July 7-9, focused on the functionalities of Fathom towards understanding the "big ideas" of data analysis, mathematical modeling, and probability and sampling distributions and inference (ASA, 2003; Scheaffer et al., 1998). Teachers participated in activities from a variety of Fathom's instructional resources, first, as students and then, discussed these tasks as teachers. These three days were designed to share Fathom activities from a variety of instructional resources (Data in Depth, Workshop Statistics, and Fifty Fathoms) in the areas of exploratory data analysis, mathematical modeling, and probability and sampling distributions, respectively (syllabus of the professional development activities for these three days is in Appendix 1). Furthermore, each day provided participating teachers with an opportunity to participate in these activities, first, as students and, then, to discuss them as teachers. The initial exploration of the activities provided teachers' opportunity to work together as students and learn how to use Fathom and to solve the statistical problems that they were investigating. During this stage of the activity I acted as a teacher who assisted them in enacting Fathom functions and interpreting tasks and instructions. Subsequent after-activity conversations allowed teachers to reflect on how the technology has helped them and/or could help their students to investigate and understand the content. During this stage of the activities, I played the role of a facilitator making sure that they shared correct interpretations of the content and shared thoughts about the technology.

Another three days of professional development, July 21-23, which four of the participants attended focused on TSIL. During these three days, teachers read and discussed education research on inquiry pedagogy and spent time planning, sharing, discussing, reflecting, and adapting TSIL with Fathom tasks to integrate within their Fall 2002 curriculum and instruction (syllabus of the professional development activities for these three days is in Appendix 2). I felt that these three days of professional development were particularly important to provide because I believe that just learning a piece of educational technology is not enough to ensure that is integrated into practice. The teachers who attended these days had time to think critically about when they could incorporate Fathom into their curriculum and instruction during the upcoming semester. Since only four teachers attended these three days of and two teachers did not, a dimension of contrast was provided between those teachers who attended all six days of professional development and those who only came for the first three. During this time, I was hands-off and allowed teachers to spend time themselves thinking about if when, where, why, and how they would use Fathom with their students, so as to not skew the data with my beliefs and ideas about integration.

The upcoming results and discussion chapter of this dissertation will follow up this section and share how this dimension of contrast corresponded with teachers' subsequent integration efforts. Field notes from each of these days professional development were gathered and analyzed as part of this project. Details of all of the data collection and data analysis procedures utilized during this dissertation will be discussed in the next sections of this chapter.

DATA COLLECTION

In order to answer the research questions of this study, data was collected about participants’ developing understandings and practices related to content, pedagogy, and technology. The practical framework model that this study utilized provided sustained opportunities, during all stages of the CERA model, for gathering data necessary to answer the research questions about teachers' understandings and practices regarding content, pedagogy, and technology. These sustained opportunities to gather data are particularly important within an interpretive study, such as this teacher development experiment. "An interpretive study employs a variety of data sources in order to increase the credibility of what is learned, and it is expected that multiple data sources would support any assertion" (Tobin, 2000, p. 489). Multiple data sources need to be collected and explored over time in order to establish the warrants for knowledge claims. When triangular data sources converge to produce a pattern or theme, there is greater confidence that the pattern is not dependent on a particular form of data, such as field notes or interviews, or idiosyncratic of events at a particular time (Tobin, 2000). The process of coordinating data analysis in order to highlight themes is discussed in the data analysis section of this chapter and subsequent findings and their implications in later chapters.

Data collection occurred before, during, and after the summer professional development. Data collected during this teacher development experiment focused on gathering information about teachers' understandings and practices regarding content, pedagogy, and technology. This project utilized surveys, observations, interviews, and document collection data collection methods in order to do so. They are explained in detail in upcoming sections.

Surveys

Surveys included items related to teachers' understandings and practices regarding content, pedagogy, and technology were administered to participating teachers Spring 2002 and at the end of the Fall 2002 semester. The reason for this data collection is two-fold. First, content knowledge has been linked to inquiry pedagogy (NRC, 1997) and differences in implementation of TSIL with Fathom may be attributed to differences in statistics content knowledge. Second, teachers' efforts to incorporate TSIL with Fathom into their instruction and curriculum may improve their understanding of statistics and pre and post surveys will be able to measure this growth. Content knowledge survey items are in Appendix 3**.**

Teachers' skills and practices related to technology also may influence how they feel about and implement TSIL with Fathom. For these reasons it was important to survey participants about their understandings and practices related to technology both before and after professional development. Technology survey items are included in Appendix 4.

General questions about participants' experiences as a teacher and their beliefs about teaching may also affect their understandings and practices related to TSIL with Fathom. For these reasons, it was important to survey participants about their experience with and perspectives regarding teaching. Pedagogical survey items are also included in Appendix 4.

Observations and Interviews

In order to have enough data to tell a complete story of the changes in participating teachers' understandings and practices regarding TSIL it is important to observe their instructional practices, and interview them about it, before, during, and after professional development interventions. Observations of teaching prior to the teaching experiment are important for collecting data on participating teachers' practices prior to any professional development. Each participating teacher was observed using technology in their instruction, and interviewed about their practice, one time during Spring 2002. Observation and interview protocols are discussed next.

It is worth noting that the process of negotiating sociomathematical norms with their students can give rise to learning opportunities for teachers. In Cobb’s (1999) work, teachers seemed to explicate and elaborate their own understanding of an inquiry form of mathematical activity as they interacted with their students. Therefore, Cobb (2000) recommends it is critical that the researchers are present in the classroom while the teaching experiment is in progress. Towards these ends, I asked teachers to plan out when they would implement Fathom activities in their classroom near the beginning, middle, and end of the Fall 2002 semester. I was present to observe teachers and take field notes as they integrated Fathom activity into their curriculum and instruction.

Data for the case studies, therefore, included field notes from observations of the participating teachers’ involvement during the summer professional development sessions, observations of their teaching practices as they integrated Fathom into their practice at the beginning, middle, and end of the Fall 2002 semester. Observation guidelines directed the field notes that were taken during these observations. After completing expanded field notes following each observation, I coded my field notes with respect to content, pedagogy, and technology and the relationship between these topics and the components of TSIL and effective learning environments. Details of the coding and analysis of data are discussed in the data analysis section of this chapter. The observations that I conduct, and their subsequent analyses, helped me to understand the Enactment and Adaptation of teacher instructional practices involving Fathom. These observations were also used in conjunction with teacher interviews in order to allow teachers to reflect on specific aspects of their practices related to the research focus of this study.

Teacher interviews were used regularly to collect data on teacher's conceptions of TSIL with Fathom. Interviews before and after each observation explored the teacher's conceptions, motivations, and thinking with respect to the instructional decisions they made within their lessons. Further understanding of the teachers' perspectives came from listening to their interpretation of their professional development experiences, classroom interactions, and individual students' behaviors. These interviews were structured so that teachers were challenged to make sense of general and particular learning-teaching interactions. Cobb's research found that "short, daily, debriefing sessions conducted with the collaborating teacher immediately after each classroom session are invaluable. A primary focus of these meetings is to develop consensual, or 'taken-as-shared' interpretations of what might be going on in the classroom (Cobb, 2000, p. 320)." Because the interviews precede or follow a teacher's lesson with a class, the interviewer can focus the discussion on specifics of the lesson or the lesson plan (Simon, 2000). "In so doing, teachers are not making general claims about their beliefs, claims whose genuineness might be questioned and whose utility and meaning to the teachers the researchers would have difficulty understanding (Simon, 2000, p. 355)". Pre and post project, pre and post observation, and post summer workshop interview protocols in Appendices 5, 6, 7, 8, and 9, respectively. These interviews are consistent with the Collaboration and Reflection components of CERA and helped me to understand the Enactments and Adaptations that I observed in teachers before, during, and after inservice and implementation.

Document Collection

Sources of documentary data included teachers' lesson plans, instructional documents (handouts,worksheets, etc…), and assessment instruments (quizzes, tests, rubrics). Samples of student work were also collected and discussed with teachers. Together, analysis of these artifacts helped to answer research questions 1, 1a, 2, and 2a.

Timeline

February 2002: Collected initial survey data from interested participants about their understandings and practices regarding content, pedagogy, and technology.
March 2002: Analyzed initial data and selected final participants.
April 2002: Collected initial observation and interview data from participants about their TSIL understandings and practices.
July 2002: Summer professional development activities regarding Fathom and TSIL. Data collection about participants' understandings and practices regarding TSIL and the professional development portion of this project.
August 2002: Interviewed participants regarding their plans for the integration of Fathom during Fall 2002.
September – December 2002: Conducted observations and interviews, and document collection regarding participants' Fathom integration activities.
January 2003: Collected final surveys and interviews about teachers understandings and practices regarding content, pedagogy, and technology.

DATA ANALYSIS

Participating teachers' understandings and practices related to technology-supported inquiry learning were analyzed during the course of this project. The research questions of this study aim to examine the relationships between teachers' understandings and practices regarding content, pedagogy, and technology. In order to gather evidence about teachers' developing understandings and practices, interactions between the participants and me were guided by the CERA model of professional development. Data was gathered from participants during all stages of the project; in the multiple contexts of their classrooms, large and small group professional development meetings, and one-on-one interviews. These occurred during the duration of this project for approximately a year.

Lee Shulman (1986), and many educational researchers since, (Grossman, 1990; Borko et al., 2000) have highlighted the importance of attending to teachers' pedagogical content knowledge, in addition to their knowledge of the subject matter that they teach, when trying to research teacher learning and practice. Consequently, this project examined teachers' understandings and practices regarding content, pedagogy, and technology and tracked development in these areas over time. Joan Hughes (1998) discusses that a diverse and complex combination of factors impact technology using teachers' path to success. Hughes (1998) suggests "we need to better understand the learning paths for teachers in context and understand such paths' possible ramifications on teacher learning and use of technology" (p. 8) and that "attention to the professional landscape, including relationships among people, contexts, and tools, and its influence on individual teacher's technology learning will tell a better story about the issues and factors that influenced these teachers' paths to technology success" (p. 9). It is the goal and purpose of this dissertation project to understand and articulate the developing understandings and practices of participating teachers regarding TSIL and meet this research need.

In order to develop these kinds of rich cases of teacher's developmental trajectories of technology integration, within-case and cross-case analysis of all sources of data will be done for all teachers. Within-case analysis seeks to describe and explain what occurs in a single case. Cross-case analysis compares multiple cases in order to deepen understanding of explanation as well as to establish the relevance or applicability of findings in one setting to similar settings (Miles and Huberman, 1984). Within-case analysis addressed research questions 1, 1a, 2, and 2a. Cross-case analysis addressed research questions 1c, 2, 2a, 2b, and 2c.

Qualitative analysis was guided by methods developed by Spradley (1980) and Miles and Huberman (1984). These methodologists have demonstrated the technique of constant comparative method, which when applied to this study, implied that the transcripts from my multiple observations and interviews were read and notes were taken that related these transcripts to factors associated with teachers' understandings and practices regarding content, pedagogy, and technology.

Spradley (1980) outlines a "developmental research sequence" that begins with organizing these notes into domains, or categories, that contain all of the units of data. Within categories, relationships among data units are organized in a taxonomy. Dimensions along which those elements of a given domain differ are constantly analyzed and compared with one another in order to ascertain the meanings of those elements. Via this process, themes emerged related to teachers' individual and group understandings and practices regarding content, pedagogy, and technology. These emergent themes included such things as their general perspectives on each of these educational topics and specific issues related individual students or classes. These themes were organized into case descriptions that demonstrated the important relationships between these teachers' understandings and practices in the areas of content, pedagogy, and technology and the criteria of effective learning environments, and are the focus of the last two chapters of this dissertation. It was the TDE methodology, as guided by the CERA practical framework model, which guided the ongoing interactions between the participants and myself and allowed continued opportunity to collect data and refine hypothesis regarding individual and group development and the effect of their social context.

> The TDE provides a dual perspective on teacher development by coordinating analyses of individual and group development. The latter is accomplished through group professional development and the former through adaptation of the individual case study. Both components involve the coordination of social and psychological analyses. The whole class (cross-case analysis) component entails looking at individuals' conceptions as well as the development of social practices. Likewise, the case study (within-case analysis) requires making sense of the social context within which the individual development occurs (Simon, 1999, p. 347).

The TDE methodology, therefore, provided a well-articulated model for gathering data related to teachers' developing understandings and practices. This data was collected and analyzed as discussed in the sections above. The CERA practical framework provided a model upon which sustained opportunity to support and research teacher learning and practice was provided. Together, the CERA practical framework, and the TDE methodology, allowed me to answer the research questions of this study and contribute greater understanding of ways that teachers' come to learn and integrate TSIL into their instruction and curriculum. Resulting case studies are shared in Chapter Four and the research questions are answered and discussed in Chapter Five.

CHAPTER SUMMARY

This project aims to examine the following research questions:

1. What are teachers' understandings as they learn about, practice with, and reflect upon technology-supported inquiry learning?
    a. What are the participating teachers' understandings regarding mathematics and statistics content?
    b. What are the participating teachers' understandings regarding pedagogy?
    c. What are the participating teachers' understandings regarding technology?

2. What do the instructional practices look like for teachers who are trying to incorporate TSIL within their classrooms?
    a. What do teachers' practices look like as they incorporate Fathom into their teaching?
    b. What are similarities and differences regarding teachers' practices involving TSIL with Fathom?

These are the types of questions that are often addressed and answered when one conducts a teacher development experiment. A teacher development experiment seeks to systematically sustain and measure the effects of teacher professional development. In this TDE dissertation study, I provided opportunity for teachers to learn about and practice integrating technology-supported, inquiry learning pedagogy utilizing Fathom educational technology. The impact of this professional development opportunity was measured by answering the research questions above about teachers' understandings and practices regarding content, pedagogy, and technology.

In order to answer these questions, close interaction occurred over a sustained duration of time between the participants and myself. The CERA practical framework model provided a model for this sustained interaction to occur. By Collaborating with teachers via observations and interviews as they learned, thought about, and discussed TSIL with Fathom, I was privy to their thoughts and ideas regarding this pedagogical philosophy and tool. I was present in teachers' classrooms as they Enacted TSIL utilizing Fathom and as they Reflected on the influence of this pedagogy. Finally, I asked them to assess the ways the have Adapted their understandings and practices regarding content, pedagogy, and technology during the course of this project.

Systematic analysis of the data that I collected during the duration of this TDE dissertation project, as guided by the CERA model, allowed me to respond to the questions of this research study. I have evidence to answer what changes in teachers' understandings and practices regarding TSIL have occurred via this professional development initiative and the importance and implications of these findings. These are shared next.

CHAPTER FOUR

RESULTS AND DISCUSSION

INTRODUCTION

This chapter introduces three teachers who have participated in this dissertation project, and describes their technology integration efforts in relation to the prior research on effective learning environments, which was discussed in Chapter Two. First, information about the contexts in which each teacher works and a vignette, based on data gathered from observations of their efforts to integrate Fathom learning technology into their practice will be shared. Next, their technology integration efforts, as exemplified by these vignettes, and supported by interview, observation, and survey data gathered during this study regarding their understandings and practices about content, pedagogy, and technology will be discussed in relation to the four components of effective learning environments; how they are learner-centered, knowledge-centered, assessment-centered, and community-centered.

The four components of effective learning environments provide a good framework to assess the strengths and weaknesses in these teachers developing understandings and practices of technology supported inquiry learning (TSIL, described in Chapter One). By discussing the ways in which these teachers utilize technology to meet, and at times fall short of enacting, what research informs us about the components of effective learning environments, a clear vision of how important each of these components is towards exemplary technology integration, and a picture of what exemplary integration of technology can and should look like, will be presented. The results and discussions shared in this chapter about each teachers technology use will be continued and expanded in the next chapter when this information is used to shed light on the research questions of this study, and final conclusions and implications of this study for teacher practice and professional development are presented.

PARTICIPANTS

Rory, Cyrus, and Patricia were three of six practicing high school mathematics teachers who participated in this professional development project. These three teachers understandings and practices surrounding the integration of Fathom into their curriculum and instruction highlight important aspects of educational practice and are examined to depth in this chapter. While all of these teachers have varied experience teaching mathematics and utilizing technology, as introduced in Table 3 below, each was interested enough in technology to spend part of their summer learning Fathom and allow me to survey, observe, and interview them about their technology integration practices.

Name	Age	Gender	Years Teaching	Subjects Currently Teaching
Rory	51	Male	18	Pre-Calculus, AP Calculus, AP Statistics, Geometry.
Cyrus	54	Male	7	Pre-Calculus, Algebra 2, Calculus, Math Alive 3 (a manipulative based Pre-Algebra class).
Patricia	31	Female	8	AP Statistics, Geometry, Pre-Calculus, Math Alive 3

Table 3: An introduction to the three case study participants discussed in this chapter.

Prior to participating in this professional development project, these teachers had all used technology personally for several years and had some experience using technology in educational settings, as demonstrated in Tables 4 and 5 below. While all of these teachers have owned a computer for several years, there use of technology in instruction was mostly limited to graphing calculator use. Cyrus and Rory had investigated computer mathematics education software before, yet they had very rarely utilized computers during instruction, (Cyrus had investigated Derive and used it during one lesson two years ago, Rory had been briefly introduced to Maple, Geometer's Sketchpad, and Fathom before but had not used them instructionally) while Patricia had not explored mathematics education software.

For how many years have you had the following experiences?[1]	**Rory**	**Cyrus**	**Patricia**
Years using computers yourself in any way	10	12	4
Years using computers yourself almost every day	7	12	4
Years assigning computer activities to students	0	0	0
Years having students use graphing calculators	10	8	4

Table 4: Participants' survey responses regarding experiences with technology.

Rory	**Cyrus**	**Patricia**
Has used graphing calculators often and fluidly for many years in instruction. Has been introduced to Fathom and Sketchpad previously but has not used them instructionally.	Has used tech for his own use for many years but after trying it in instruction earlier in his career has been using it very rarely with students in recent years.	Has used mathematics technology personally and in instruction for the past four years. This has been mostly limited to allowing students to use graphing calculators.

Table 5: Summary of participants' experiences with technology.

Rory had access to computer hardware and software at his school prior to this professional development project. His school had a couple of computer labs available

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[1] This item came from Becker, 2000.

for teachers to check out and use with their classes. Cyrus and Patricia’s school did not have a computer lab available for teachers prior to this project.

All three teachers spent three days during the summer of 2002, July 7-9, learning how to use Fathom and participating in and discussing Fathom-based mathematics and statistics activities in the areas of Exploratory Data Analysis, Mathematical Modeling, and Probability and Sampling Distributions and Inference. These areas were selected because they are commonly acknowledged as crucial components of the subject and practice of statistics (ASA, 2003; Sheaffer, 1998) and because Fathom lends itself well to the development of these ideas.

Rory and Patricia also attended three more days of professional development two weeks later, July 21-23, in which they read about and discussed technology-supported inquiry learning. During these latter three days they were provided the opportunity to think about, discuss, reflect upon, and begin planning when and how they could integrate Fathom into their curriculum and instruction during the subsequent school year. Cyrus was not able to attend this second week of professional development, choosing to go camping instead.

