Investigating the Nature of Teacher Knowledge Needed and Used in Teaching Statistics

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ABSTRACT

This thesis explores the knowledge needed for teaching statistics through investigations at the primary (elementary) school level. Statistics has a relatively short history in the primary school curriculum, compared with mathematics. Recent research in statistics education has prompted a worldwide move away from the teaching of statistical skills, towards a broader underpinning of statistical thinking and reasoning. New Zealand's nationally mandated curriculum reflects this move. Consequently, little is known about the types of knowledge needed to teach statistics effectively. Ideas from two contemporary areas of research, namely teacher content knowledge in relation to mathematics, and statistical thinking, are incorporated into a new framework, for exploring knowledge for teaching statistics.

The study's methodological approach is based on Popper's philosophy of realism, and the associated logic of learning approach for classroom research. Four primary teachers (in their second year of teaching) planned and taught a sequence of four or five lessons, which were videotaped. Following each lesson, a stimulated recall interview, using an edited video of the lesson, was conducted with the teacher.

The video and interview recordings were analysed in relation to the teacher knowledge and statistical thinking framework. The results provide detailed descriptions of the components of teacher knowledge in relation to statistical thinking that are needed and used in the classroom. Included in the results are profiles of each teacher's knowledge. These profiles describe 'missed opportunities', which were defined as classroom incidents in which teacher knowledge was needed but not used, and consequently resulted in the teachers not taking advantage of chances to enhance students' learning.

A number of significant themes were revealed, linked to knowledge for teaching statistics. The themes include: problems associated with teacher listening; the need for the teacher to be familiar with the data; students' difficulties with various components of the statistical investigation cycle; and understanding variation and the development of inference.

The study concludes that for effective teaching of statistics through investigations, it is necessary for teachers to have knowledge in each of four categories as related to each component of statistical thinking. If any aspect of knowledge is not available or not used, teachers will not enhance, and could disadvantage, students' learning. Implications from the findings are considered for initial and on-going teacher education.

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Chapter 1
Introduction

1.1 Broad aim of this research

This thesis examines the nature of knowledge that teachers need and use to teach statistics through investigations at the primary school level.

1.2 Background and rationale

1.2.1 Statistics in the curriculum

New Zealand is acknowledged as a world leader with regard to statistics in the school curriculum (Watson, as quoted in Begg, Pfannkuch, Camden, Hughes, Noble, & Wild, 2004), especially with respect to the primary school curriculum. Statistics has been included as part of the primary school mathematics curriculum (although not at all levels) since 1969, and in the mathematics curriculum at all levels of schooling from Year 1 to Year 13 since 1992. At the senior secondary school level, statistics was introduced as an option available within the subjects of Applied Mathematics and Additional Mathematics from the early 1970s through to the 1980s, when the Year 13 subject options were changed, one of the new subjects being Mathematics with Statistics (Begg et al., 2004). In contrast, other countries, such as Australia and USA, have typically only included statistics in curricula since the early 1990s (Watson, 2006). Even then, those curricula were not necessarily mandated nationally as was the case in New Zealand.

Currently, New Zealand is involved in a 'stock-take' and revision of its national Year 1 to Year 13 school curriculum. The stock-take commenced in 2003, and in 2006 the draft curriculum was released for consultation. This revision of the national curriculum, which includes the learning area of Mathematics, is the first to be undertaken since the previous review. That previous review resulted in a series of curriculum statements, the first of which (*Mathematics in the NZ Curriculum*) was released in 1992. As part of the current stock-take, and as a result of a number of submissions from interested groups, the Ministry of Education decided, in 2004, to re-name the learning area from Mathematics, to *Mathematics and Statistics.* Such a re-naming signals an increased prominence for Statistics as being a domain of learning related to, but not contained within, Mathematics. Moore (2004) applauds such recognition for statistics, as important for the development of statistics education in its own right, rather than being constrained under the umbrella of mathematics education.

