Developing Students' Understandings and Representations of Statistical Covariation

by

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Abstract

Statistical covariation refers to the correspondence of variation of two statistical measures that vary along numerical scales. Reasoning about covariation commonly involves translation processes among three representations: (1) numerical data, (2) graphical representations, and (3) verbal statements such as “taller people tend to be heavier.” Two well-known translations are *graph production* and *graph interpretation*. Less well known is the process of *speculative data generation*, involving translating a verbal statement into a possible graph or other data representation. This study explored school students’ reasoning involving these three translation skills through various tasks in surveys and interviews. Evidence is presented concerning methods to assess these skills, and concerning how students as young as third-grade can engage covariation tasks involving familiar contexts. Interviews involved prompting for cognitive conflict using responses from other students, and provided evidence of limited engagement of ideas that were slightly more sophisticated than their own responses.

Responses for each of the three translation skills were described within assessment frameworks involving four levels – Nonstatistical, Single Statistical Aspect, Inadequate Covariation, and Appropriate Covariation – distinguished by the structure of combining correspondence and variation. Distinguishing features of the levels suggested stages of development that may inform instruction. For development from prior beliefs to data-based judgements, tasks involving counterintuitive covariation were designed to prompt students to engage data. For development from single variables to bivariate data, time was observed as a natural covariate, implicit in statements such as “it’s getting hotter,” with a connotation of order that supported pattern recognition of passing time being associated with

corresponding change in a measured variable. For development from single cases to global trends, many students represented correspondence in a single pair of values, at the expense of representing variation. Tasks involving discrete data with few cases, and the use of case labels in responses, were observed to support the view of two data values each linked to the same corresponding case label. This consolidated view of correspondence supported consideration of additional bivariate cases involving variation. Students tended to articulate covariation using the language of comparison and change.

Findings were related to issues in the historical development of coordinate graphing, to findings from educational research in statistics, algebra, science and psychology, and to recommendations within curriculum documents. Student representations of statistical covariation were observed to provide a window into statistical reasoning, and are advocated as a valuable basis for classroom discussions to help develop statistical literacy.

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