These three teachers are further introduced below. The background and instructional context for each, and a vignette that represents their technology integration efforts, is shared. These vignettes will later be supplemented by survey, observation, and interview data, and these results will together serve to frame a discussion in the remainder of this chapter about how these teachers understandings and practices regarding technology, content, and pedagogy meet the criteria of effective learning environments.

These three teachers, as will be demonstrated, represent weak, developing, and strong integrators of Fathom data analysis software. In early analysis of the six teachers, two teachers were categorized as strong, two developing, and two weak Fathom integrators. I decided to develop full case studies of these three teachers over the other three teachers because I wanted a case study in each group, weak, developing, and strong, to be shared and discussed. Each of these three teachers was more forthcoming and articulate with their thoughts and feelings during interviews, which made for richer case studies and, hence, were the ones included as full cases. Cyrus, even though he did not attend the last three days of professional development, was still included as a case study because, as will be discussed later in this chapter and the next, his understandings and practices align with what research informs us about many other teachers and, thus, provides a basis for interesting and meaningful discussions and conclusions.

Rory

Rory is a fifty one year old white male. Rory attended a major western university, where he was on the diving team and majored in applied mathematics. After graduating college, he spent several years doing seasonal outdoor work such as leading kayaking and hiking trips before deciding to return back to school and obtain his teaching license. He began his teaching career at the age of thirty-three and has now taught high school mathematics for eighteen years. He has taught in the same city in northern New Mexico for fifteen years; eleven at the old high school and the last four at the new high school, where he is the head of the mathematics department. He is still an avid outdoorsman and enjoys running, bicycling, and kayaking.

Rory has taught AP calculus courses for a dozen years and AP Statistics for eight years. He leads summer workshops for AP Statistics in the southwestern United States, in which he helps other teachers to better understand and prepare their students for the AP Statistics course and examination. This year, Rory teaches 10th - 12th grade students the subjects of Geometry, Pre-Calculus, AP Calculus, and AP Statistics. His classes have between twenty-two and thirty students in each class. His calculus class meets on a block schedule, while his other classes do not.

The following vignette serves as an example of Rory's use of Fathom technology. This vignette, and the ones that come next describing Cyrus' and Patricia's practices with Fathom, comes directly from actual classroom observations. The vignettes describe, in narrative form, what the teacher and the students were doing during a class in which Fathom was incorporated. These vignettes provide a picture of what these teachers' classrooms looked and felt like during times that Fathom was utilized and raise several key issues with respect to effective learning environments will be supported with additional observation, interview, and survey data and discussed later in the chapter.

Rory has a computer and projector set up at the front of class to demonstrate how to open Fathom and begin to explore the effect that different variables have on equations within various families of functions. Before he releases the students to investigate on their own computers, which he has collected himself and dispersed around the outside of his classroom, he has the students watch what he is doing and 'speculate' what happens when different values of a and of p are used in the equation $y=ax^p$.

All of the students in the class are carefully watching Rory's demonstration and appear to be thinking about his question. One student quickly offers that when p=2, a parabola is formed. Another student states that this is true for any even exponent. Rory asks whether everyone agrees with this speculation. Most students show that they do by nodding their heads up and down and saying yes. Rory asks if this is true for negative even exponents?

Students think about this for a minute. Again, all of the students appear on task, many jotting down ideas in their notebooks. A different student offers that a negative even exponent would form a hyperbola. Rory demonstrates that this is in fact true by plotting some positive and negative even values for p in Fathom on the projected computer.

Another student excitedly offers that a cubic equation looks like this (he traces his hand in the air, snaking down from the top left to the bottom right). Yet another student says that all odd integers look similar to that.

Rory asks them to also think about negative values and non-integer values. He says that he wants them to 'be in the habit of conjecturing'. 'Hopefully messing with the computer will give you a better feel.' He says that they need to be able to describe in words what happens when different values are used for the variables in the equations.

He mentions that they would not be able to do this investigation as neatly on a calculator and demonstrates how the sliders in Fathom allow one to fluidly investigate this problem on the computer.

Rory says that on their own computer he wants the students to look at families of functions (power, exp, trig) and see what happens on the graph as the parameters vary. 'What are the properties of this family?' 'I want you thinking about what you are going to see before you do it.' He points out their assignment, which is written on the board, that for each function they are to address the concavity, rate of change, increasing or decreasing nature, and shifts that occur when different values are used for the variables in the functions. He wants them to complete write-ups of what they investigate and learn.

Students quickly move over to the computers that are arranged around the back and sides of the classroom. The students don't take very long to open Fathom on their computers and to begin investigating these families of functions. As students work on this task, Rory walks amongst the groups to check on their progress and ask them about their problem solving strategies and the results they are finding.

One group makes one slider and investigates one variable at a time. Another group immediately makes multiple sliders that they can manipulate in order to look at all of the different variables. As Rory interacts with the groups he doesn't tell them that one way is correct and one is incorrect. He is looking to make sure that each group is systematically investigating the effects of different variables on the shape of the graphs and that they are gathering the kind of data they need to complete their task. He asks them to be sure to investigate each of the properties they were asked to examine and to summarize what they are finding in writing.

It will be demonstrated throughout the remainder of this chapter that Rory's technology integration understandings and practices strongly align with all of the components of effective learning environments.

Cyrus

After dropping out of high school due to a dispute with his mathematics teacher, Cyrus worked in the construction business, where he eventually owned and operated his own small business. At the age of 39, he decided that he could not do this job forever due to the fact that his body was aging and he had no money for retirement. He decided to fulfill a promise he had made more than twenty years before during his argument with his mathematics teacher, that he would himself teach mathematics one day. He went back to school part time for four years, while continuing to work full time, and earned an Associate Degree at a New Mexico community college. He then went on to get his teaching certification from a major New Mexico university in secondary mathematics education.

Cyrus has now taught high school mathematics for eight years. He has taught all eight years of his career at the same high school, where he is in his fourth year as the head of the mathematics department. This school has a reputation as a high crime, low achievement population and Cyrus has been working to change this notion by becoming involved with many professional development initiatives around the state and encouraging other teachers at the school to do the same. This year, Cyrus teaches 9th - 12th grade Math Alive 3, Algebra 2, Pre-Calculus, and AP Calculus. His classes have between fifteen and thirty students in each class of "low to medium to high ability (survey, 12/15/02)".

Cyrus did not attend the three additional days of professional development that was focused on technology integration. As a result of not participating in these discussions, it will be argued that his conceptions of the role of technology were not challenged. Although Cyrus personally enjoyed learning about Fathom software, after doing so he still had only a very vague idea of when, why, and how he would utilize Fathom in his curriculum and instruction (interview, 7/29/02).

The following vignette serves as an example of Cyrus' use of Fathom technology and raises several key issues that will be discussed later with respect to effective learning environments:

Cyrus takes attendance in his classroom and then announces, "today you are going to go into the computer lab and use Fathom, a statistical program". He asks the students if they remember when they made stem and leaf plots with their test scores the other day. A few students nod their head. He says that they will not make stem and leaf charts on Fathom but that they will make ball charts. He says that if you turn a stem and leaf sideways it looks like a ball chart or a histogram. He draws a diagram of the board of some x's in rows as if they were the values in a stem and leaf chart and then, while pointing at the graph he has just made, uses his arm to motion that he is rotating this graph ninety degrees. He then draws a new graph that shows the same amount of x's now in columns and calls it a histogram. Most students have distant looks on their faces as Cyrus lectures to the class and draws on the board.

Cyrus tells the students to leave their personal belongings in the classroom because they are going to go to the computer lab and that he will lock the classroom *door. Students slowly get up and file towards the computer lab. As they are walking into the computer lab he instructs them "not to touch anything". Once there, Cyrus tells the students to pay attention when they get in the computer lab. As the students are settling into their seats behind the computers, Cyrus hands out a worksheet on exploring census data from Data in Depth, a workbook of Fathom activities, and says to "follow the directions".*

The students have trouble logging onto the computers and finding the census data files that they are supposed to work with, and look as though they are in no particular hurry to do so. If and when they ask Cyrus how to open Fathom he says, "you guys have to learn how to read!" Cyrus is having difficulty logging on himself so I help him and his students log in and find the census data files. Once everyone has found a census data set that they are interested in, Cyrus again instructs them to "follow the directions on their worksheet".

Some students work on the assignment as per the instructions, most just talk with each other about individuals in the data set (i.e. what race they are, what their income is, what their profession is). Cyrus stands around and watches, occasionally trying to help students to understand what to do or interpret what they are looking at, which he does by asking them to carefully read and follow the directions. He often does not know how to do the things students are asking him about (i.e. how to make certain types of graphs or to look at the bottom left hand corner of the screen in order to receive information about the graphs).

After 30 minutes of intermittent activity, Cyrus tells the students "make sure you finish answering the questions and have your name on the paper". They are *instructed to return to their regular classroom, where he says, "Hush. It's still my turn to talk. Homework is section 2.3 from your book. Make sure your name is on the classwork and turn it in." He asks the students "did you enjoy it"? Several students say "yes. He asks them "did you learn"? A couple of students respond "yes". He asks them "what did you learn". One girl says she learned that "there were more boys than girls" where her data was from. Most students have not answered any of his questions and do not appear to be engaged in the discussion. Cyrus asks if anyone was frustrated and there are no answers. He asks if they like the software. A few heads nod and then Cyrus says that it is "a pretty slick statistics software". The bell rings. He has them sit down and be quiet and then lets them go.*

It will be demonstrated throughout the remainder of this chapter that Cyrus' technology integration understandings and practices weakly align with the components of effective learning environments.

Patricia

Patricia, a thirty-one year old Hispanic American female, was born and raised in northern New Mexico. She went to college at a major New Mexico university and has been teaching mathematics since she graduated college eight years ago. She has been teaching all eight years at the same school that Cyrus teaches at. She currently teaches AP Statistics, Geometry, Algebra II, and Math Alive 3. She really cares about the students she works with and strives to get to know them as much as possible. She coaches tennis and chaperones dances. She made it a goal this year to learn how to integrate technology-based and hands-on activities into her teaching.

The following vignette serves as an example of Patricia's use of Fathom technology and raises several key issues that will be discussed later with respect to effective learning environments:

Patricia greets each of her students as they enter her class and after the bell rings says to "turn to page 74 on your Workshop Statistics homework. You are going to start on question 3 on your worksheet, which deals with mean and median." Patricia introduces the task by saying "remember when we did box plots for the midwestern and western states and their was lots of yelling? We are going to do that again." Most of the students say yes they remember and are excited when she says "go to the computer lab. Don't touch the computers. Pick one. Bring your worksheet. Leave all of your other stuff." They walk across the hall to the computer lab.

Students pass around two floppy disks that I have saved the data onto and load it onto each of their computers and begin on their assignment, which uses box plots to examine and data about movies that were voted by American Film Institute as the Top 100 American movies of all time. (Before class as Patricia was getting ready to download the data from the internet she discovered that most of the computers were not able to download data because of a firewall, so I hurriedly found one that did and saved it onto the disks.)

Students get to work quickly and work steadily on the assignment investigating the problems and discussing them with one another and recording their answers. As the students work, Patricia talks with the student aides about the new Harry Potter movie that just premiered in town. She asks the students to work *through part c of the worksheet and says that they will discuss the questions after they have all gotten to that point. Occasionally, students will ask her questions like, "Mrs. Gutierez, what is the mode?" She responds by asking back "I don't know, what is the mode?" Student:"I don't know". Patricia:"See if you can figure it out."*

After twenty-five minutes Patricia announces to the class "we are going to have a little extra credit contest. Part c says to compare and contrast the two box plots." Patricia randomly puts the students into groups by counting the students off by fours and announces that they are "to compare each box plot and use the best language, vocabulary, and description" that they can and "then she will decide which is the best answer".

Each group jumps right in and gets busy working on their interpretations of the box plots. Patricia again does not walk around to visit the groups while they are working. She waits about seven minutes and then tells them they have three more minutes to finish their write-ups and then they are to share them with the class.

Patricia asks who wants to go first. After groups volunteer, she asks everyone to be quiet and listen. Several groups compare the properties of the box plots (i.e. the maximum and minimum values, and the median are higher in box plot B than in box plot A) and therefore conclude that the movies that have won Oscars are lower in age. All of the groups participate without any extra prodding. Patricia then asks if they have any questions. One student asks if all the Academy Award winners are on the Top 100 list. Patricia says probably and another student says probably not. She asks him why and he explains that a second place movie one year might be better than a first place movie another year. Patricia nods her head in agreement and then *announces "OK, go back to class now". Once they have returned to the classroom she asks "did you learn about comparing box plots today?" Students nod their head and say yes they did as they are turning off their computers and heading back to their classroom.*

Back in the classroom she asked students how they liked it. The majority of the students raise their hands to answer. Among the comments, some say that they didn't like that they couldn't download data off of the internet. They did like that the computer was able to help them do the calculations. They said that it was easy to move the data around and put it in the correct place. Some liked the data they investigated and some didn't. Some think Fathom is confusing and some think it is easy. Patricia asks students what they learned about mean and median? A student comments that they vary according to outliers. She nods her head and asks the students if they found an outlier and they say yes. Patricia says "we'll continue to work on the computer tomorrow but I'll make sure the aids have it set up so we don't have to go through that again (not having access to the data on all the computers)."

It will be demonstrated throughout the remainder of this chapter that Patricia's technology integration developing understandings and practices do not consistently align with the components of effective learning environments, meeting some criteria while falling short on other aspects.

EFFECTIVE LEARNING ENVIRONMENTS

Recently, prior research on cognition and learning, as discussed in Chapter Two, has been synthesized in such a way as to focus on four components of effective learning environments (NRC, 2000). These four aspects, which are fundamental towards the successful development of any learning environment, are learner, knowledge, assessment, and community. These components are explored below. For each tenet, research and theory based perspectives from *How People Learn* (NRC, 2000) and other literature are given, and evidence is provided regarding how well each of the highlighted participating teachers' understandings and practices regarding content, technology, and pedagogy meet these effective learning environment criteria are shared.

These components of effective learning environments are not mutually exclusive. There is alignment and overlap among the four perspectives of learning environments (see Figure 7). "They all have the potential to overlap and mutually influence one another (NRC, 2000, p. 154)."

Figure 7. Effective Learning Environment Framework (NRC, 2000, p. 134)

For this reason, within the sections on learner, knowledge, assessment, and community, data presented within one tenet may also relate to another tenet and be discussed again later. However, whichever component data about content, technology, and pedagogy is presented in, important insights into practice will be revealed. Throughout this chapter, comparisons between the theoretical connections between TSIL and effective learning environments posited in Chapter Two (Table 1) will be compared to teachers' actual technology integration practices and understandings found during this project. The final chapter of this dissertation will offer conclusions and implications regarding the important contributions that these results and discussion of the relationship among effective learning environments, teachers' understandings and practices regarding content, technology, and pedagogy, and teachers' technology integration practices provide for teacher practice and professional development.

Learner-Centered

Learner centered aspects of effective teaching environments pay careful attention to the knowledge, skills, attitudes, and beliefs that learners bring to the educational setting. Theodore Sizer (1992) states "it is a truism that we learn well only when we are engaged. That is, if we do not pay attention, we will not 'get it'. Our attention is caught by things that interest us, that so intrigue us, that we are compelled to find out more about them, that we believe we had better attend to or we might miss something (p. 85)". One theme that this quote suggests, and is explored in this section, is the ways in which these three teachers strive to actively engage their students in learning.

For a century, researchers have been advising that for effective learning to occur, education needs to take into account the fact that all we come to "know" is filtered through our own identities, experiences, and perspectives (Greeno, 1998;

John Dewey, 1902). Students build new knowledge and understanding on what they already know and believe. Students formulate new knowledge by modifying and refining their current concepts and by adding new concepts to what they already know (Lave, 1988). Another theme that is explored in this section is the way that these participating teachers build on their students' prior knowledge and experience during technology supported learning activities.

These two learner-centered themes of actively engaging students and connecting their learning to prior knowledge, experiences, and interests are explored herein by drawing on the vignettes introduced above, and supporting these pictures of teachers' practice with data gathered via surveys, observations, and interviews regarding their understandings and practices about content, pedagogy, and technology. While some of the data presented also relates to other overlapping components of effective learning environments, the focus of this section is the way that these teachers' practices relate to learner-centered aspects of effective learning environments.

Rory

Rory strives to cultivate student attention and interest in mathematics and statistics. His passion for the subject and for getting students involved in learning the subject were very visible whenever I visited his classroom and subsequently talked with him about education. He models excitement about and engagement with mathematics and statistics and often shares ways that one part of mathematics relates to other parts of the discipline and to "real-life". In the vignette above this was exemplified, in part, by the way that he excitedly asked the students questions to get them thinking about the problem and then shared examples with students on the projector and continued to prompt their reflection and participation.

Rory gives his students a sense that what they are doing is important, to him and for them. In the vignette above, when students were in groups, he actively hopped from group to group to ask them probing questions, which let them know that he cared about what they were doing and thinking, and furthered their understanding. When asked about his beliefs about learners, he reflected on the importance of the teacher being energetic about the subject (Interview, 8/31/02).

> To me, you've got to have a passion for what you're doing! If you're not excited about it or hyper about it then why in the hell are you going to expect some 14, 15, 16, 17 year old kid to want to study statistics? I mean, why would they? Pretty sick, you know, it's like they said, 'get a life'!
> I like statistics. I just think statistics is neat and I have a high-energy class. I think you have to be willing to show a passion for what you're doing.

The energetic classroom environment that Rory fosters encourages student interest, participation, and engagement.

Another important way that Rory focuses on actively engaging his students and developing their interest in learning is by working to make connections between the content and students' lives. Many times that I was in his class, I witnessed Rory facilitating discussions about some applications of the mathematics and statistics that they were studying to other things that the students had studied in their math class, or in other classes, or to the real world. After one class period when he had spent time talking about what osteoporosis was because students had questions about that data, and also had one student who knew a lot about history share information about plagues since that was related to a data set they were examining, I asked him why he felt it was important to spend time having these kinds of conversations (Interview 11/15/02).

> I believe that the only way kids are going to retain stuff is if it becomes of interest to them. And if they are examining data or doing an activity that is interesting to them they are going to leave that remembering and never forgetting an idea. It is something that they can tie the concept to, those ideas stick in their head a lot more than just a formal math term, like a memory device.

Rory further incorporates learner-centered aspects into his practice by recognizing and building on students' prior mathematical experiences. In the vignette example above, he knew that his students had previous experience solving and graphing functions and challenged them to recall and build on their prior understandings of this concept in a flexible and general fashion by thinking about functions within families rather than as individual equations. The ways that Rory identifies key ideas that he wants his students to know, and works to build an environment in his classroom for students to develop these understandings, will be further discussed in the sections regarding the other components of effective learning environments. They are first introduced here because having clear goals for actively engaging student learning, and understanding, respecting, and building on what they know and bring to the learning situation are important aspects of learner-centered environments. At this point, what is important to note is that all of Rory's practice is developed with furthering students' mathematics interest, engagement, and understanding in mind.