Statistics education research, although relatively young in comparison with mathematics education research, has grown significantly in recent years, as evidenced by the number of international conferences and journals that are devoted to such research. Recent research in statistics education includes a strong thread focusing on the nature of statistical thinking, statistical reasoning, and statistical literacy. Wild and Pfannkuch's (1999) description of what it means to think statistically has made a significant contribution to the statistics education research field, and has provided a springboard for research that further explores and contributes to an understanding of statistical thinking and its application. Internationally, there is a marked trend in school curricula away from a focus on statistical skills (such as graphing, and finding measures to represent a set of data) towards reasoning and thinking statistically.

As part of the curriculum review process, the New Zealand Ministry of Education commissioned a literature review of research in statistics education. The comprehensive review by Begg et al. (2004), which drew on the available research literature, responses to a questionnaire from international experts in statistics education, and the expertise within the review team itself, included recommendations for the curriculum revision. These recommendations included statements regarding the place of statistics in the curriculum, the content to be taught, approaches to teaching, and the inclusion of aspects relevant to statistical thinking. The 1992 mathematics curriculum included a strand named the Mathematical Processes (which included problem solving, communication skills, and logic and reasoning) along with five content strands (number, algebra, measurement, geometry, and statistics). Begg et al. recommended that the new curriculum should emphasise conceptual understanding, and thinking – both mathematical and statistical. They also advocated for investigations and problem solving to be major themes for statistics in the new curriculum. Such recommendations were focused on building on what the 1992 mathematics curriculum had initiated with statistical investigations being a component of the statistics strand through all the levels of the curriculum from Year 1 to Year 13.

The latest curriculum draft for Mathematics and Statistics was published for consultation in 2006. It includes three strands, namely Number and Algebra, Measurement and Geometry, and Statistics. In line with the recommendations of Begg et al. (2004), the Statistics strand includes a strong emphasis on statistical investigations (thinking), by requiring students at all levels to be involved in conducting investigations. The explicit reference to statistical thinking provides implicit acknowledgement of some significant, contemporary research from the statistics education field.

1.2.2 Teacher knowledge

Debate about teacher knowledge and its connections to student learning has had a long history. At one level, anecdotal comments from secondary school students have often lamented the fact that their teachers have had the mathematics background ('they knew their subject'), but did not know how to get it across, at a level and in a way that contributed to the development of students' understanding. At another level and using findings from research, Ball and Bass (2000) strongly argue that without adequate mathematical knowledge, teachers will not be in a position to deal with the day-to-day, recurrent tasks of mathematics teaching, and as such, will not cater for the learning needs of diverse students. They assert:

> No matter how committed one is to caring for students, to taking students' ideas seriously, to helping students develop robust understandings, none of these tasks of teaching is possible without making use in context of mathematical understanding and insight. (Ball & Bass, 2000, p. 94)

Research on teacher knowledge has had various foci. Ball (1991a) identifies three phases of research on teaching, in which teacher knowledge was the focus during the first phase (through to the 1960s and 1970s) and the third phase (from the 1980s). The first phase was generally concerned, according to Ball, with identification of the characteristics of good teachers. Although subject matter knowledge was claimed to be an important characteristic, either the research did not test this against student outcomes, or, if tested, the measures used for teacher knowledge were later adjudged to be inappropriate. Some of these measures included the number of courses taken by teachers as part of their qualifications, the length of teaching experience, teachers' personal enjoyment of mathematics, or some aspect of their teacher education programmes. It is recognised that using such measures as proxies for teacher knowledge yields little with regard to explaining differences in student achievement (Rowan, Correnti, & Miller, 2002). The second phase of research on teaching (Ball, 1991a), characterised as investigations of what teachers do, studied general pedagogical strategies, such as questioning, use of praise, use of groups, and pacing of lessons. With regard to mathematics, this phase of research examined student gains in terms of the mastery of skills through drill and practice. Because such a conception of mathematics was seen to be limited and simplistic, and therefore disregarded the complexity of classrooms and of teaching, changes in research characterised the onset of the third phase. Similar to the first phase, the focus during the third phase was teacher knowledge, particularly with regard to teachers' thoughts and decisions. For mathematics teaching, it was mathematics knowledge that was believed to be critical. However, Ball acknowledges that varied conceptions and definitions of subject matter knowledge in mathematics have threatened to "muddy our progress in learning about the role of teachers' mathematical understanding in their teaching" (1991a, p. 5).