Cyrus

Cyrus also discussed the importance of getting students engaged in learning. However, while he speaks about the having the desire of fostering student excitement, enthusiasm, and engagement when he is interviewed, Cyrus did not appear to succeed in enacting this during his observed teaching. He keeps a very rigid structure in his classes, which stifles student involvement. Cyrus described his struggle to get students engaged, while still covering all the content that is expected (Interview 5/1/02):

> Every day I'm looking to get the kids engaged. So, my biggest problem, the thing I think about the most is how to get somebody tuned in and paying attention and trying to learn the material. And I don't do that well. I'm a little old fashioned in that I'm kind of stuck on the book. I have been ever since I started. I mean, they tell me that I'm supposed to teach them this, this, this, this, this, and that. And so that's what I try and teach them. And I haven't found good ways to teach exactly what it is I'm supposed to teach them that's different than what's right there in front of me and what they've got to take home to work with in the book. I've been trying for the last 7 years, ever since I've been a teacher, to find other ways to do this that I'm comfortable with.

Cyrus chose to participate in this Fathom professional development project due to his goal of finding other ways to help get students engaged in the material. He made time over the summer to learn how to use Fathom with the hope that this tool would help him engage his students. He did not, however, take time to come to the second week of professional development and critically think about the nuances of integrating this technology in a student-centered way. Subsequently, when he did use Fathom with his students, as exemplified in the vignette above, it did not satisfy learner-centered aspects of effective learning environments. Although Cyrus espouses the goal of cultivating student engagement, he regularly expresses disappointment in his ability to do so. When asked about the activity described in the vignette, he felt that students were engaged and learning. However, there was no discussion of what a census is or why it might be of interest to them to explore, and very little about how this activity related to their lives inside or outside of school. The students did enjoy the break from their normal routine (described more later) but were not really engaged in learning.

Besides not working to actively engage the students in the technology-based activity, the first theme in this section, their work did not fit with their usual curriculum and instruction and therefore did not satisfy the second theme either. When Cyrus' students used Fathom, it did not build on their prior experiences and understandings. These activities contrasted greatly with he and his students' normal classroom routine. Cyrus' classroom is usually very structured. Students are expected to come to class with their homework completed and ready to ask questions about any problems they may have had difficulty with. Students generally have an opportunity to ask questions on the homework for the first ten to twenty minutes of each class. After reviewing the previous night's homework, Cyrus will then give a lecture about the next section of the book, and conclude by using any time left over for guided practice on the next homework assignment (Interview, 5/1/02; Observations, 5/1/02, 10/10/02). When asked to discuss why he chose to have his students participate in Fathom computer activities he responded (Interview, 11/14/02):

> I was interested in trying to incorporate technology into my teaching and this project gave me an opportunity to do so. I sat down and calculated that if every section of book took one day, then we had twelve days left over and that is why I felt alright about doing fathom activities some of these times. I would love to do more of it (technology activities), but in order to do more of it we have to tack more days on the year. If we had class twelve months of the year instead of nine months of the year and the same curriculum that we have, I could do it and I think the kids would walk out of there with a real bang up idea of what it is they are doing and how to do it. But

> trying to cover everything it is that I need to cover and find time to do this kind of stuff, if I hadn't said I'd do this with you, they wouldn't be in there.

The content and structure of the activity described by the vignette, and of other technology based lessons, deviated drastically from what students did before or after being on the computer. The students were never told if or how their worksheets would be assessed, just to do it and turn it in, nor were they told why they were even working on the worksheets, and their end of class homework assignments were not related to what did on the computer. In the context of his curriculum and instruction, the vignette exemplified how Cyrus' educational technology use was not learner-centered, since it did not engage students in connecting these activities to students' prior content learning, classroom routine, or to their lives.

Patricia

Patricia talked about the importance of her role in modeling enthusiasm about the subject being taught and the methods being used. She reflects that she has been energized by participating in this professional development about TSIL with Fathom, and her subsequent use of this pedagogy in her practice (10/15/02).

> Well, I'm more enthused about teaching now than I was before I learned to integrate inquiry learning, and that shows. Whatever teaching method you are using, if you are enthused about the subject and the method, then it's going to show. I'm feeling great. It's really working out. They like it. They get more enthused about mathematics this year than last year. They understand it well.

Patricia strives to engage and motivate her students to learn. She tries to get to know her students and make them feel comfortable being in school and feel that it is important for them to be there (discussed more in community-centered section). She utilizes Fathom in her curriculum and instruction more than any of the participants and feels that the Fathom data sets, like the one in the activity described in the vignette above, interest students enough to motivate them to think about and try to understand the information they are investigating. In this sense, she meets the first theme of learner-centered environments, she actively engages her students. Students are clearly comfortable with Patricia. Every time I visit her classroom between classes or after school there are students hanging around. The fact that Patricia did not interact closely with the students as they were doing these technology-based activities, however, will be examined during the discussion of the other components of effective learning environments later in this chapter.

Patricia organized her course around what she feels are the big ideas of AP Statistics and frequently provided opportunities for students to explore activities from the textbook *Workshop Statistics with Fathom* (Rossman, 2000) that focused on these ideas (explored more in the knowledge-centered section). Each week, she would have her students spend time exploring concepts with and without Fathom. In this sense, her integration of Fathom was learner-centered. She strove to engage her students and to connect their technology-supported activities to her overall goals for student learning. However, as will be demonstrated soon, Patricia’s lack of understanding of, and focus on, content interfered with her ability to deeply engage student learning. Hence, as a result, her technology integration practices do not fully meet learner-centered criteria of effective learning environments.

Learner-Centered Discussion

It was posited in Chapter Two (Table 1) that ‘TSIL with Fathom activities give students opportunity to pursue interesting problems, and to make conjectures, gain and weigh evidence about, and consider multiple solutions to these problems. In this way, the TSIL process respects the tenets of learner-centered environments by allowing students to build off of their prior experiences and understanding and to actively be involved in constructing their own knowledge. TSIL statistical activities utilize authentic data in order to connect school to the real world that surrounds the learner’.

While TSIL with Fathom activities have the potential to create an energetic social atmosphere whereupon students can become engaged in striving to understand how mathematics and statistics relate to their world, it was discovered by examining these teachers’ understandings and practices that this can be difficult to actually do.’ The responses of the participants to the survey items[2] about pedagogy and technology help to shed light on factors that confound teachers’ ability to align with learner-centered tenets of effective learning environments (in tables 6, 7, and 8 below).

	Rory	**Cyrus**	**Patricia**
Teachers know that different approaches sometimes work for different types of students and that a mix of approaches is often the best. Between the two basic approaches shown, what percentage of lesson time do you think is best for each of these types of students? Giving students background factual knowledge and directly teaching concepts. Using active learning approaches like student discussions, projects, and presentations.			
For a “gifted” or “advanced” class	50 - 50	90 - 10	10 - 90
For a “basic” or “remedial” class	50 - 50	30 - 70	10 - 90
For an enthusiastic learner in one of your classes	50 - 50	10 - 90	10 - 90
For a slow learning and unmotivated student whom you teach	50 - 50	10 - 90	10 - 90

Table 6: Participants' survey responses regarding direct versus active learning.

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[2] These items from Becker, 2000c.

	Rory		Cyrus		Patricia	
Rate beliefs and practices regarding technology integration (From 1=Strongly Agree to 5= Strongly Disagree)	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*
I integrate computer activities into the curriculum.	4	1.5	4	4	5	2
Incorporating technology into instruction helps students learn.	1	1	3	2	1	1

Table 7: Participants' survey responses regarding their technology integration beliefs and practices.

	Rory	Cyrus	Patricia
On how many days last semester did a typical student in your class use a computer while you were teaching their class?	21-40 times (about once a week)	1-5 times	41-62 times (about two times a week)

Table 8: Participants' use of computers with students last semester.

Rory believes the appropriate balance when teaching is 50% of the time to directly teach and 50% of the time to use active learning strategies for all kinds of learners. He stated that about once a week, his students' engage in active learning activities including computer use. This sounds like a reasonable representation of his practice, in which he makes sure he spends time introducing facts and concepts, and that students have time to investigate and discuss these ideas. Rory gained confidence during the course of this project and now strongly agrees that he is able to meaningfully integrate technology in order to help students learn. He provides us an example of an energetic classroom environment that successfully integrates technology in order to meet learner-centered aspects of effective learning environments.

Cyrus does not feel that "advanced" classes should engage in active learning. He also does not believe in giving "slow", unmotivated, or enthusiastic students much background factual knowledge. It appears that no matter the type of student, they are missing out. “Advanced” students’ are not actively engaged and we witnessed how when students do engage in active learning approaches, it runs the risk of not being connected to the direct instruction they do receive. Cyrus makes sure he covers all of the content but does not feel that he knows how to integrate technology in order to help students learn. He rarely incorporated computer technology into his instruction and disagrees to the statement that he integrates computer activities into curriculum. When he did use the computers with his students, it was to take a break from the structured routine and “play around”, a phenomena that will be discussed more in later sections. He also offers conflicting evidence that he feels that technology can help students to learn. In the above survey response, he said that he agreed that incorporating technology into instruction helps students learn but in interviews he said that he was not sure about this (Interview, 10/15/02).

> I still think technology definitely has a role in math education. I’m still sketchy on what it is though. I think that if you use technology to help students have a visual picture before they know the procedures they won’t go back and learn it by hand and won’t understand it. If you wait until they know how to do something by hand first, they may never understand it and get to learn it with the technology.

His understandings and practices regarding the relationship between technology integration and students understanding of content will be further explored in subsequent sections. For now, it suffices to say that Cyrus’ is not able to integrate technology in order to meet learner-centered aspects of effective learning environments.

The fact that Patricia believes that she should only spend 10% of the time giving students background factual knowledge and directly teaching concepts further supports the premise that she misses opportunities to further her students’ understandings. Patricia's focus is not first and foremost on content. It is on making students feel comfortable. Math is the class that she teaches and wants students to know, but her first care is that they are comfortable. When I asked her after one observation why she didn't probe more about student understanding, she responded that she did not want to make them feel uncomfortable (Observation, 5/1/02).

> I didn't ask them to do that because I didn't want to get into it with them. I would have lost them. There are only some classes that you can take further, sometimes you just have to stop right there and see if they know how to do it (the procedure) and that's good enough. If I push them further they are going to push against me. They wouldn't have liked that at all.

Patricia misses opportunities to deepen students' content understanding, partially due to her pedagogical actions and partly due to her content knowledge. Both of these may be attributed to her lack of content focus, which will be further explored in the next sections of this chapter. For now, we can see that while Patricia uses computer activities most often of all of these teachers and meets some aspects of learner-centered learning environments, her understandings and practices limit her ability to do so effectively.

Participants' learner-centered understandings and practices are summarized in Tables 9 below.

Rory	Cyrus	Patricia
He aims for students to be able to make connections between mathematics content across the discipline, among other disciplines, and to "real-life" and he gives them opportunity to do so.	Although he espouses the importance of developing understandings and connections, his practice shows little opportunity for students to develop these robust understandings.	Depends on curricular resources to link content to applications. She wants her students to make these connections but does not always understand them fully herself.

Table 9: Summary of participants' learner-centered pedagogical understandings and practices.

Rory	Cyrus	Patricia
Technology supported activities develop students' conceptual understandings and inquiry abilities, which align with his overall goals and practices.	Technology based activities not connected to the "regular" instruction and curriculum that occurs in his class.	Activities are aligned with the big ideas in her course. They give students the opportunity to make connections, collaborate, and communicate.

Table 10: Summary of participants' learner-centered technological understandings and practices.

Knowledge-Centered

This section draws on data from observations, interviews, and surveys regarding participants understandings and practices about content, technology, and pedagogy in order to shed light on how their technology integration practices, as exemplified by the vignettes above, align with knowledge-centered aspects of effective learning environments. Knowledge-centered aspects of effective teaching environments help students develop well-organized bodies of knowledge and organize that knowledge so that it supports planning and strategic thinking. In these kinds of environments, students "learn their way around" a discipline (NRC, 2000). Like experts, they are able to make connections among ideas. In these kinds of learning environments, teachers help students think about the general principles or "big ideas" in a subject. When they learn new knowledge, students also learn where it applies and how. They have opportunities to practice using it in novel situations. Students should be able to use what they learn, understand major concepts, build a strong base of supporting factual information, and know how to apply their knowledge effectively. They should be able to describe a problem in detail before attempting a solution, determine what relevant information should enter the analysis of a problem, and decide which procedures can be used to generate descriptions and analyses of the problem (NRC, 2000).

Themes related to teachers ability to foster these kinds of knowledge-centered learning environments emerged from the data gathered in this project regarding participants perspectives on the disciplines of mathematics and statistics, their own understanding of the content, and their content and process goals for their students. As will be discussed at the end of this section, for teachers to be able to facilitate deep and broad knowledge-centered understandings in their students, they themselves need to have a rich, connected view of the subject and believe that they can foster these understandings in their students.

Rory

Rory's facilitation of the activity outlined in the vignette demonstrates that his instruction respects important knowledge-centered components of learning environments. An explicit part of the described activity was for students to think about and speculate on what kinds of behaviors they would expect the graphs to demonstrate and for them to go about implementing a strategy that would assist them in discovering and describing the effects of changes of the variables on the behaviors of different families of functions. This aligns with the goals of fostering organized, strategic student understanding. He also helps his students to deeply understand the "big ideas" of mathematics. The activity outlined in the vignette above happened early in the year in a calculus class. Rory not only wanted to use this opportunity to help his students review properties of families of functions but to help prepare his students to understand the fundamental theorem of calculus, derivatives, of which they would spend much of the year developing a conceptual understanding. He explained (8/31/02):

> I'd like to think they could tell you every single thing about a horizontal and vertical shift and compression but when we started playing with the power itself that led to some real differences. As soon as it went from being like squared or cubic, to the negative values and they started asking what happens and investigating the presence of the asymptotes and things like that some real differences appeared. I would like to think they would be able to describe a general exponential function so that they have it down well enough in their heads so when we start talking about how does the concavity change as the function goes, and stuff like that they can then start using their knowledge of the function already to start doing that. To me, the better they have it, the more we can start talking about advanced ideas like derivative and stuff like that. That was what I was kind of hoping the idea of the different concavities would lead into.

In this example, Rory worked to support his students' developing understanding of the "big idea" of derivative, and is another example of the way in which he meets the knowledge-centered principles of effective learning environments.

Rory's knowledge-centered goals for his students did not focus exclusively on mathematical content, but also on mathematical processes. This was undoubtedly related to the way that he himself thought about the discipline of mathematics. Rory demonstrated a robust and connected understanding of mathematics and statistics and spoke about the discipline of mathematics as a process. He believed that math is about thinking and problem solving, and determining ways that you can model and describe (5/2/02).

> I believe math is "thinking". It is trying to solve a problem by developing an approach that can be used for more than one problem and be extended to help solve different problems. It is taking a situation and being able to "model" it in such a way that you can describe the situation in a general structured manner that can be applied to other situations. I like math because it is a continuous

> challenge to attempt to solve problems and to be challenged intellectually.

True to the way that he himself thinks about mathematics as a process, when Rory was asked what was important for students to know and understand about mathematics when they finished his class, he mentioned process-oriented, problem solving goals for his students (Interview, 5/2/02).

> As a teacher it is a continuous challenge to try to teach students to learn to problem solve and how to learn to look at a problem from more than one perspective. I feel that by studying math, students can learn and build confidence in themselves and their own abilities. Life is basically a series of decisions for a person, and by learning to think mathematically and problem solve we are better prepared to make those decisions. The whole goal to me is for a person to be a critical thinker and a critical user of information - to be statistically literate.

Rory does not, however, guide his instructional goals for his students, solely on his own beliefs. He is very familiar with the Standards and Principles for School Mathematics (NCTM, 2000), as well as the guidelines articulated by College Board for the Advanced Placement courses that he teaches. He is quick to cite how these organizations hold content and process goals for student learning and that he uses these to guide his practice (Interview, 5/2/02).

> I don't claim to know what's best for teaching statistics. I have looked and seen that the statistics community has said this what we want students to know and understand and this is what we want Advanced Placement to be. I don't dare challenge it. My issue is if what they want us to teach is upper level thinking and original problem solving, then you need to simulate it and build towards it.

Rory helps his students to develop problem solving and reasoning skills by giving them opportunities to do so as much as possible. A major part of supporting students' developing understandings of mathematics content and processes is Rory's ability to engage students in investigation and communication. The pedagogical strategies that Rory utilizes in order to advance student understanding are further discussed in the assessment section of this chapter, but an essential component of his ability to facilitate student content and process understandings is related to his rich perspectives and understandings of the subject and practice of mathematics and statistics. Rory answered every content survey item correctly, and was the only one of these three teachers who did so. His understanding of the central limit theorem, and the implications of this theorem to understand variation and inference were particularly impressive. For instance, Rory was the only participant who answered the following two items[3] correctly, selecting (b) for the first question, about the Central Limit Theorem (Figure 13 below) and (d) for the second, about the normal distribution (Figure 14 below).

Which of the following statements is NOT predicted by the Central Limit Theorem?

a. A larger sample size will produce a smaller standard error for the sampling distribution.
b. The mean of the sampling distribution is equal to the population mean divided by the square root of the sample size.
c. The larger the sample size, the more the sampling distribution will resemble a normal distribution.
d. The mean of the sampling distribution for samples of size n = 15 will be the same as the mean for the sampling distribution for samples of size n = 100.

Figure 8: Content Survey Item S13 about standard deviation

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[3] These items were adopted with permission from Garfield et al. (http://www.gen.umn.edu/faculty_staff/delmas/stat_tools).

Weight is a measure that tends to be normally distributed. Suppose the mean weight of all women at a large university is 135 pounds, with a standard deviation of 12 pounds. If you were to randomly sample 9 women at the university, there would be a 68% chance that the sample mean weight would be between: *(circle one)*

a. 119 and 151 pounds.
b. 125 and 145 pounds.
c. 123 and 147 pounds.
d. 131 and 139 pounds.
e. 133 and 137 pounds.

Figure 9: Content Survey Item S14 about normal distribution.

Rory's response to the first item demonstrated that he understood the central limit theorem theoretically, while the second proved that he could apply what he knew about this formula in order to solve a complex problem.

His deep understanding of sampling distributions manifested in the way he talked about statistics with his students during class, and with me before and after he taught. He repeatedly emphasized the primary role that understanding variation played in developing an overall understanding of statistics (Interview, 7/24/02).

> I always tell the kids if somebody ever asks you what is the biggest theorem in statistics, you tell them the central limit theorem. The central limit theorem allows you to understand that there is variation, but that the variation that is there is predictable. And I always tell the kids if somebody ever asks you what the biggest idea about statistics is, say that it is that no matter what you do you have variation. 'Error, error, error', at least humor me and say it. Understanding that variation and error is inherent is essential to be able to make predictions. Decisions can be made from the data about what is a reasonable amount of variation and data analysis helps you to organize and understand the data to be able to begin to make these decisions.

With these rich personal understandings and robust content and process goals for students in mind, Rory was able to utilize the affordances of Fathom to help students to deepen their learning. He worked hard to design and structure tasks that provided opportunities for his students to develop the understandings that he articulated. Instead of only relying on pre-made Fathom activities (although many are exemplary), Rory often scavenged many resources to find just the right task, or help him to design his own (he designed the activity in his vignette on his own), to further the understandings that he felt were most important for students to develop. As witnessed by his response to the following item[4], in Table 11 below, Rory strongly agreed that he could use technology to design new learning experiences for students.

	Rory		**Cyrus**		**Patricia**	
Rate beliefs and practices regarding technology integration (From 1=Strongly Agree to 5= Strongly Disagree)	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*
I use technology to design new learning experiences for students incorporating the unique capabilities of technology.	**2**	**1**	3	4	3	3

Table 11: Participants' use of technology to design new learning experiences.

Rory discussed how he utilizes the powerful capabilities of Fathom software to help his students investigate and develop understandings (10/13/02).

> Inherent randomness and variation in data is an essential concept that I try to emphasize and weave throughout the year. Fathom really lets you explore this. Being able to rerandomize data and quickly look at repeated samples is really amazing. To be able to look at the variation is so powerful. This enhances their understanding of probability and inference. I think one of the biggest things is the ease and speed in which you can examine this important concept. Whenever we have the opportunity, I change the group that we take a measure on and investigate the variation. One of the most powerful things about Fathom is that you can simulate this and look at those distributions and see that no matter what the population looks like, you can go in and as you increase the size of your random samples, you start seeing this variation become very predictable.

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[4] This item adopted with permission from Becker (2000c).

When looking at how Rory's practice aligns with knowledge-centered aspects of effective learning environments, what really stands out is how thoughtful and thorough he is in considering the content and process goals he has for students. His stated goals for students align with those that research informs us are necessary for strong content understanding; deep, flexible, organized, and adaptable knowledge. Upon closer inspection, this seems intimately related to the rich understanding of content he has himself, and his belief that all students can succeed in achieving these understandings themselves. More about the ways that he facilitates their learning community and deepens their understandings will be explored in upcoming sections of this chapter.

Cyrus

Cyrus' technology-based instruction, as exemplified by the earlier vignette, failed to live up to the deep, organized knowledge-centered goals articulated in the research on effective learning environments. The content that the students were supposed to be investigating, the idea of making and interpreting graphs, was not part of the normal Algebra 2 curriculum, the course these students were from. Instead of investing time to have students develop deep understandings of the course in which they were enrolled, Cyrus "supplemented" their usual curriculum with this data analysis assignment because he knew that statistics was an often ignored, yet large part of the state mathematics standards. However, as mentioned earlier, without connecting this activity to their regular curriculum and instruction, the students had nowhere to hang these quickly introduced and then forgotten data analysis ideas.

While he demonstrated a pretty strong understanding of mathematics and statistics himself, answering all of the content survey questions correctly except the two demonstrated in the discussion about Rory above, Cyrus expressed dismay at how to facilitate these understandings in his students. Perhaps, related to this pedagogical dilemma is that fact that philosophically, he believes the discipline of mathematics is not as a process that students can come to understand, but one that must be discovered (7/29/02).

> Math is the recognition of patterns, the abstraction of those patterns into symbols, and the ability to make predictions and achieve greater understanding of objects and events around us through this abstraction and to make predictions based on these recognitions.
> I like it because it is challenging, because there is a great sense of order and beauty in the understanding of mathematics, because in some greater sense a get a glimpse of the divine with each new understanding.

Since he feels that mathematics is a set of divine relationships that are to be discovered rather than a process, this does not fit well with his emphasis on having students learn the disconnected facts and skills. Cyrus does not articulate to his students that he wants to help them to develop an appreciation for the beauty of mathematics and his instruction provides little opportunity for students to do so. In fact, when asked what was important for students to know and understand when they finished his course, Cyrus offered a checklist of ideas, which he admitted that didn't know how to best help his students to learn (Interview, 5/1/02).

> Students need to know the essential rules for abstraction and manipulation of mathematics, the difference between linear and non linear relationships, basic budgeting, general rules of probability, the essentials of statistics, and have a general appreciation for how the world can be abstracted, manipulated and then returned to the real world so that predictions can occur.

> I don’t know how they learn. I mean I know how I learned when I was in high school and it was easy. About every 10 minutes I’d look up at the board and go, ‘oh, okay’ and I’d go back to reading my book. When I was in college, I wrote down everything that was up on the board, I wrote down as much as what I could remember of what he said, and I went home and I did all the homework all the time. And I did okay but I couldn’t repeat most of it.

A real mismatch, as exemplified by the quote above, appeared between the way that Cyrus learned mathematics, by working on the problems by himself, and the type of instruction forwarded by the state and national standards. While Cyrus covers every section of the book, he doesn’t discuss holding the goal that students develop process-oriented, problem solving and reasoning skills. In fact, knowing that Cyrus had been involved in writing the new state mathematics standards, which emphasize these mathematical processes, I asked him what he thought about the problem solving process standards. And while he was familiar that this was a goal in the standards, he was at a loss of how he could incorporate this goal into his instruction (9/1/02).

> The new Standards are written by subject rather than by strand at the high school level. The process standards got mixed in and tacked on to each subject. Where we saw an example of how we could tack a process standard in we did so but I cannot remember an example. I have a hard time seeing how we can fit problem solving process skills into a public school. It is really hard when you are being held accountable to goals of students being required to know things in certain amount of time. I have a hard time seeing how you can fit a set list of standardized process goals and say you have met them.

Upon talking with Cyrus about the Fathom activity described in the vignette about him, it actually sounded like he had some process-oriented goals for it, but did not really know how to put them into practice, and did not devote time to really trying to integrate them in a meaningful way into his curriculum and instruction. Although he never emphasized this fact to the students during the lesson described in the vignette in Chapter Four, Cyrus said in the post observation interview that he had hoped that students would be interested in trying to investigate and understand the data. He also held the competing and contradictory goal of just wanting to let them "play" (Interview, 9/1/02).

> I have had enough people hit me over the head with 'you have to let the kids play too'. So my thought was I'll get them in the program, I'll get them the data on the thing, I'll give them some directions that they are supposed to follow, and I'll stand back and let them play. I was hoping I would have time today that I could ask them about what they actually saw with their data – did you have a lot of old folks in your population? Do you have a lot of young folks? You know, where did you pick? Why did you pick this place? Is it this someplace you want to move to? One of the girls pointed out there is more men than women where she had picked. Isn't that an interesting concept that you might consider where you might live based on how many men versus women there were. If I can get them to have some concept of what statistics can do in their life, even if they don't know how to do anything with it, I will feel like I accomplished a lot. The census data is good for that; the students were so surprised that this was real people with real answers to real questions.

So, while Cyrus did discuss some problem solving and reasoning types of ideas with me, he did not emphasize these goals to his students, nor structure the learning activity in such as a way as to develop student investigation of these ideas. Overall, the pattern emerged that while Cyrus held some strong mathematics content and process knowledge himself, he did not believe that his students could develop these rich understandings, and did not provide them with opportunity to do so. Another example of this occurred when I asked him about his views of mathematical modeling and the ways that Fathom helps to understand this concept (7/29/02).

> Modeling means that anything that goes on in the world you can take any object, any action, any motion, and describe it, and write an equation that describes or very closely approximates it. You can put it together and describe the entire world if you got it in a mathematical equation. That means you can turn it around and reapply it to the

> world. When you start talking about word problems and mathematical modeling and stuff you're really kind of getting over their heads. Well, Fathom is a fascinating program. I really like it. I'm looking forward to being able to use it. There is a lot of stuff that you can do with it. The activities themselves were very powerful in terms of showing the value of statistics. I think more than anything else the value of technology and of programs like Fathom is that it shows you and gives you some clues of what is really possible in terms of analysis and understanding something through data analysis. I love technology. I love the way it works. It really makes things helpful for me.

It is difficult to accept that Cyrus can truly know that this is over students head when he rarely gives students opportunity and support to investigate, problem solve, reason, and communicate mathematically. More details about Cyrus' perspectives on these practices follow in later sections of this chapter. For now, however, suffice it to say that while he enjoys mathematics and Fathom himself, his technology integration practices are not grounded in what research informs us about knowledge-centered aspects of effective learning environments.

Patricia

Last year, Patricia followed the same procedure as Cyrus, doing one page of the book at a time and making sure to cover every page in the book. This year, she moved from just doing the next page in the book to thinking about the big ideas in the classes she was teaching and how to get her students to understand the main concepts in these classes. For example, last year in statistics, she chronologically followed the order of the statistics book that she had been given to teach the course with. This year she is using a variety of textbooks and incorporating technology-based and hands-on activities into her AP Statistics course. She is also flexible about the amount of homework that she assigns this year, which she bases on what she wants her students to practice and when. Before, it was just give them twenty problems a night. Now, she thinks about the important concepts and picks out the most useful problems. Her main concern is that her students develop understandings of the main concepts in AP Statistics. She and her students have responded positively to this change (10/12/02).

> Rory mentioned this summer that he structures his AP Statistics course around some main ideas in the class. I thought that sounded like a good idea, so I outlined what I wanted them to learn and then found activities that would help my students to learn it. I tell them that 'computers are fun, but we are not going to do our whole class based on it. I'm trying to get you guys the best statistics education I can so that doesn't mean computer work all the time. Sometimes we are just going to have to buckle down and read the book and do homework from the book. Other times we are going to have plenty of days where we are in here in the lab, which means we will have to finish the projects in class.' I told all my classes at the beginning I'm not here to assign you homework every night, I'm here to have you learn math. So sometimes homework is necessary for you to learn math other days if we end at the end of class period it is not necessary for you to take anything home, just let it be. So, they always have that in mind and I always remind them of it. So they are excited about stuff and I think what they are really excited about is that they are learning now.

In the vignette above, Patricia, like Cyrus, had students work directly from a Fathom worksheet. Patricia's assignment, however, fit in with and was an integral part of developing student understanding of the unit on univariate data that was in the midst of being taught. Patricia reminded her students that they had previously been using box plots to investigate data and that they would be doing that in the described activity. The way in which she set the content focus of this activity was more clear than what Cyrus had done, but less focused than Rory' introduction about what students were supposed to investigate and try to understand. While Cyrus' students didn't really know what they supposed to be doing or why, Patricia's students knew they were working within their unit on univariate data and were supposed to examine and discuss the similarities and differences between the box plots of two different sets of data. But, while Rory' students were clearly focused on what attributes of functions they were to conjecture and test, Patricia's students did not know before, during, or after their activity what they were supposed to examine within these graphs or whether their understandings were correct or not.

Patricia is very interesting because she has taught secondary math for eight years but just taught AP Statistics for the first time last year. She did not understand the content very well and said it was a case of "the blind leading the blind". While she worked hard this summer to gather resources from other statistics teachers, think about the big ideas of the subject, and plan a curricular sequence to help students understand these big ideas, her lack of deep content knowledge still showed up in her instruction. Patricia's limited understanding of this topic limited the depth of questions she could ask students and, hence, limited their investigation and understanding of the topic. This was exemplified in the vignette by the way in which neither Patricia nor her students examined the intersection of movies that were in the American Film Institute Top 100 and had won Academy Awards. They only looked at the box plots of each category separately and did not explore how they related to one another. If they had, they would have easily seen that it was clearly labeled that only 32 of the Top 100 movies were Academy Award winners. Also, Patricia allowed only a very cursory mention of outliers and did not discuss the fact that outliers may or may not affect various measures of central tendency and spread of data.

While Patricia's lack of deep content knowledge limits the knowledge-centered aspects of her classroom learning environment, she still believes that her use of Fathom technology as a means to develop big ideas in the subject has helped her and her students to develop more comfort and understanding regarding AP Statistics (10/12/02).

> The inquiry nature of Fathom resources makes me more optimistic (than last year). I can go forward with more confidence in teaching the kids rather than having to say I don't know to them so many times. It enhances my instruction because it takes the burden off of me to try to explain it to them in old-fashioned lecture method because they can formulate their own ways of understanding it without me having to try and explain it when it's something that I can barely grasp myself right at this point in time. Plus it keeps the students pretty intrigued working with the computer and it helps me to learn statistics better because it takes some of that burden off me as far as just sitting there and lecturing and trying to explain it to them and then they ask a question that I don't know. I can say 'this activity kind of covers that one, why don't you go through that and maybe it might become clear'. I can go through it with them rather than trying to find some lame explanation where I know I'm not saying it right. So that's how it helps my teaching quite a bit.

While Patricia alludes to following along in the Fathom activities herself as a way to develop her own content understandings, in the vignette above, as well as in other observations, she did not participate in the activity before or during class. She was not prepared to help students engage in significant issues in these assignments and missed opportunities to further her and her students' understandings.

Patricia's understandings regarding exploratory data analysis (EDA) and variation were weak and shallow prior to the professional development project (Interview, 5/1/02).

> EDA is just looking at the data and putting it on the calculator. It involves looking at what pattern the data makes and talking about 'is it exponential, linear, quadratic or what'. Being able to do this is

> important to be able to make predictions. “To tell you the truth I really don’t understand variability quite well”.

Her instructional practices and post professional development survey responses highlighted this knowledge gap still existed after her initial learning of Fathom too, contrary to her statements above about it helping her to understand it better. The teaching episode described in the vignette demonstrated that Patricia did not understand the nuances of EDA. She was satisfied having her students discuss the surface features of box plots (i.e. which group of movies had the lower minimum or higher maximum), but did not concentrate on the fact that the goal of making these kinds of EDA graphs was to be able begin to critically examine the data. The graphs are a means to better understand the data not an end in itself.

Her course outline, structured around the big ideas of statistics, included a couple of weeks on ‘mean, median, and mode’ and many separate weeks on ‘univariate data’. Mean, median, and mode could definitely be included in any unit on univariate data and EDA techniques would facilitate understanding of these concepts. She even missed a very simple EDA question on her end of project content knowledge survey. When asked to answer the following question[5], shown in Figure 10 below, she did not stop to check the reasonableness of her answer and stated that the average weight was 64.65 grams.

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[5] This item adopted with permission from Watson et al. http://www.educ.utas.edu.au/users/watsonjm/tdg/mercindx.htm

A small object was weighed on the same scales separately by nine students in a science class. The weights (in grams) recorded by each student are shown below.

6.3 6.0 6.0 15.3 6.1 6.3 6.2 6.15 6.3

The 'average' value could be calculated in several ways.
How would you find the average? *(Please show and explain your work?)*

The average weight is _____ grams.

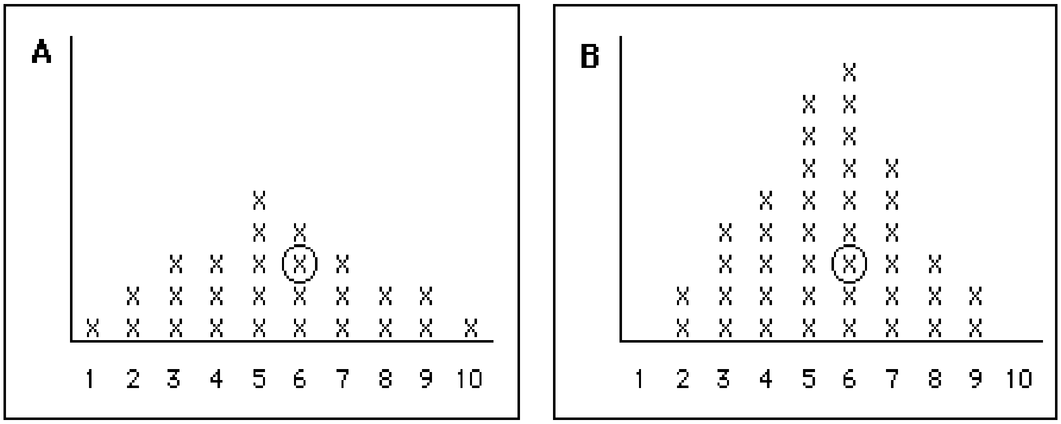
Figure 10: Content survey item S3 about average.

On her work she showed that she was finding the sum of all of the weights and dividing this sum by nine. Besides the fact that she did not take account of the obvious outlier in her solution, she also did not recognize that there is no way that the 'average' value could be a factor of ten larger than the measures that it was representing the average of.

During her initial interview, Patricia missed almost every problem that dealt with looking at sampling distributions, understanding the Central Limit Theorem, and making inferences. When interviewed and asked what was important for students to know and understand about sampling distributions and inference (Interview 5/1/02), Patricia responded "last year we did some of that type of stuff but once again we only looked at how and didn't get into the why. It's been difficult for me because I haven't taken statistics since 1990 when I was in college. All of statistics has been hard for me and totally new at this point".

Patricia still demonstrated a particularly poor understanding of the fundamental concepts of sampling distributions and inference at the end of the project. This was demonstrated in her responses to the following two survey questions[6], in which her answers showed a real lack of comprehension. In the first problem, about sampling distributions as shown in Figure 11 below, Patricia did not even recognize the fundamental difference that the second graph represented the mean of samples of size three while the first graph was the distribution of samples of size one. She said that there was (a) no difference between what is represented by the X in the two graphs. Without understanding this difference, there is no way that she could teach the central limit theorem and inference successfully. In the problem about confidence intervals, as shown in Figure 12 below, Patricia did not even know that a confidence interval is centered around the sample mean, answering (d). Patricia had not yet made it to the section on inference in the statistics course that she was instructing during this study, but it is difficult to imagine that she could successfully learn and teach inference without understanding these fundamental ideas.

In the graphs below, Figure A represents a sample of 26 weights. Figure B represents a sampling distribution of mean weights for samples of size 3. One value is circled in each distribution.



Is there a difference between what is REPRESENTED by the X circled in A and the X circled in B? *(circle the letter for only one answer)*

a. No.
b. Yes (please explain what you see as the difference)

Figure 11: Content Survey Item S9 about sampling distributions.

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[6] These items adopted with permission from Garfield et al.

Which of the following values will ALWAYS be within the 95% confidence interval limits?

a. The population mean
b. The sample mean
c. The sample size
d. The standard deviation of the sample

Figure 12: Content survey item S15 about confidence intervals.

Not only was Patricia's content knowledge weak, her perspectives on the discipline were shallow too. Whereas both Rory and Cyrus, when asked what about mathematics is important and why, share an appreciation for the way that mathematics can describe the world, Patricia does not talk about the discipline in this same manner. She only mentions that she feels that mathematics is useful in many fields and cares about her students and wants them to succeed in life (5/1/02).

> According to the New Mexico State benchmarks, students have to take Algebra 1, Geometry, and Algebra 2 to graduate high school. Math is also important because you are going to use it in life. Life statistics have also shown that how successful you are in math determines how much money you make. The more math you take the more money you make regardless if you are an engineer or not.

Patricia feels that she is helping her students to improve their life chances by helping them to feel comfortable in school and complete their mathematics courses. As discussed previously in the learner-centered section of this chapter, and more so later in the community-centered section, she does succeed in making them feel more comfortable. This section has described however, many ways that she does not succeed in meeting knowledge-centered aspects of effective learning environments. While she has made some progress from last year by structuring her course around big ideas in the subject, such as summarizing data, correlation and regression, and hypothesis testing, by not involving herself in preparing for and studying Fathom investigations and statistics content, she misses opportunities to further her and her students' content understandings. Furthermore, after the summer workshop, Patricia has focused much more on having students investigate and analyze real data using Fathom. In so doing, she has been amazed at her students' ability to reason and justify their solutions after she adjusted her pedagogy to give them more opportunity. This phenomenon will be discussed more in the in assessment-centered section of this chapter.