Contemporary research literature recognises that effective teaching is dependent on teacher knowledge. The research literature includes examples of both positive outcomes for students arising from strong teacher knowledge, and negative outcomes resulting from inadequate and/or inappropriate teacher knowledge. Anthony and Walshaw (2007) summarise an extensive range of research literature that supports the importance of various types of teacher knowledge in relation to students' development of understanding, the establishment of communities of effective mathematical practice, and the implementation of effective pedagogy. Some of that literature found negative outcomes in relation to classroom discourse and students' learning, stemming from teachers' inadequate use of particular categories of knowledge. For example, the use of certain scaffolding strategies by teachers resulted in students merely being kept busy to achieve task completion rather than in encouragement of student learning (Myhill & Warren, 2005).

Teacher knowledge is often targeted at a system level, for instance through the provision of professional development programmes. For example, one of the stated aims of the Numeracy Development Programme in New Zealand is to develop teacher knowledge:

> Much of the focus of the work in literacy and numeracy is on increasing teacher content knowledge and their knowledge of how students learn in these areas along with the teaching practice most likely to create conditions for success.
>
> http://www.tki.org.nz/r/literacy_numeracy/index_e.php

Another example of the drive towards enhancing teacher capability comes from the New Zealand Teachers Council, the government organisation responsible for the accreditation and auditing of initial teacher education programmes, and the registration of teachers. The Teachers Council recently published (in 2007) a set of standards for teachers graduating from initial teacher education programmes. These standards, which become mandatory from 2008, include:

> Standard 1: Graduating teachers know what to teach: a) have content knowledge appropriate to the learners and learning areas of their programme; b) have pedagogical content knowledge appropriate to the learners and learning areas of their programme.
>
> (NZ Teachers Council, 2007)

The publication of such standards raises the important question as to what content knowledge and pedagogical content knowledge is considered adequate and appropriate. Although much is known about teacher knowledge pertinent to particular aspects of mathematics, the situation for statistics is less clear. Arguably, the mathematical knowledge needed for teaching and the statistical knowledge needed for teaching do share some similarities. Yet, there are also differences, due in no small way to the more subjective and uncertain nature of statistics compared with mathematics (Moore, 1990). Pfannkuch (2006, personal communication) claims that, because of the relatively brief history of statistics education research in comparison with mathematics education research, there is still much that is unknown about the specifics of teacher knowledge needed for statistics. As one example, it is her belief that few teachers have a sufficient grasp of the difficulties that students experience with developing understanding in statistics.

1.3 Research hypothesis and questions

Two key themes underpin this research. These themes are: (i) the increasing status of statistics in the curriculum, and (ii) a lack of shared understanding about the knowledge needed to teach statistics to students of diverse abilities. The following research hypothesis provides the focus for this study.

1.3.1 Hypothesis

All aspects of teacher knowledge in relation to the components of statistical thinking are necessary for the work of teaching statistics through investigations, and the absence of any aspect will impact negatively on the learning opportunities for students.