Knowledge-Centered Discussion

A major goal of TSIL with Fathom is to respect knowledge-centered aspects of effective learning environments by helping students develop deep conceptual statistical understandings and the empowerment to engage in statistical investigation themselves. A TSIL framework should allow students to be able to take advantage of the multiple representations and interactive graphics within the Fathom environment so that they can gain deeper understandings of statistics concepts and procedures. The goal for their statistical activity is for students to be confident and able to wade through the abundance of information that exists in this day and age, and to be able to represent, understand, and communicate the important aspects of this data.

By examining teachers' understandings and practices regarding content, technology, and pedagogy, it has been demonstrated that there are many factors that relate to teachers' ability to facilitate knowledge-centered aspects of TSIL pedagogy. In particular, teachers perspectives on the discipline of mathematics, their content knowledge, and their content and process goals for student learning all influence the level of their implementation.

Rory was demonstrated deep understandings of the content and processes in the discipline of mathematics and believed his students could successfully obtain these abilities too. He thoroughly planned how to facilitate curriculum and instruction to meet these goals and succeeded in giving a picture of what this practice can look like. Cyrus' practice fell short of knowledge-centered descriptions because although he understood the content himself, he did not feel as though his students could develop deep, connected, conceptual understandings and he did not know how to facilitate their learning. Patricia's own content knowledge and perspectives on the subject are shallow and interfere with her ability to develop thorough understandings in her students.

Tables 12, 13, and 14 below, summarize participants' knowledge-centered understandings and practices relating to their own content knowledge, pedagogy, and technology, respectively.

Rory	Cyrus	Patricia
Rich, deep, connected content knowledge. He was articulate and thoughtful about each of the big ideas that he was asked about by me. Held content and process understandings about the discipline.	Weaker content knowledge than Rory but stronger than Patricia. He is familiar with the big ideas he was prompted about, but doesn't believe that students can grasp all of these concepts. He believes that mathematics is a divine creation to be discovered, not a process to be investigated and created.	Weak content understanding with large knowledge gaps. Unconfident in her own knowledge. She is trying to develop an understanding of big ideas of statistics. She was not familiar with many of the ideas prompted about by me. She does not study the concepts before she teaches them.

Table 12: Summary of participants' knowledge-centered personal content knowledge.

Rory	Cyrus	Patricia
He discusses valuing student understanding of content and process. His practice matches his stated values	He discusses valuing student procedural and conceptual knowledge. His practice shows little evidence of helping students to connect these understandings.	She discusses valuing student understanding of content and process. Her practice strives to support these understandings although she misses opportunities to do so.

Table 13: Summary of participants' knowledge-centered pedagogical understandings and practices.

Rory	Cyrus	Patricia
Has clear goals for when, how, and why he uses technology activities, which are based on Standards.	Does not have clear goals other than to "let them play" and take a break from structured routine.	Wants the students to enjoy class and do good work. Has goal of covering the big ideas in AP Statistics.

Table 14: Summary of participants' knowledge-centered technological understandings and practices.

Assessment-Centered

Assessment-centered aspects of effective teaching environments help students learn to monitor and regulate their own learning. Students learn to question "why it is they believe what they believe, and whether there is sufficient evidence for their beliefs" (White and Frederiksen, 1998, p. 13). These environments provide students with opportunities for feedback and revision. Effective learning requires that students take control of their own learning. Students need to learn to recognize when they understand and when they need more information. Good learners articulate their own ideas, compare and contrast them with those of others, and provide reasons why they accept one point of view rather than another (NRC, 2000). They are "metacognitive", that is, they are aware and capable of monitoring and regulating their thoughts and their knowledge (APA, 1993; White and Frederiksen, 1998). As Black and William note, it is only when students are trained in, and given opportunities for self-assessment, that they "can understand the main purposes of their learning and thereby grasp what they need to do to achieve (1998, p. 143)." Engaging students in assessment of their own thinking and performance allows them to be more self-directive in planning, pursuing, monitoring, and correcting the course of their own learning. Self-assessment nurtures discovery, teamwork, communication, and conceptual connections (NRC, 1997, p. 80).

This next section draws on data from observations, interviews, and surveys regarding participants understandings and practices about content, technology, and pedagogy in order to shed light on how their technology integration practices, as exemplified by the vignettes above, align with assessment-centered aspects of effective learning environments. Important themes related to participants' assessment-centered practices emerged during data collection and analyses. How teachers planned and implemented technology-supported tasks, their understandings and practices regarding inquiry-based instruction, and their understandings and practices regarding communication in the classroom all related to how deeply teachers met assessment-centered criteria of effective learning environments.

Rory

Rory works hard to plan and facilitate tasks that provide opportunities for his students to develop deep mathematics content and process understandings. He works to strike a balance of the right amount of structure, which will allow the students room to investigate, problem solve, and reason, yet focuses what they are thinking about and leads them towards building deep, organized, and flexible understandings. For example, in the activity described in the vignette, Rory structured student investigation in a manner that directed student investigation on aspects of families of functions that he felt were important towards developing deep understandings (i.e. changes in concavity and shifts in the functions and the rates of these changes in various families), yet left them room to problem solve and reason about how they should approach this investigation (8/31/02).

> So I think it really allowed the kids to reinforce that the ideas that I had been kind of emphasizing but in the same breath, allowed them freedom in which to investigate the problems. I was pleased that there was enough structure but in the same breath allowed them openness to problem solve so they were not stymied by having to follow a cookbook.

Dynamic simulations afforded by Fathom nicely illustrate assessment-centered features of effective learning environments. They provide a medium through which students can repeatedly make conjectures, test applications, and evaluate their results. By questioning what results they believe will occur when they enact simulations, and continuing to run simulations while positing different outcomes, students are able to test and refine their developing understandings. Rory often reflected that he really latched onto this idea of having students speculate and conjecture prior to their Fathom investigations and then reflect on their conjectures afterwards during the summer project and that he worked to incorporate into his practice during the subsequent school year. He felt that this was a valuable way to help students develop their problem solving and reasoning abilities and upper level thinking. In the activity described in the vignette, Rory specifically asked students to make conjectures about the effects that changes in the values of different variables would have on the graphs of the equations that they were a part of and then to reflect on the results of their investigations. By so doing, the students participating in this activity were able to “metacognitively” assess their understandings of the characteristics of different families of functions. Rory reflected on the value of this kind of inquiry in general and about another TSIL with Fathom activity that he facilitated later in the semester specifically (11/15/02).

> I just think the stuff that we read this summer in a couple of the articles of trying to foster inquiry learning and played with this summer with Fathom is such an important idea. I think if they have an idea or conjecture of what they are going to see then they have to really think about it. And if they see it and they were right it's like reinforcement, but if they are wrong then it going to really hit them and they'll have to problem solve what was wrong and why. So I think maybe it forces them to stop as they go through material. I think a lot of kids have never been asked why and unless they are interpreting they do not really understand. So I think the inquiry can help lead to that and I'm trying to do more inquiry based instruction. This activity was really successful. It was reinforcing what they were learning but it was also addressing things they may have missed. So I think it really allowed the kids to reinforce the ideas that I had been emphasizing but in the same breath allowed them freedom to investigate the problem and deepen their understandings.

Another key aspect of Rory's assessment-centered practices was that students were not conducting individual isolated investigations, but were expected to communicate their reasoning orally and in writing. By involving students in sharing their ideas aloud with the class, modeling how to investigate conjectures, and continuing to give students opportunities to speculate and test their ideas, Rory incorporated assessment-centered tenets of effective learning environments into his practice. When ideas are exchanged and subjected to thoughtful critiques, they are often refined and improved (NRC, 2000). Research points out the fact that communication is often motivational and helps students to persist in completing tasks and striving for understanding (NCTM, 2000; CTGV, 1997). Rory builds in opportunities for he and his students to formatively assess their developing conceptual understandings. In the described activity, Rory allowed students to make and test conjectures in order to refine and develop their understanding of families of functions. Their understanding of this concept, and of other concepts throughout the course of the year, were further challenged and articulated by the focus that Rory places on communication and discussion in his classes (10/13/02).

> I always ask the students to share why. I ask them what they think? Why? What do you think is going to happen? Why does it do that? My class discussions are an interaction of questions. This interaction of questions is a way of helping students to speculate, communicate, and think. I think making conjectures in something really makes kids want to start wondering how something works and why. From looking at the write-ups and seeing their activity, I thought there was a fair amount of conjecturing and discussing back and forth and trying to explain to each other what did happen and why. So I guess that objective was accomplished and I thought it was very beneficial.

Rory values student investigation and inquiry and provides opportunities for his students to problem solve, reason, and communicate in his classroom. These practices align with assessment-centered aspects of effective learning environments by helping students to become metacognitive about their own learning. Students have opportunity to get feedback on their thinking and reflect on what they know and how they know that they know. Cultivating this community of learners takes practice and work and the community-centered section of this chapter explores more about how Rory facilitates this pedagogy in his practice.

Cyrus

Cyrus' practice, whenever I observed his class or talked with him about his practice, was either very authoritarian and drill and practice based, or, as in the vignette, open-ended and unstructured. Students are mainly expected to do their homework and "keep up" with the lecture, and on occasion can take a break to "play around". Hence, his students seldom have opportunity to develop and formatively assess their conceptual understandings. While they may be able to check whether their procedural textbook solutions are correct, without opportunity to investigate and communicate their thinking these students are not able to test, revise, and develop robust understandings. These observations were supported by Cyrus' response to the pedagogy survey item shown in Table 15 below.

	Rory	**Cyrus**	**Patricia**
About how often did students in this class part in the following types of activities?			
Do hands-on/laboratory activities	1-3 times per week	Sometimes	1-3 times per week
Work individually answering questions in the textbook or worksheets	Sometimes	Almost Everyday	Sometimes

Table 15: Participants' use of hands-on versus textbook and worksheet activities.

While Cyrus theoretically feels that there is some value to having students construct their own knowledge, he does not feel that it is practically possible or worth the time it would take (7/29/02).

> I'm intrigued with constructivism, but still have problems with it in a timeframe. Clearly when you've figured things out for yourself you understand them much better than just rote memorization and drill and practice. But I have a hard time in my mind justifying how much time it's going to take. I think a lot of the connections that you make from doing it in a constructivist point of view are ultimately made in the memorization and drill method and just not as quickly as with the rote and drill. Your understanding happens further down the road. It seems like to me that in a fifty-minute period there is hardly enough time for them to spend experimenting and have reflection and conversations and in that time frame. I would love to have the kids for twice as long every day, not every other day like a block schedule, then I could easily do constructivist teaching and really make good progress.

Given the time constraints that Cyrus shares, he bases his practice on covering one section of the book each day. There is not time nor place in this schedule for lengthy investigations or conversations. Other than asking procedural questions about the homework, communication in the form of collaborative discussion does not have a place in Cyrus' class. Communication "slows down" the pace of the class. There is a lot to do and it needs to get done (5/2/02).

> I mean I'm a business person. My time is worth money and you walk in my door and you are here to learn math and here we go, boom, boom, boom. The whole hour I'll give you math. You should be looking for it, that's what you're here for. I don't stop and I don't slow down. One of the things I discovered real early on in teaching is that the slower I went, the slower they went. You know, if I go back over a lesson, they don't learn any more the second time around then they did the first time. And so, I just keep right on bookin', you know? 'Here we go, if you guys want to walk with me, you have to keep up'.

Cyrus's answers to the survey questions below correspond to his interview excerpts above. Students rarely have the opportunity to do hands-on work in his "regular" classes. Usually they spend the majority of their time at their desks working individually.

Cyrus does not feel as though inquiry learning has a place in the constraints of his normal curriculum and instruction. He does not design tasks to facilitate student experimentation and investigation. He does not take the time for them to communicate their reasoning. He is left as the authority that doles out assignments and grades. His students spend the majority of their time individually answering questions from their textbooks and listening to his lectures. Under this pedagogy, his students cannot formatively assess their understandings or become metacognitive about their own learning in this environment. His practice, therefore, does not meet assessment-centered components of effective learning environments.

Patricia

Patricia has been working to incorporate more investigation and communication in her instruction this year. She was tired of the lecture followed by homework method of instruction that she used last year and wanted to have more variety and activity in her classes. During interviews that took place following the summer professional development, Patricia often stressed the importance of fostering student inquiry and communication as a means to develop her students' understandings of the subject (10/12/02).

> If they can investigate problems and formulate their own wording then they'll understand it. I mean, I could talk about it here until I'm blue in the face and if they don't understand what I'm talking about with my vocabulary it's not going to sink in. If they formulate it using their own language then it will stay with them longer.

Patricia tries to provide her students with opportunities to communicate their understandings, both verbally and in writing. She often assigns Fathom activities, like in the vignette above, where students must investigate data and then explain what they have found. In opening up her practice to include higher expectations of having students sharing and justifying their thinking, she has been amazed at her students' ability to do so. Last year, she would often avoid asking why questions for fear of losing student interest, effort, and understanding. This year she more frequently asks them to defend and justify their reasoning. Patricia explains the ways that she has been working to incorporate investigation and communication into her practice (11/14/02).

> I try to use a lot of why questions. Like when they say 'because of this', I'm all 'why', 'because why'. So I'm trying to refrain from actually showing them and I'm trying to get them to do a lot of it themselves, generate a lot and explain it to me. I've also been trying to do more journal entries with them, having them formally write it up on a piece of paper and say what they learned or what they liked, what

> they didn't like, and things like that. Explaining and journaling has helped them to consolidate their thinking and get clear on their opinion.

The consistency in which Patricia involves students in Fathom investigations, her efforts to have them justify their thinking, and the richness of some of these materials makes her feel as though her students are developing strong content understandings. She is sure that they know statistics more this year than last year due to their use of Fathom (11/14/02).

> I am using Fathom in conjunction with *Workshop Statistics* and it helps them a lot more than the old-fashioned textbook. When I go in there and give them a worksheet to do on Fathom there is no hesitation and their answers come back from compare and contrast, or explain what this means, pretty concise. That freaks me out half the time. I have no clue where they are getting it from although I know it has a lot to do with the computer lab. They are getting it some way, somehow.

Her last comment, that 'they are getting it some way, somehow' alludes to the fact that Patricia could be doing more to help develop her students' understandings. While she discusses the importance of facilitating investigation and communication in her practice and often attempts to do so, she often misses opportunities to focus the students' inquiry at a conceptual level. As discussed in the knowledge-centered section above, some of this is due to her limited content knowledge. Part of this is also due to her unfamiliarity with this pedagogical technique. For example, in the vignette above, Patricia chose to sit in the middle of the computer lab and discuss the new Harry Potter movie with her student aids rather than circulate around the room and see what her students were doing and thinking. When a teacher circulates around the room, they are able to learn a lot more about their students thinking and can often use questions that one student has to stimulate the thinking of the whole class. Patricia's missed opportunities here were pedagogical, not conceptual.

While she is still learning how to effectively integrate assessment-centered components into her practice, Patricia has found merit in her attempts to do so. She has been willing to try out more student-centered instructional practices, unlike Cyrus, and found that although it sometimes takes longer to allow students to develop and articulate their own understanding, the payoff is that students try harder and learn more. Patricia has seen that involving her students in investigation, collaboration, and communication result in increased motivation, interest, and understanding (10/12/02).

> Sometimes it takes longer, something that I used to be able to whip through in half an hour just showing them, now sometimes takes an hour or takes a little bit into the next day. I'm not concerning myself with it because I can always try to make up time later and it can't be any worse than what happened last year. So I figure I'll just take my time and let it all flow nice and easy and not pressure them and some days, like I told them 'you will have a whole bunch of homework you are going to have to do and other days none'. They are learning it though. It is cool to see.

Even though she just largely leaves them free to explore on their own during class before having brief discussions with them afterwards, she still feels as though her students are learning more this year than last. Her openness to allowing students to investigate, problem solve, and communicate has shown progress, and continued effort on her part to develop stronger content knowledge and pedagogical practices should help her practice continue to improve. If she prepares more for the activities before she assigns them, and attends more to students thinking during the activities, she should be able to even better help her students' understandings.

Assessment-Centered Discussion

At the broadest level, assessment of inquiry "measures the capacity of students to evaluate the kinds of questions that scientists investigate, understand the purposes of investigation, and assess the qualities of data, explanations, and arguments (NRC, 1997, p. 76). Assessment of TSIL activity should be both formative and summative and assist both the students and the teacher in determining whether students can generate and/or clarify questions; develop possible explanations; design and conduct investigations; and use data as evidence to support or reject their own explanations. Rubrics that clearly state the objectives that teachers hold for student TSIL activity and the way that these objectives are assessed should be provided to students and discussed with them before, during, and after they work on activities.

By examining teachers' understandings and practices regarding content, technology, and pedagogy, it has been demonstrated that there are many factors that relate to teachers' ability to facilitate assessment-centered aspects of TSIL pedagogy. In particular, teachers planning and implementation of tasks, their understandings and perspectives regarding inquiry-based instruction, and their understandings and practices regarding communication in the classroom all related to how deeply teachers met assessment-centered criteria of effective learning environments.

Rory valued the role of inquiry for enhanced student understanding. He took time to plan how to structure and facilitate technology-supported tasks that focused students on important ideas but allowed them room to problem solve and assess their own thinking. An important component of their ability to become metacognitive was their opportunity to communicate and receive feedback on their ideas. His practice gave us a picture of strong assessment-centered practices. Cyrus' practice was not so aligned with assessment-centered descriptions because although he theoretically valued constructivist learning, he did provide students opportunity to problem solve, reason, and assess their understandings. He did not feel as though he could take the time to plan and integrate student-centered, inquiry-based tasks in his curriculum and instruction. Patricia met some aspects of assessment-centered learning environments. She has come to value creating opportunities for students to become involved in investigation, communication, and justification and seen an increase in student understanding as a result. However, her lack of attention to content, pedagogical, and technological nuances of the tasks she assigns limits the depths of student understanding and point to areas that she must continue to improve if she is to better model effective assessment-centered teaching environments.

These assessment-centered findings are summarized in Table 16 below.

Rory	Cyrus	Patricia
Focuses on providing opportunities via structured inquiry for students to discover the outlined core understandings and processes from AP and NCTM.	Focuses on delivering content to students in daily doses from the book such that they can receive necessary skills.	Focuses on providing supportive environment for students to be comfortable and able to participate mathematically. Makes sure that she covers the big ideas in the course.

Table 16: Summary of participants assessment-centered understandings and practices.

Community-Centered

Community-centered learning environments build an atmosphere where students are students are encouraged and able to articulate their ideas, challenge those of others, and negotiate deeper meaning along with other learners. Such environments encourage people to learn from one another. They value the search for understanding and acknowledge that mistakes are a necessary ingredient if learning is to occur. Such environments are open to new ideas and ways of thinking, as the community members are both encouraged and expected to provide each other with feedback and work to incorporate new ideas into their thinking (NRC, 2000). A comfortable and welcoming community becomes the setting in which effective educational activities can be enacted. The learner, knowledge, and assessment centered aspects of effective learning environments are all necessary, yet exist within, and depend upon the facilitation of a community of learners (Figure 7). Without fostering environments that align with learners and the knowledge that is to be learned, and that invites participation, communication, and collaboration, educational activities are doomed. Therefore, some components of community-centered environments have already been discussed within other sections. Here, however, the focus is on the overall learning community.

This section draws on data from observations, interviews, and surveys regarding participants' understandings and practices about content, technology, and pedagogy in order to shed light on how their technology integration practices, as exemplified by the vignettes above, align with community-centered aspects of effective learning environments. Important themes related to participants' community-centered understandings and practices emerged during data collection and analysis. How teachers fostered a supportive, collaborative, student-centered environment, and how they focused this community on learning corresponded to how deeply teachers met community-centered criteria of effective learning environments.

Rory

Rory strives to cultivate a comfortable environment for his students, but ensures that it is one that is focused on learning. He believes that all students can succeed, a perspective that is supported by his many years of experience and practice. He is particularly proud of the fact that he makes almost all of his students feel that they can succeed if they try and an overwhelming number of his students successfully pass the AP Calculus and AP Statistics exam each year. He attributes his positive results to his ability to emphasize the learning process rather than just the results. In so doing, his students learn how to think and develop strong understandings.

Rory fosters an environment where it is expected that every student will learn, and where they are safe to take intellectual chances and make mistakes. Rory's classroom is community centered in that students are expected and enabled to participate in discussion and problem solving activities. Thinking, asking questions, making conjectures, and problem solving is encouraged. It is this process that is encouraged and supported (10/13/02).

> Some kids are very structured and just want the answer but I don't just give you answer. I want to help them to see the big picture. It s is almost like it becomes a game, is there an answer that I can give them without giving the answer. I don't know where that comes from but I have had some principles observe and say that I have a way of saying your wrong without saying your wrong. It's like your wrong but that is not the point and I think that just goes back to the process of how to get there. To me, it is a way to interact with kids and maybe that is

> just part of the enjoyment I get out of teaching is interacting with kids like in coaching. If I don't interact with them in what I'm doing, it's not any fun. I try to make them wonder why and have this done in a friendly atmosphere.

Rory fosters an environment where it is expected that every student will learn, and where they are safe to take intellectual chances and make mistakes. His classroom environments models core community-centered concepts in that his students are able to participate in collaborative problem solving and meaning making activities. Rory has clear content and process goals for his students and takes advantage of multiple tasks and contexts to advance his students' interest, engagement, and understanding. An example of this is the way that Rory learned to fluidly integrate Fathom technology into his curriculum and instruction, as demonstrated in Table 17 below, which shares participants' use of computers with students.

[7]	**Rory**	**Cyrus**	**Patricia**
Where do students use computers during your classes? Typically, how many students operate any one computer at one time during this class?	Classroom has 11 computers, so students worked on pairs on these. The computer lab has 28 computers, so there is one student per computer here. I also used one computer with projector to demonstrate concepts.	Computer lab has 25 computers. Students work individually on these.	Computer lab has 25 computers. Students often worked individually on activities and then discussed ideas in pairs or groups.

Table 17: Participants' use of computers with students during the semester after the Fathom with TSIL inservice.

Although he had no previous experience utilizing computer software in his practice prior to this project, he learned to flexibly take advantage of the affordances Fathom provides for student learning. In addition to having access to the school computer labs, Rory obtained eleven older computers when his school got new ones

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[7] This item adopted with permission from Becker, 2000c.

and arranged them around his classroom. He also made arrangements to have a computer projector and laptop computer available for classroom use when desired. Depending on the context, he learned to adroitly move from having students work in pairs on computers in his classroom for short investigations, to having them work on longer activities individually in the computer lab, or utilizing a projector to demonstrate, model, and stimulate discussion of important concepts. In fact, the advantage of being able to quickly use a projector for demonstration and discussion purposes is something that he grew to appreciate over the course of the project (11/15/02).

> I really like using the projector. I have been using it a lot. It is really nice to be able to quickly turn to and use. It provides a different medium than just having me talking. Demos are very powerful and provide a great way for me to introduce a concept or demonstrate or emphasize a point. I wouldn't have thought that the display was a very powerful means of teaching before this project. It is lot more powerful than I realized.

Rory's practice really does align with all aspects of effective learning environments and is a very safe, dynamic, and exciting place to learn mathematics and statistics. He goes to great lengths to make sure that his students understand the purpose, goals, and expectations of the technology supported activities that they engage in. His classroom learning community is supported in developing mathematical content and process understandings. With his solid base of attending to important learner, knowledge, and assessment centered aspects of effective learning environments, which operate within a healthy learning community, Rory is able to refine his understandings and practices regarding TSIL and support his students mathematical intellectual development.

Cyrus

As discussed in the sections above, Cyrus' technology integration practices do not foster a learning environment that supports the learners, the knowledge that is to be learned, nor that invites inquiry and communication. The learner, knowledge, and assessment centered aspects of effective learning environments described in this paper are all necessary components associated with facilitating of a supportive community of learners context, therefore Cyrus' practice falls short of meeting community-centered tenets. This section examines more of the factors that relate to the lack of community reflected in his technology-based practices.

Cyrus does care about his students and wants them to be able to succeed. He was a part of a group of teachers from across the state of New Mexico who pushed for high standards for all students. He feels that it is important that all students are successfully prepared from their high school experience to go on and be successful in college (5/1/02).

> I think that it's important that they're prepared to go to college and be successful at college mathematically. And for that, I think, at the bare minimum they need to be able to walk in and take college Algebra and be successful at college Algebra. And, now I can't tell you why they have to be successful at college Algebra except that you need college Algebra to get just about any degree. So they should be able to walk in there and take that course even if they do nothing else. So I feel that all the Algebra I and Algebra II and Geometry curriculum is important regardless of whether you'll ever use it again in your life or not.

Cyrus subscribes to the "tough love" philosophy and feels that his high standards are best for students. He does not believe that it is right to lower his standards so that more students can succeed, but that he should keep high expectations for his students. He keeps very regimented and structured classes and expects his students to be prepared with questions and ready to follow along as he shows them how to do the math. His views on keeping a brisk pace and flow to his classes were shared in the assessment-centered section earlier. He takes pride in the fact that he models the types of attitudes and behaviors he would like to see in his students (5/1/02).

> I think all of them respect me in that I walk the walk too. I mean I do my work. They get their grades every Monday and know exactly where they stand. They know how they got that grade, what they can do to make it better, and what will happen if they don't? You know, I say, 'this is what I'm going to do', and then I pretty much do that. The nice thing is I come in an hour early in the morning and I'll sit there through lunch and I'll help anybody who walks in the door who is willing.

While he holds high expectations for his students and wants them to succeed, his common instructional practices are not successful towards developing a holistic learning community. Over half of his class gets below a 'C' each year and when students fall behind, they have difficulty catching up. It was already discussed how students have little opportunity to communicate and formatively assess their reasoning. Cyrus, as demonstrated by his responses to the pedagogy survey items demonstrated in Tables 18, 19, and 20 below, feels that allowing time for students to collaborate, problem solve, and communicate with one another is not a valuable use of time and does not provide opportunity to do so.

	Rory	**Cyrus**	**Patricia**
Indicate how much you disagree or agree with each of the following statements about teaching and learning. (From 1=Strongly Disagree to 6= Strongly Agree)			
A quiet classroom is generally needed for effective Learning	1	4	1

Table 18: Participants' beliefs about quiet classrooms.

	Rory	Cyrus	Patricia
About how often did students in this class participate in the following types of activities?			
Work in small groups to come up with a joint solution or approach to a problem or task	1-3 times per week	Never	Almost Everyday

Table 19: Participants' use of small group work.

	Rory	Cyrus	Patricia
During the last unit that you taught, roughly what percentage of the time students spend in each of the following activities?			
Teacher led a whole-class discussion (students listened and answered questions)	25%-50%	50%-75%	Under 25%
Students led a discussion or gave a presentation	25%-50%	Under 25%	25%-50%
Students worked on their own on assignments at their desks	Under 25%	25%-50%	Under 25%
Students worked together in small groups to complete an assignment as a team	25%-50%	Under 25%	50%-75%

Table 20: Participants' use of collaboration and discussion.

The above survey responses[8] demonstrate that Cyrus does not encourage collaboration. Students spend the majority of their time listening to him and working on their own at their desks. He feels that the classroom should be a quiet place, not one that wastes time and energy having students communicating and collaborating with one another. He explains (11/14/02):

> I've gone from having them work in partners all year long to hardly ever letting them work in partners because one of the things I discovered is that they each learned half.
> Which, that's not so good. You know? 'I got half the knowledge, you got half the knowledge and if we're working together we got the test'. And if I make them take the test separately they both do poorly.

With these general beliefs that investigation, communication and collaboration are not valuable, it should be no surprise that when students are allowed participate in

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[8] These items adopted with permission from Becker, 2000c.

technology-based activities, that these are not goals, as demonstrated in Tables 21 and 22 below.

	Rory	Cyrus	Patricia
Which of the following are among the objectives you have for student computer use? (Check all that apply.) Learning to work collaboratively	Yes	No	Yes

Table 21: Participants' objectives for student computer use.

	Rory	Cyrus	Patricia
Rate beliefs and practices regarding technology integration (From 1=Strongly Agree to 5= Strongly Disagree) I use technology to support project- and problem-based learning in my classroom.	1	4	2

Table 22: Participants' beliefs regarding integration of technology for problem-based learning.

Cyrus used educational technology, in general, and Fathom, in particular, the least of all of the participants. As discussed earlier, when he did use Fathom it was to allow him and the students to go to the computer lab to “play around” and have a break from the regular structured classroom routine. While these technology-based activities may lighten the atmosphere somewhat, they do not satisfy community-centered components of effective learning environments. Overall, his practices do not succeed in cultivating a supportive, engaging, community of learners. It is too often authoritarian and procedural, on one extreme, or unfocused and loose on the other, to fully align with the learner, knowledge, assessment, or community centered criteria of effective learning environments.

Patricia

Patricia really enjoys being a teacher and purposely spends time getting to know her students and making them feel comfortable. She cares about her students and enjoys the time she spends with them. She enjoys the social aspects of high school and commits to getting to know her students social interests. She also spends time during the school day talking with them about things that are going on in the students' lives such as what they did over the weekend and their hobbies, dating, attire, etc…. This, in turn, she says, helps her students feel more comfortable and engaged (7/24/02).

> I like to get to know my kids in school and keep their classroom comfortable. I get excited for them during prom and for homecoming. We do a lot of joking and teasing and talking about social issues in between math. It just makes a comfortable atmosphere so that I try to keep them coming back and become interested in school and getting an education. If you don't know what is going on in their lives or what they feel about this or that then you are not really in touch with them. It's important to show that you care about them and that you care enough to care about what is going on in their lives. That might be especially important with these students, because many of them come from families that don't value education. That's how I think learning goes on. If they are comfortable in their environment than they learn more.

This year, Patricia has found that she can still make her students comfortable and can push them to develop and articulate their understandings. She has found that incorporating TSIL with Fathom into her curriculum and instruction can make her and her students more motivated about teaching and learning (11/14/02).

> I was getting tired of the way I was teaching and I felt it was time to change because it was the same thing year after year and I just wasn't happy with what the kids were learning. There was no enthusiasm with them. The (TSIL) workshop helped renew my interest and enthusiasm in teaching. I think my whole philosophy changed as far as we are here to learn math. It just wasn't fun teaching anymore so

> now I have to make fun of the way it was. I am amazed at how well TSIL works. I knew in back of mind it would, even common sense says it should, but you really don't know it until you do it. The students are more engaged and they have more retention. Using more hands-on activities and journaling has helped them to be more confident in themselves and willing to take chances. They are a lot calmer. You could feel the tension before and now they are "let's just do it". They do not complain and whine about it. Last year, I had to hold their hands more last year and you would see their books close after my lecture, this year they go straight to doing work. It is different than the way I felt last year. The kids are calmer and I am calmer. It is a feeling you get.

Prior to this project, Patricia believed that her students did not enjoy mathematical reasoning and were not able to do so. She accepted this view and just told her students that one day they would understand. She would settle for not pushing her students to explain their thinking as long as she knew that they had tried to do some work and were coming to and participating in school. She was concerned that pushing them more would upset the balance and ruin their effort and enjoyment. This year, however, she has learned that she can expect more from her students without compromising having a caring community. She has always met this first community-centered theme of facilitating a safe and comfortable environment, but is now learning how to meet the second theme, that of making it a true community of learners. A major part of this shift has been due to her focus on incorporating TSIL with Fathom into her practice (Interview, 11/14/02).

> When I go in there and give them a worksheet to do on Fathom there is no hesitation and their answers come back from compare and contrast, or explain what this means, pretty concise. That freaks me out half the time. I have no clue where they are getting it from although I know it has a lot to do with the computer lab. They are learning it.

Patricia used computer technology more than any other participant. Approximately twice a week, students would explore Fathom activities to reinforce or apply concepts related to the unit they were currently covering. Although she responded in her survey that student worked individually on computers, there was a lot of collaboration and communication in her class in general, and in these technology-supported activities, in particular, as was demonstrated in Tables 8 and 17.

As Patricia's responses to the two survey items about group work and collaboration and discussion, demonstrated in Tables 17 and 18 respectively, she encouraged students to collaborate with one another to think about and answer problems and tasks. As she was quoted earlier as saying, you felt that this opportunity to work together kept them more interested and engaged and the opportunity to explain their thinking to one another aided their understanding through the process of putting it into their own vocabulary. What was too often missing in the learning environments that I saw her facilitate was a sharp focus on the knowledge that the students were developing. To be effective, she should have always had one eye and ear focused on steering the investigations and discussions towards the understandings the students should be obtaining. These students participated in active learning 90% of the time, by her account, and seem to rarely have received direct feedback on the concepts, probably due to the lack of clear understanding that Patricia had herself.

Overall, Patricia has theoretically embraced the idea of TSIL. She really cares about her students and strives to build a comfortable learning community. She still has many areas that of her practice that she can continue to improve in order to fully meet criteria of effective learning environments. She is working to deepen the knowledge component of her practice by restructuring her course around big ideas, but needs to focus more on, and better understand, the content herself. She is trying to utilize assessment-centered strategies of inquiry and communication in her practice, but again can attend more to students developing understandings and how to further them. Due to her novelty in understanding the content, managing reform-based instruction, and utilizing and integrating technology, she has a lot of good things happen and also things that can be improved. However, the important thing at this time is that she and her students are enjoying the teaching and learning of mathematics and statistics. Their classroom community is comfortable and caring and spends time thinking and learning about and furthering their understandings of mathematics and statistics. With continued practice with TSIL, the results can only continue to improve, since the motivation clearly exists.

Community-Centered Discussion

Chapter Two (Table 1) forwarded the vision TSIL with Fathom activity would allow students to collaboratively investigate statistical phenomena. Students will utilize Fathom in order to learn statistics and come to understand ways that they can use statistics to better understand the world around them. When they enter into the world of statistics by investigating content that is meaningful to themselves, and collaborate with others to gain understandings they are building learning communities within their classroom and participating in a scientific community of meaning making. When students communicate the results of their investigations with authentic audiences of peers and teachers, they further build community. In so doing, they are also able to facilitate the classes' understandings of the relationship between statistics and the world, and also appreciate the fact that they are able to participate in the process of scientific inquiry.

By examining teachers' understandings and practices regarding content, technology, and pedagogy, it has been demonstrated that there are many factors that relate to teachers' ability to facilitate community-centered aspects of TSIL pedagogy. In particular, teachers' understandings and practices around developing a supportive classroom environment and their ability to cultivate a community of learners each related to how deeply teachers met community-centered criteria of effective learning environments.

Rory succeeded in giving a picture of a learning environment that made students feel safe and comfortable to participate and share their ideas while constantly maintaining a focus on what was to be learned. In his class, the process of mathematical investigation is as important as the final knowledge obtained, and TSIL with Fathom plays a key role in this process. This teacher and his students flexibly integrate technology in order to meet learner, knowledge, and assessment centered goals within a supportive community centered environment.

Cyrus did not succeed in building a supportive student-centered community. In this classrooms normal structure, he is very much the authority and the students are not actively involved or listened to. When technology is utilized, it does not support knowledge-building goals related to the course curriculum and instruction, but is solely an aside. Students in Cyrus' classes often are not involved in formatively assessing their own learning. They rarely have opportunity to problem solve, collaborate, or communicate. Overall, the lack of strong alignment with none of the criteria of effective learning environments provide a contrast to Rory and show a picture of what it can look like when technology is not utilized to support the development of a community of learners.

Patricia cares deeply about her students and holds a primary focus of making them feel safe, comfortable, and cared about. She has embraced the idea of TSIL with Fathom and is working to utilize this pedagogy to transform her classroom into a community of learners. While she incorporates rich technology supported tasks into her curriculum and instruction, her lack of emphasis on her own understanding of the content and the technology, and the nuances of students developing understandings produce weaknesses in her alignment with the tenets of effective learning environments. Her practice provides a picture of how a teacher can begin to integrate technology to support the development of a learning community but needs to continue to improve in each area to truly exemplify a community-centered knowledge-building environment.

Tables 23, 24, and 25 below, summarize participating teachers' community-centered understandings and practices regarding pedagogy involving technology, content, and technology, respectively.

Rory	**Cyrus**	**Patricia**
Has computers and projector in his classroom, as well as access to computer lab. He flexibly utilizes the different mediums of teacher demonstration, quick student investigation in class, or extended activity in computer lab based on his instructional goals. Utilizes pre-made and hand-made activities.	Uses only the computer lab. Rarely incorporates technology. Technology based activities consist of having students work individually on worksheet activities	Uses only the computer lab, having students work on activities and then discuss them after they have done so. Consists solely of pre-made activities from *Workshop Statistics with Fathom*, which she aligns with scope and sequence of the course.

Table 23: Participants' community-centered understandings and practices regarding pedagogy involving technology.

Rory	**Cyrus**	**Patricia**

He aims for students to be able to make connections between mathematics content across the discipline, among other disciplines, and to "real-life" and he gives students opportunity to do so.	Although he espouses the importance of developing understandings and connections, his practice shows little opportunity for students to develop these robust understandings.	Depends on curricular resources to link content to applications. She wants her students to make these connections but does not always understand them fully herself.

Table 24: Participants' community-centered understandings and practices regarding content.

Rory	**Cyrus**	**Patricia**
He scours resources, develops, and practices with the technology. He clearly knows how to do what he asks students to do	Not prepared to model activities with students. He does not know how to use the computer lab nor do things he asks students to do.	Not prepared to model activities with students. She has not reviewed the technology nor the content before activities and does not know how to do things she asks the students to do.

Table 24: Participants' community-centered understandings and practices regarding technology.