In order to put this hypothesis to the test, the following research questions will be investigated:

1.3.2 Questions

1. *What types of teacher knowledge in relation to the components of statistical thinking are needed and/or used in the work of teaching statistics through investigations?*

2. *What are the features of such knowledge in relation to aspects of statistical thinking?*

3. *Are there types of teacher knowledge in relation to components of statistical thinking that are not in evidence in the classroom and, although absent, do not impact on the potential learning opportunities for students?*

4. *Does teacher knowledge grow in the course of teaching? If so, what are the conditions or events that lead to the growth of teacher knowledge?*

1.4 Overview of this thesis

Following this introductory chapter, Chapter 2, *Literature Review*, examines the research literature pertinent to this study. It looks at the broad teacher knowledge research domain, in particular the research of Shulman (1986), which provided an impetus for a vast amount of subsequent research, particularly research that focused on what he termed 'pedagogical content knowledge'. From research on teacher knowledge in general, research on teacher knowledge in relation to mathematics is then reviewed.

Statistics education research reveals a number of issues that arguably differentiate the needs of statistics from those of mathematics, with regard to teaching and learning. For example, in mathematics one counterexample is sufficient to disprove a hypothesised generalisation; in statistics, however, one counterexample may merely illustrate the inherent variation in data without discrediting the hypothesised generalisation. Such differences point to a need to consider those differences when developing research on statistics teaching.

The literature review synthesises various research approaches from the mathematics education and statistics education domains, and from that synthesis, a new framework is proposed for investigating teacher knowledge in relation to statistics. This framework, which integrates ideas from the two domains, will then be used to analyse the knowledge needed and used in teaching statistics.

Chapter 3, *Methodology in Theory*, considers broad questions in relation to conceptions of knowledge, through to those specific to conducting research in the classroom. As a result of considering these epistemological issues as they relate to this study, a theoretical approach is argued as being appropriate for this research study. It is broadly derived from Popper's philosophy of post-positivist realism. Methodological issues with regard to the conduct of such research, including those in relation to data collection, and the ethical conduct of research in the classroom, are discussed.

The details of the conduct of the research are outlined in Chapter 4, *Methodology in Practice*. This chapter describes the selection of teacher participants, the acquiring of informed consent from the teachers, the students, and the students' parents or caregivers, the data collection processes (for video of the classroom lessons and audio of stimulated recall interviews with the teachers), and the coding and analysis of the data.

Chapter 5, *Results* describes in detail the teacher knowledge needed and/or used in the classroom, in relation the framework, and provides some examples from the data to support the knowledge descriptions. Each teacher is then 'profiled' with regard to his or her knowledge, particularly in relation to 'missed opportunities' in the lessons. These missed opportunities, which represent classroom incidents in which teacher knowledge was needed but not used, are identified and discussed in relation to their links to the teacher knowledge framework.

A number of themes were identified from the *Results* as being common across a number of teachers. These themes are discussed in Chapter 6, *Discussion: Significant Themes* in relation to the teacher knowledge framework, and with research literature links where available and appropriate.

In Chapter 7, *Conclusions and Implications*, each research question is addressed, along with the overall research hypothesis. This study's contribution to the research field is outlined, as are possible limitations of the study, and suggested implications arising from this study. The main implications include recommendations regarding initial teacher education and the professional development of practising teachers. The recommendations, for both groups, focus on developing the various categories of teacher knowledge in statistics, through the use of investigations. The connections between the categories of teacher knowledge, as shown in this study, indicate that these categories must be targeted in an integrated way.

1.5 Usage of terms

In this thesis, the term 'primary' is used in preference to 'elementary' to refer to pre-secondary levels of schooling. Overseas research conducted in, or pertinent to, 'elementary' school is discussed in this thesis in relation to a 'primary' school focus. In New Zealand, students attend primary school from Year 1, with starting age generally five years old, through to Year 8. In the majority of cases Year 7 and Year 8 students attend an 'intermediate school', a type of middle school which is considered part of the primary system, with mainly generalist classroom teachers, rather than subject specialists as in secondary schools. Secondary schooling in New Zealand is generally from Year 9 through to Year 13.

With regard to teacher education, the terms pre-service teacher education and initial teacher education are used interchangeably in this thesis, as both are used in the research literature.