CHAPTER SUMMARY

This chapter introduced three teachers who have participated in this dissertation project, and describes their technology integration efforts in relation to research on effective learning environments. Situating these teachers' technology-supported activities within this effective learning environment theoretical framework highlights how technology can provide the opportunities to see this theory put into practice in exciting, innovative, and effective ways, and the nuances that this entails.

By examining teachers' understandings and practices regarding content, technology, and pedagogy, it has been demonstrated that there are many factors that relate to teachers' ability to facilitate learner, knowledge, assessment, and community-centered aspects of TSIL pedagogy. Effective technology-supported learner-centered environments depended on teachers' facility in actively engaging students and connecting their technological activities to students' knowledge, experiences, and interests. Teachers' perspectives on the discipline of mathematics, their content knowledge, and their content and process goals for student learning all influenced the level of their implementation regarding knowledge-centered environments. Effective technology-supported assessment-centered environments depended on teachers' level of planning and implementation for the tasks they utilized, their understandings and practices regarding the inquiry-based instruction and classroom communication. How deeply teachers met community-centered criteria of effective learning environments depended on all of the above components of effective learning environments in addition to their ability to cultivate an overall supportive community of learners classroom environment.

The NCTM PSSM (2000, p. 26) state that "using technological tools, students can reason about more general issues and they can model and solve complex problems that were heretofore inaccessible to them". The four components of effective learning environments provide a good framework to assess the strengths and weaknesses in these teachers developing understandings and practices of technology supported inquiry learning (TSIL). By discussing the ways in which these teachers meet, and at times fall short of enacting, what research informs us about the components of effective learning environments, a clear vision of how important each of these components is towards exemplary technology integration, and a picture of what exemplary integration of technology can and should look like, has been presented.

Examining these teachers uses of technology within an effective learning environment framework has provided an opportunity to demonstrate that technology-supported activity can increase student motivation, connection, and understanding. However, for this to occur, all aspects of effective learning environments must be aligned. When they are not, technology use is just an add-on that does not result in desired student motivation, engagement, and understanding. Through their own opportunities to observe, practice, and reflect TSIL, Rory and Patricia have discovered for themselves that technology can help facilitate effective learning environments that are learner, knowledge, assessment, and community centered. Rory has given us a vision of how all of these components can give a truly dynamic and engaging, knowledge-building learning community. Cyrus' practice shows ways that technology can be integrated non-substantively without meeting effective learning environment criteria. Patricia demonstrates a teacher in between these two, one whose practice last year was more similar to Cyrus' but who is striving to move towards the time of environment Rory manages.

By examining their TSIL practice, I have been able to tie theory to practice by providing actual examples of technology use in mathematics education that illustrates the effective learning environment framework. Technology, in and of itself, is complex. Yet even more complicating is ways in which to use it as a tool for teaching and learning. Guided by this theoretical framework, technology supported activities can meaningfully work in a learning environment that research proves effective. The next chapter will build off of these examples of technology use, and these discussions of effective learning environments, to answer the research questions of this study and offer final conclusions and implications that this study provides for teacher practice and professional development.

CHAPTER FIVE

CONCLUSIONS AND IMPLICATIONS

The emphasis and driving force of this study was a desire to research teachers' understandings and practices regarding content, pedagogy, and technology as they learned to integrate technology-supported inquiry learning with Fathom into their curriculum and instruction. Many findings regarding teachers' understandings and practices about content, pedagogy, and technology were shared in Chapter Four in order to contribute to results and discussions about the ways that their technology integration practices aligned with research on effective learning environments. This dissertation chapter will build off of the results and discussion in the previous chapter and come full circle to answer the research questions of this study:

1. What are teachers' understandings as they learn about, practice with, and reflect upon technology-supported inquiry learning?
    a. What are the participating teachers' understandings regarding mathematics and statistics content?
    b. What are the participating teachers' understandings regarding pedagogy?
    c. What are the participating teachers' understandings regarding technology?

2. What do the instructional practices look like for teachers who are trying to incorporate TSIL within their classrooms?
    a. What do teachers' practices look like as they incorporate Fathom into their teaching?
    b. What are similarities and differences regarding teachers' practices involving TSIL with Fathom?

These research questions are complex ones to answer. Teachers' understandings affect their practices and adaptations to their practices can affect their understandings (Putman & Borko, 1998). Furthermore, teachers' understandings and practices regarding content, pedagogy, and technology are not mutually exclusive and can and do relate to one another (NRC, 2001a; Becker, 2000a).

Within distinct sections of this chapter, I will address what has been learned about the participating teachers' understandings and practices in the areas of first content, next pedagogy, and then technology. Since knowledge about teaching and teaching itself are inexorably linked (Shulman, 1986) and inform one another, within each of these sections I will discuss teachers' developing understandings and practices together, rather than separately. Furthermore, connections between the findings regarding content, pedagogy, and technology and other education research will also be shared in each section, respectively. While, undoubtedly, overlaps between teachers' understandings and practices regarding content, pedagogy, and technology will occur among their respective sections, a separate discussion about how these areas come together to influence teachers' ability to integrate technology will be presented later during this chapter. Finally, implications of this research for teacher education and professional development will be discussed.

CONTENT

Themes associated with teachers' mathematics and statistics content knowledge, their general perspectives on the subject of mathematics and statistics, and their strategies for developing student content knowledge emerged as content-related factors associated with teachers' TSIL understandings and practices. Findings regarding where each participant fit within these themes were introduced and discussed in Chapter Four, and are summarized in Table 26 below.

CONTENT		
Rory:	**Cyrus:**	**Patricia:**
• His content knowledge is rich, deep, and connected. • He was articulate and thoughtful about each of the big ideas that he was asked about by me. • Feels that mathematics is a process of problem solving. • Holds content and process goals for student understanding. • Aims to make connections across the discipline and to "real-life" with students. • Focuses instruction on the big ideas from AP, MAA, ASA, NCTM	• Has pretty strong content knowledge. • Was familiar with all of the big ideas he was prompted about. • Feels that mathematics is a divine process to be discovered. • Believes that students cannot really appreciate the beauty of mathematics and must learn procedures. • Content goals for students often get actualized in a rote, procedural manner.	• Has weak, unconfident content knowledge with large gaps. • Was not familiar with many of the big ideas prompted about by me. • Did not talk about the process of mathematics, only that it is important to understand math in order to be able to make more money in life. • Structures her curriculum so that she covers the big ideas of the subject. • Does not emphasize content with students.

Table 26. Summary of participants' understandings and practices regarding content, which were introduced and discussed in Chapter Four.

Patricia has weak content knowledge. She incorrectly answered many items on her content knowledge survey and exhibited many gaps in understanding during observations and interviews. Although she cares deeply about her students and has a sincere desire to cultivate their mathematical engagement and understanding, her lack of content emphasis and understanding get in the way of her being able to thoroughly achieve this goal. She provides us with a picture of how it is not enough to hold content and process goals for students for TSIL to work, teachers must also have strong content knowledge themselves and emphasis content in their practice.

Research has established evidence that corresponds to findings about Patricia’s weak content understandings and practices interfering with her ability to implement effective instruction (NRC, 2001a; Ball, 1991; Borko et al., 1992; Thompson and Thompson, 1994,1996). NRC (2001a) states “teachers are unlikely to be able to provide adequate explanation of concepts they do not understand and that “not surprisingly, these teachers [with weak conceptual knowledge of mathematics] gave students little assistance in developing an understanding of what they were doing (p. 378)”. Patricia’s case presentation and discussion highlight that for students to be able to develop strong content knowledge, the teachers needs to have robust understandings themselves and focus on the concept development.

Cyrus has a pretty firm grasp on the big ideas of statistics. He was comfortable talking about each of these ideas and got almost every problem correct on his content knowledge survey. There is a gap in the way that Cyrus sees the discipline of mathematics and the way that he believes students come to know it. His instruction, like that other research has documented in teachers who have strong content knowledge without corresponding pedagogical content knowledge, is very formal and structured and does not convey any of the underlying inherent beauty of the subject or make clear the processes for discovering and understanding this beauty. Research supports what this study has demonstrated with Cyrus, that while

> a strong grasp of mathematics can help to make it possible for teachers to understand and use constructively students’ mathematical solutions, explanations, and questions … some teachers with strong conceptual knowledge did not necessarily use that knowledge to understand their students’ mathematical explanations, preferring instead to impose their own explanations (NRC, 2001a, p. 378).

Cyrus’ practice is predominately limited to procedurally covering each section of the textbook with the occasional use of using technology to “play around”. Cyrus provides us with a picture of how although teachers may understand and enjoy mathematics themselves, their TSIL practices are weak without knowledge of or ability to use or implement strategies and practices that cultivate this same interest and understanding in students. Again, the relationships found in this study between Cyrus’ own understandings about mathematics and his instructional practice is not uncommon. NRC (2001a) explains that “teachers’ knowledge is of value only if they can apply it to their teaching; it cannot be divorced from practice (p. 379)”. In articulating a knowledge base necessary for teaching mathematics well, NRC (2001a) emphasizes that teachers’ mathematical content knowledge includes not only an understanding of mathematical facts, concepts, procedures, and relationships, but also an appreciation of how mathematical knowledge is produced and the importance of problem solving, reasoning, and communication for developing understanding. In discussing this often apparent gap between knowledge and practice, they proclaim that the traditional advanced mathematics courses that teachers take as part of their teacher preparation do not emphasize the “ideas needed by teachers whose use of mathematics are to help others learn mathematics (NRC, 2001a, p. 375)”. Given this prior education research it should not be surprising that Cyrus’ own knowledge of mathematics and statistics does not correspond with a robust practice that supports students to develop deep interest and understanding of the discipline and implies that teachers must be educated no only in content knowledge, but in pedagogical content knowledge too.

Rory demonstrates how strong content knowledge and pedagogical content knowledge can be utilized to cultivate student engagement and understanding. He has a rich, deep, connected understanding of mathematics and statistics. He got every problem correct on his content knowledge survey and thoroughly understood each of the big ideas of statistics that he was asked about. To Rory, mathematics is thinking and problem solving and this perspective bellies the goals that he holds for his students. He bases his goals for his students on the fact that NCTM, AP, MAA, and ASA articulate that students should be original thinkers and problem solvers, and he facilitates a learning environment that cultivates these content and process understandings in students. Rory provides us with a picture of how when a teacher has strong content knowledge, broad perspectives on the discipline, and rich understandings and practices regarding cultivating student content and process understandings, TSIL practices can be strongly implemented.

Rory's case aligns with prior research, which articulates that teaching for mathematical proficiency requires knowledge of mathematics for oneself, as well as an understanding of how mathematical knowledge is produced in others (NRC, 2001a; Shulman, 1986; Ball and Cohen, 1999). NRC (2001a), in outlining a knowledge base for teaching mathematics, includes not only mathematical knowledge, but also knowledge of students and knowledge of instructional practices as crucial. This foreshadows the interactions between content, pedagogy, and technology, which are discussed later in this chapter. In order to fully understand the relationship between each of these areas, I will first conclude findings regarding pedagogy and technology within their own respective sections. For now, suffice to say that Rory's robust understandings and practices regarding content align with prior research, which states

> to be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks. They need to understand and be committed to their students as learners of mathematics and as human beings and be skillful in choosing from and using a variety of pedagogical and assessment strategies (NCTM, 2000, p. 17).

PEDAGOGY

Many aspects of pedagogy have already been introduced in this chapter and the previous one too. Here they will be summarized. Themes associated with teachers' understandings and practices regarding inquiry, and inquiry-based practices of communication, collaboration, and assessment, as well as how teachers planned for and implemented tasks, emerged as pedagogy-related factors associated with teachers' TSIL understandings and practices. Findings regarding where each participant fit within these themes were introduced and discussed in Chapter Four, and are summarized in Table 27 below.

PEDAGOGY		
Rory:	**Cyrus:**	**Patricia:**
• Centers his practices on student understanding. • Uses structured inquiry supported by direct instruction. • Is balanced between student-centered and teacher-led instruction. • Values and supports bi-directional communication.	• Centers his practices on covering the whole book. • Instruction is delivered by teacher and received by the student. • Instruction is usually teacher-led. • Relies on unidirectional communication. • Does not value or	• Centers her practice on covering the big ideas of the subject. • Primarily utilizes unfocused student investigations during instruction. • Instruction is predominately student-centered, little teacher-led. • Values and supports bi-directional communication although she misses opportunities to

• Values and supports collaboration. • Values student engagement and developing a community of learners. • Incorporates TSIL. • Provides students opportunity to formatively assess their developing understandings. • Utilizes rich tasks. • Draws from multiple resources in planning how to develop student understanding.	support collaboration. • Does not develop a community of learners. • Does not incorporate TSIL. • Does not provide students opportunity to formatively assess their developing understandings. • Utilizes procedural tasks. • Plans to cover each page of the book in order.	develop student thinking. • Values and supports collaboration. • Values student comfort and community. • Tries to incorporate TSIL activities. • Is not able to provide substantive opportunity for students to formatively assess their developing understandings. • Uses rich pre-made tasks • Draws from multiple resources in planning how to cover big ideas.

Table 27. Summary of participants' understandings and practices regarding pedagogy, which were introduced and discussed in Chapter Four.

Cyrus does not believe that inquiry, communication, and collaboration are important aspects of curriculum and instruction. He feels that investigation takes longer than direct instruction and that there is no time for it when there is so much content that needs to be covered. Likewise, communication slows down the class and wastes time that he could be telling students what they need to know. He feels that collaboration only results in each student knowing part of what they need to know. Cyrus mainly utilizes tasks that come directly from a traditional textbook because he feels that is what he is supposed to cover. His technology-based activities did not fit in with his regular curriculum and instruction and were add-ons designed to let students play around. Neither these technology-based activities nor his traditional ones allow students to formatively assess their understandings since they do not provide rich opportunities for problem solving, communication, and reflection. His practice gives us a picture of how when a teacher does not value pedagogical practices involving inquiry, communication, and collaboration, TSIL activity will not fit into curriculum and instruction and, hence, will not benefit student learning.

Cyrus is not alone in questioning the value of inquiry-based instruction. “Many teachers question why they should reorient their teaching towards inquiry based methods (NRC, 1997, p. 115).” However, a substantial body of research has pointed out that inquiry based teaching can substantially improve student learning (NRC, 2000). In particular, “research underscores the value of self-assessment in developing their understandings of scientific concepts as well as their abilities to reason and think critically (NRC, 2000, p. 119)”. Furthermore, research indicates that learning is enhanced when students are able to communicate and collaborate (NRC, 2000; NCTM, 2000). NRC (2001a) emphasizes that effective mathematics teaching must include understandings and practices that manage classroom discourse and support the creation of a community of learners. While this substantial body of research supports inquiry-based instruction, many barriers to successful implementation exist (NRC, 1997). One documented barrier that resonates in considering Cyrus is the “preparation ethic, an overriding commitment to coverage because of a perceived need to prepare students for the next level of schooling (NRC, 1997, p. 142)”. As Borko and Putman (1998) have posited, it is these kinds of underlying beliefs that must be addressed if changes in teaching practice are to be truly internalized and actualized. Patricia’s case however, discussed below, demonstrates that believing in inquiry-based practices is not sufficient to ensure successful implementation.

Patricia values inquiry, communication, and collaboration. She encourages students to work together to solve problems and regularly asks them to explain their thinking, verbally and in writing. She chooses tasks that provide students opportunity to investigate, problem solve, reason, and communicate. However, she does not understand the content well enough herself to assess and build on students' developing understandings, and hence does not fully allow her students to assess and deepen their understandings. Her practice provides us with the picture that having teachers hold TSIL process goals is not enough to ensure rich implementation. Teachers must develop strong content and pedagogy understandings and practices in order to support deeply meaningful TSIL integration.

Patricia's case is not unique. Many teachers do not have strong content knowledge (NRC, 2001a). However, she is not only handicapped by her content knowledge in effectively incorporating TSIL, but also by her lack of focus on continuing to develop her own understanding and that of her students. "The more teachers know about inquiry and about subject matter, and the more they themselves are effective inquirers, the better equipped they are to engage their students in inquiry. It generally does not work for teachers to stay one step ahead of the students when using inquiry-oriented practices (NRC, 1997, p. 137)." Patricia felt as though using rich tasks alone was sufficient to develop student learning. However, she did not model inquiry and engagement for her students, or work to build on and deepen their thinking and, hence, did not meaningfully enhance her students' understandings. Patricia's pedagogical gaps combine and overlap with her lack of deep content knowledge to limit her ability to effectively integrate TSIL practices. It is hoped that her instructional understandings and practices, like all teachers, becomes generative (Franke et al., 1998). Generative teachers “think that mathematics, their understandings of students thinking, and their teaching practices fit together to make sense and that they are capable of learning about mathematics, student mathematical thinking, and their own practice themselves by analyzing what goes on in their classes (NRC, 2001a, p. 384). Rory, discussed next, demonstrates a teacher who does believe and do so.

Rory strongly believes in cultivating students’ ability to engage in inquiry. He works to provide a structure for student investigation so that they attend to the important concepts that he wants them to build understandings of. He believes that collaboration and communication are important aspects of this developing understanding, and that students need to share their conjectures verbally and in writing, and then reflect on their merit, in order to deepen their content and process knowledge. He works hard to develop tasks that provide opportunity for students to reason mathematically, share their thinking, and formatively assess their developing understandings. Likewise, Rory continually reflects and revises his curriculum and instruction based on students’ understandings and selects the tasks and strategies will best help him support his students.

NCTM (2000) discusses that “worthwhile tasks alone are not sufficient for effective teaching. Teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge (p.

19)". Rory continually reflects on these aspects of his pedagogical practices based on what he is learning about students' learning. He has clear mathematical goals for his students and structures tasks that will help his students reach these goals, but is sure to listen carefully to their ideas and explanations and use this information to make instructional decisions. By so doing, his understandings and practices provide us with a picture of how when teachers believe in inquiry and assessment, communication, and collaboration, and work hard to plan for and assess tasks that utilize these processes towards student understanding, rich TSIL practices can result.

TECHNOLOGY

Themes associated with teachers' experience with technology, their perspectives on the role of technology in education, and their perspectives and facility integrating Fathom emerged as technology-related factors associated with teachers' TSIL understandings and practices. Findings regarding where each participant fit within these themes were introduced and discussed in Chapter Four, and are summarized in Table 28 below.

TECHNOLOGY		
Rory: • Had used graphing calculators often and fluidly for many years in instruction. Had been introduced to Fathom and Sketchpad previously but had not used them instructionally. • Values the role that technology can play in mathematics	**Cyrus:** • Had previously used technology personally, but very seldom for instruction. • Is not clear on what the role of technology in math education should be, feels it can inhibit student comprehension. • Does not have	***Patricia:*** • Had use technology for a few years personally and in instruction, though instructional use was mainly limited to procedurally using graphing calculators with students. • Values instructional technology. She believes it enhances student motivation

education for student interest and understanding. • Has clear goals for when, where, how, and why he uses technology activities. Facilitates activities that develop students' inquiry abilities and build students' understandings. • Utilizes both teacher-created and pre-made Fathom instructional resource materials. • Practices with technology and knows how to do what he asks students to do. • Utilizes different delivery methods (i.e. one computer demonstration, student computer pairs in classroom, individual computer lab exploration.	clear goals for when, how, and why he utilizes instructional technology with students other than to "let them play" for a change of pace. • The technology-based activities he utilizes entail going to the computer lab to complete Fathom worksheet activities, which are not connected to the instruction and curriculum he and his students regularly experience. • Is not prepared to model activities with students – does not know how to use Fathom to do things he asks the students to do.	and understanding. • Has the goal of using instructional technology in order to develop student investigation, interest, and understanding but has difficulty developing students' content understanding due to her own content and pedagogy gaps. • Her Fathom activities are limited to using pre-made instructional resources with students in the computer lab, which are selected for their connection to the big ideas she wants her students to know • Often, is not prepared to model activities with students – does not know how to do things she asks students to do.

Table 28. Participants' understandings and practices regarding technology, which were introduced and discussed in Chapter Four.

While none of the participating teachers' had previously integrated computer technology into their instructional practice, they all had used computers themselves and had tried out using graphing calculators with their students. They were all interested enough in teaching and in technology to spend time over the course of a year participating in surveys, observations, and interviews about their educational technology understandings and practices. Each, however, had unique understandings and practices regarding where, when, how, and why to incorporate Fathom software into their practice.

Although Cyrus likes and uses technology, including Fathom, himself, he is conflicted about how to integrate it meaningfully into his practice. He rarely practiced with Fathom or tried to embed TSIL activities into his curriculum and instruction. He ended up only utilizing Fathom when he wanted students to have a break from their normal structured routine and be able to play around. He provides us with an example of how it is not enough for a teacher to learn how to use and like technology themselves, without robust perspectives on reasons and ways to integrate it, teaching and learning are not meaningfully enhanced.

In many ways, Cyrus is more the norm than the exception. Based on national survey data, Becker (1994) estimated "that, nationwide, only 3-4% of computer-using secondary mathematics teachers would be classed as exemplary users of computers for math instruction (p. 321)". Almost all of the non-exemplary computer using teachers in Becker's national studies (1994, 2000) said that they used computer technology with their students as a reward for completing other work or so they would master basic skills. By contrast, the few teachers who were listed as exemplary, based on responses to Becker's national surveys about instructional practices with technology (1994, 2000), used computers regularly with their students so that they could learn to appreciate and apply mathematics. Moving teachers towards holding these meaningful goals for technology integration is difficult, as witnessed by small amount of teachers who do so, and will be discussed in more depth later this chapter. Once teachers do hold these ideals for the use of technology with their students, achieving effective TSIL practices are still difficult to obtain, as Patricia's case, discussed next, demonstrates.

Patricia values the role that technology, in general, and Fathom, in particular, can provide for enhanced student motivation and understanding. Although she frequently utilized technology-supported activities in her curriculum and instruction, she rarely practiced with Fathom herself or knew how to do the things she asked her students to do. While she had high ideals for the value of TSIL, she did not have strong practices to match. Dias and Atkinson (2001) emphasize that "educators should not be overly impressed by the mere use of technology in instruction. It is not the media that influences learning; rather, it is the instructional design utilizing effective teaching strategies that makes an impact. Good teaching combined with appropriate and effective uses of technology makes for a dynamic, rich learning environment (p. 1)." This dissertation outlines how TSIL can be integrated within an effective learning environment framework. It takes more than just a desire to use technology to make the use of it meaningful for student learning. Patricia provides us with an example of how a teacher can have robust perspectives regarding the value of technology, but without strong facility with technology and goals and practices that support student investigation and understanding with it, teaching and learning, as a result, are only moderately enhanced. Rory, discussed next, demonstrates how technology and instruction can come together to provide a dynamic, successful learning environment.

Rory turned out to be the only participant who had a clear, strong vision for when, where, how, and why he could integrate Fathom into his curriculum and instruction. He clearly felt, and demonstrated, that technology-supported activities could be utilized to deepen students' engagement with, and understanding of, mathematics and statistics, and always integrated Fathom with these goals in mind. Towards this end, he developed and utilized rich technology-supported tasks that simultaneously helped to meet learner, knowledge, assessment, and community centered aspects of effective learning environments. He provides us with an example of how a teacher who has experience and facility with technology, and robust perspectives on reasons and ways to integrate it, can use these understandings to strongly enhance teaching and learning.

The previous sections have demonstrated the importance of teachers have strong understandings and practices regarding content, pedagogy, and technology, respectively. As mentioned previously, it is the fact that Rory brings robust understandings and practices in all of these areas together that makes his technology integration practices so strong. He deeply understands the subject matter that he teaches and holds clear content and process goals for his students. He knows how to use technology and flexibly integrates it to meet his teaching and learning goals for his classes. He incorporates a variety of pedagogical strategies to develop student interest and understanding. The following section of this chapter situates this key finding of this study about the important relationship among content, pedagogy, and technology within education research literature.

RELATIONSHIPS AMONG CONTENT, PEDAGOGY, AND TECHNOLOGY

Previous education research in the areas of mathematics education and educational technology have not previously examined content, pedagogy, and technology in relation to one another. Prior research has examined the relationship between instructional technology and mathematics content (NCTM, 2000; Kaput, 1986; Dugdale et al., 1995) or mathematics content and pedagogy (Ball & Bass, 2000; Nathan & Koedinger, 2000; Ball, 1991) or pedagogy and technology (Cuban, 1986; Becker, 1994, 1999; Norton & Wiburg, 1998; Etmer et al., 1999). However, the interactions between teachers' understandings and practices regarding content, pedagogy, and technology have not been examined in the level of detail of this study in previous education research. It is this overlap between content, pedagogy, and technology, and the ways they mutually influence participants' abilities to integrate TSIL practices within an effective learning environment that is essential for effective technology supported practices to occur. This topic is expanded on below.

Educational technology research has outlined a number of factors that inhibit teachers from learning about instructional technology (OTA, 1995; Wetzel, 1998; Becker, 1999, 2000). These factors, aforementioned in Chapter Two, such as time, money, and support were systematically addressed in this study. Teachers who participated in this project were provided with financial and technical support, as well as time to practice with, plan for, and reflect on their use of the technology. However, even given this time and support, some participants did not meaningfully integrate the technology into their practice. Ertmer et al. (1999) propose that once these kinds of external first-order barriers are addressed, internal second-order barriers often limit technology integration efforts. They describe second-order barriers teachers may face when they begin implementing technology, which resonate with the difficulties that Cyrus had being willing to emphasize the integration of technology in his practice. They discuss,

> these barriers relate to teachers' beliefs about teacher-student roles as well as their traditional classroom practices including teaching methods, organizational and management styles, and assessment procedures. Add to this an unclear vision regarding what is expected of them and their students as well as a general uncertainty about the relevance of technology in their prescribed curricula, and teachers are likely to experience a severe case of cultural incompatibility (p. 51).

Educational technology research, however, has been unclear about how to best support teachers in overcoming incongruence that occur when they hold pedagogical understandings and practices that differ from best practices with technology.

Research in the area of educational technology does discuss the relationship between teachers' pedagogical beliefs and their technology integration understandings and practices (Cuban, 1986; Becker, 1994,1999, 2000; Norton & Wiburg, 1998; Etmer, 1999). However, this research, though, hypothesizes that when teachers begin integrating technology, their practices will shift to a more constructivist, student-centered pedagogy as a result. It is difficult to believe that if Cyrus were solely to begin using technology more often that his practice would become more student-centered as a result. The fact that Cyrus chose not to attend this project's latter three days of professional development, which centered on discussing, thinking about, practicing, and reflecting on technology integration, suggests that for second-order barriers relating to teachers' pedagogical understandings and practices to be influenced that they must specifically addressed. Researchers have proposed that helping teachers to develop constructivist-based visions and enactments of technology integration can be forwarded via providing them with modeling, reflection, and collaboration opportunities (Harper et al., 2001).

Addressing second-order barriers and helping teachers to adapt student-centered perspectives regarding the role of technology in curriculum and instruction, though, is not a sufficient result if effective integration practices are to result. Patricia, for example, believes that technology supported inquiry learning activities that provide students opportunity to problem solve, collaborate, and communicate are valuable and she provides opportunity for her students to engage in these kinds of tasks. However, as has been documented and discussed, these student-centered practices have not resulted in an effective knowledge-centered learning environment. These findings emphasize the importance of simultaneously supporting teachers' content, pedagogy, and technology understandings and practices through sustained, ongoing professional development.

Prior mathematics education research has emphasized that technology can and should be used to help students better understand mathematics (NCTM, 2000; Dugdale et al., 1995; NRC, 2001a). NCTM (2000), for instance, states "the teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways (p. 26)". However, little to no mathematics education research has closely examined the interplay between teachers' understandings and practices with content, pedagogy, and technology as they relate to teachers efforts to integrate technology. Stohl et al. (2002) have recently hypothesized that teachers' pedagogical knowledge of teaching and learning mathematics with technology, as well as their ‘techno-mathematical knowledge’, are both important factors associated with helping teachers to effectively integrate technology into mathematics classrooms. This research project begins to do so and, therefore, makes an important contribution to the education community. The results of this study point to the fact that in order for meaningful TSIL practices in mathematics education to result, teachers need sustained, ongoing professional development in the areas of content, pedagogy, and technology. Teachers need to be able to have time to develop their own rich understandings of the content that they teach (Usiskan, 2003; NRC, 2001). They need to be able to participate in reform based educational situations as learners and practice facilitating them as teachers (Romagnano, 1995; NRC, 2001b). They need to be able to practice with technology and see how they can integrate it into their curriculum and instruction (OTA, 1995; Cuban, 1986). This project began to provide teachers with these kinds of opportunities and documented their understandings and practices as they did so. The final section of this paper continues to build on the implication of this research for teacher education and professional development.

Final Conclusions and Implications

Many national organizations (RAND, 2001; NCTM, 2000; NCMST, 2000; NRC, 1997) forward policies that emphasize integrating educational technology into teaching and learning. It has been well documented that although millions of dollars are spent annually on technology hardware and training, rarely is technology integrated in such a way as to meaningfully enhance teaching and learning (OTA, 1995; Becker, 1994, 2000; Cuban, 1986, NCMST, 2000). In order to begin to articulate and demonstrate what effective technology integration in mathematics education can look like in theory and practice, I conceptualized, organized, facilitated, and researched this dissertation mathematics education professional development research project.

Three teachers' understandings and practices regarding mathematics and statistics content, pedagogy, and technology were deeply examined and the ways that these areas interacted with one another to influence the ways these teachers integrated technology into their curriculum and instruction were demonstrated and discussed. The goal of studying and sharing these findings were to be able to contribute to mathematics education and educational technology research and practice in order to better understand how to help other teachers meaningfully integrate technology to enhance student learning. What was learned was that in order for effective TSIL practices to occur, teachers must have strong understandings and practices in all of these areas; content, pedagogy, and technology.

These findings and implications build on previous education research in the areas of mathematics education and educational technology, which have not previously examined all three of these areas; content, pedagogy, and technology in relation to one another. Prior research has examined the relationship between instructional technology and mathematics content (NCTM, 2000; Kaput, 1986; Dugdale et al., 1995) or mathematics content and pedagogy (Ball & Bass, 2000; Nathan & Koedinger, 2000; Ball, 1991) or pedagogy and their technology (Cuban, 1986; Becker, 1994, 2000; Norton & Wiburg, 1998; Etmer et al., 1999). However, the interactions between teachers' understandings and practices regarding content, pedagogy, and technology have not been examined in the level of detail of this study in previous education research. It is this overlap between content, pedagogy, and technology, and the ways they mutually influence participants' abilities to integrate TSIL practices within an effective learning environment, which offers the most powerful implications for future research.

The case studies that resulted from this work, and the extensive discussion of them, should benefit the education community as it seeks to better understand effective technology integration practices, in general, and mathematics education, in particular. These kinds of case examples and discussions are a way to help teachers and teacher educators build deeper understandings of the complex relationships between understandings and practices regarding content, pedagogy, and technology. The goal is for others to "analyze cases not to figure out how they can do what the teacher in the case example did; instead, the case discussions provide models for inquiry that teachers may apply to their own students' mathematical thinking and their own teaching practices (NRC, 2001a, p. 395)". I sincerely hope that is the result of all of this time and effort and these cases are utilized to further discussion in the education community about educational technology, teacher education, and mathematics education.

I believe, however, that this and all kinds of teacher professional development must be systemic. In thinking about Cyrus, for example, his dilemma of not feeling though he had the time to integrate problem solving, collaboration, and communication supported by technology could be related to local, state, and national assessment and accountability pressures. I feel that teachers need to spend time aligning their curriculum, with local, state, and national instructional standards, and assessment practices. I think that it is only when teachers can think about the content that they are teaching and that their students are being tested on in such a way as to see the breadth and depth and connections of the subject, as Rory does, that they can begin to try to really integrate technology to help students to develop deep, connected understandings. As long as teachers think that they have to check off a long list of topics they will not effectively utilize reform-based pedagogy, with or without technology. Patricia has started to make this movement by using technology to support the teaching and learning of "big ideas", and is now poised to continue to refine her understandings and practices in order to do so more effectively. I believe that working with all parts of the system; teachers, parents, administrators, and policy makers to understand that less topics, taught more deeply, broadly and connectedly will lead to stronger understanding is essential, so that teachers can have the time, motivation, and support to learn how to teach in the ways encouraged in this dissertation and TSIL can become a reality.

My next goals are to build off of this research by continuing to provide teacher professional development, and to make sure that it is ongoing and supports teachers understandings and practices regarding content, pedagogy, and technology, and is embedded within systemic reform. I would like to incorporate video case studies of exemplary technology using teachers like Rory into teacher professional development. By so doing, the complex interactions between content, pedagogy, and technology can be explicitly modeled and discussed and, hopefully, lead other teachers to deeper understandings and enactments of TSIL. I have enjoyed, and will continue to enjoy, working with preservice and inservice teachers and sharing theories and practices that can make mathematics education more meaningful, enjoyable, and comprehendible.

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Articles:

By July 7: Finzer, William. (2000). "Design of Fathom, a Dynamic Statistics Environment, for the Teaching of Mathematics"

Day 1: Velleman, P. and Hoaglin, D. (1992). Data Analysis. In *Perspectives on Contemporary Statistics*. MAA: Washington, DC.

Day 3: Gordon, F. & Gordon, S. (1992). Sampling + Simulation = Statistical Understanding: Computer Graphics Simulations of Sampling Distributions. In *Statistics for the 21st Century*. MAA:Washington, DC.

By July 22: White, B. and Frederikson, J. (1998). Inquiry, Modeling, and Metacognition: Making Science Accessible to All Students. *Cognition and Instruction*.

Minstrell, J. (2000). Implications for Teaching and Learning Inquiry: A Summary. In *Inquiring into Inquiry Teaching and Learning and Teaching in Science*. AAAS: Washington, DC.

Day 1: Focus on Exploratory Data Analysis

(Please note that all activities will alternate between having participants work independently to complete activities as students and having them demonstrate their learning of the technology and the content. After the completion of the activities, participants will discuss the activities as teachers.)

8:30 am. Introduction to Schedule for the Day and for the Week and to each other.

9:00 am. Data in Depth Activities from *Come to your Census.*

11:30 am. Discussion of Fathom, Data in Depth, and Census activities

12:00 am. Lunch

1:00 pm. Fifty Fathoms Demos: *The Meaning of Mean* and *Mean and Median*

1:30 pm. Workshop Statistics Activity: *Exploring Data pg. 170 (1,2) and 8-1, 8-2, and 9-1*

3:30 pm. Discussion of Fathom, Fifty Fathoms, and Workshop Statistics.

4:00 pm. Feedback forms and Adjourn

Day 2: Mathematical Modeling:

(Please note that all activities will alternate between having participants work independently to complete activities as students and having them demonstrate their learning of the technology and the content. After the completion of the activities, participants will discuss the activities as teachers.)

8:30 am.	Data in Depth Sonata: *Tall Buildings*
9:00 am.	Data in Depth Activity: *Fitting the Planets*
11:15 am.	Fifty Fathoms Demo(s): *Devising the Correlation Coefficient, Correlation Coefficient of Samples,* and *Regression Towards the Mean.*
11:45 am.	Lunch
1:00 pm.	Workshop Statistics Activity: *Least Squares Regression II (11-1, 11-2, 11-3)*
2:30 pm.	Data in Depth Activity: *Modeling Mauna Loa*
4:00 pm.	Feedback forms and Adjourn

Day 3: Simulations, Probability, Sampling Distributions and Inference:

8:30 am. Data in Depth Activities: *Exploring Sampling I and II*

9:30 am. Fifty Fathoms Demos: *Law of Large Numbers* and *Sampling Distributions and Sample Size*

10:00 am. Break

10:15 am. Workshop Statistics Activity: *Sampling Distributions I: Proportions (16-2, 16-3)*

11:30 pm. Fifty Fathoms Demo: *The Central Limit Theorem*

11:45 pm. Lunch

1:00 pm. Workshop Statistics Activity: *Tests of Significance II: Proportions (21-2)*

2:00 pm. Fifty Fathoms Demo: *Capturing Props with CIs*

2:15 pm. Closing Discussion regarding Fathom and the ways that it can support teaching and learning of EDA, Modeling, and Inference and strengths and weaknesses of the technology. Homework assignment: Continue to look at activities and at your curriculum and think of areas/ways that you would like to use Fathom next semester with your students. Read articles on Technology Supported Inquiry Learning and be prepared to discuss TSIL and Fathom. Bring your curricular materials and ideas and be prepared to play and to share.

2:45 pm. Final Paperwork

3:00 pm. Feedback forms and Adjourn

July 22

8:30 - Welcome back and "How you doin?" (collect vendor questionairs.)

8:47 - Discuss teaching and learning of Technology-Supported Inquiry

10:15 - Break (Note for Jeff – make photocopies of reading and evaluations)

10:30 - Conclude today's discussion of teaching and learning Technology-Supported Inquiry.

11:00 Work Time : Please use your curricular materials along with your Fathom materials and continue to think about how you will integrate them together.

12:00 - Lunch

1:00 - Work Time: Continue what you were doing before lunch.

(Please note that I will be touching base with each of you to hear about what you are thinking, brainstorm ideas with you, and set up times for us to do interviews and presentations later in the week.)

3:30 - Wrap-up, Brief Sharing, Feedback, Next Steps

4:00 - Adjourn

July 23

8:30 - Workshop Statistics Activity

9:00 - Discussion of Activity and How to Link it to Fathom. Importing Data into Fathom

9:15 - Break

9:30 - Work Time: Please continue your planning.

11:45 - Lunch

1:00 - Work Time: Prepare to share with your colleagues what you’ve been working on.

2:15 - Presentation and Discussion with Mrs. Donovan

3:15 - Closure

3:30 Adjourn (Note: Remember to get evaluation data to/from Christalina before she goes home.)

July 24:

8:30 - Work Time

9:30 - Break

9:45 - Presentation and Discussion with Mr. Williams

10:45 - Break

11:00 - Presentation and Discussion with Mrs. Suazo

12:00 - Lunch

1:00 - Presentation and Discussion with Mr. Clawson

2:00 - Follow-Up Plans, Evaluation data

2:30 - Adjourn