Chapter 2
Literature Review

2.1 Introduction

This thesis examines the nature of the knowledge that teachers use while teaching statistics. The literature review initially examines the field of teacher knowledge from the 'ground-breaking' work of Shulman (1986). Shulman's classification of components of teacher knowledge provided an important base from which subsequent researchers developed their work.

From a general look at teacher knowledge, the review then proceeds to the literature pertinent to teacher knowledge and mathematics education. Various domains of knowledge, each with their unique characteristics, have been the focus of research, and mathematics is generally considered to present its own challenges as far as knowledge relevant to teaching and learning is concerned.

Irrespective of whether statistics is viewed as a sub-domain of mathematics or as a domain in its own right, the literature identifies differences between mathematics teaching and learning, and statistics teaching and learning. As a discipline, statistics is much 'younger' than mathematics, and because it is new to many teachers, its status, in relation to the teaching and learning of it will be examined in this literature review.

The broad teacher knowledge literature base and the specifics of teaching and learning in statistics both contribute to the development of a framework that might be useful for researching teacher knowledge as it is used and develops in the teaching of statistics. The framework is broadly described, and its potential for investigating teacher knowledge is discussed.

2.2 Knowledge bases for teaching

It is a truism that a teacher needs to know something in order to be able to teach. Just what knowledge or how much is needed is much less clear, and has been the focus of a significant amount of research. The work of Shulman (1986; 1987) has been influential in classifying and defining aspects of teacher knowledge that had not previously been part of the lexicon of teacher knowledge.

Shulman (1986) describes three categories of teacher content knowledge, namely subject matter knowledge, pedagogical content knowledge, and curricular knowledge. Subject matter knowledge refers to the knowledge of facts and concepts, and understanding the structure of the subject. The second category, for which Shulman coined a new term of 'pedagogical content knowledge', refers to the aspects of subject matter knowledge that are specifically required for teaching. Shulman claims that this knowledge goes beyond that of the subject specialist, such as the mathematician. Pedagogical content knowledge includes knowledge of the most useful forms of representation of ideas within a topic,

> the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject to make it comprehensible to others … [and] includes an understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p. 9),

and consequently knowledge of how learners may be assisted in their learning of these concepts. The third category of content knowledge, curricular knowledge, includes knowledge of the sequence of topics or concepts to be taught and the materials and resources suitable for a particular topic.

Subsequent to his original work of describing the categories of teacher content knowledge, Shulman (1987) outlines seven categories of knowledge that he claims to be the requisite knowledge base for teaching: content knowledge, general pedagogical knowledge (such as principles and strategies for classroom management and organisation), curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts (such as the workings of a group or classroom through to the character of the community in which a school is situated), and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. There is a certain discrepancy between Shulman's 1987 listing of the categories of the knowledge base for teaching, in which content knowledge is listed alongside pedagogical content knowledge and curriculum knowledge, and his 1986 description that classifies pedagogical content knowledge and curriculum knowledge as sub-categories (along with subject matter knowledge) or components of content knowledge. It would appear that although his 1987 paper lists content knowledge as one of the seven knowledge areas, Shulman might have intended it to be the same as, or representative of, subject matter knowledge.

Several researchers have developed models and frameworks for teacher knowledge based on Shulman's work. For example, following Shulman's work with classifying teacher knowledge, and in particular his definition of pedagogical content knowledge, Marks (1990) further refined pedagogical content knowledge into four components: subject matter for instructional purposes; students' understandings of the subject matter; media for instruction in the subject matter; and instructional processes for the subject matter. In spite of developing these categories of pedagogical content knowledge, Marks acknowledges the difficulty in classifying aspects of a teachers' knowledge into one particular category of pedagogical content knowledge, or even into one of the three broader categories of pedagogical content knowledge, subject matter knowledge, and general pedagogical knowledge. The indistinct boundaries between the various categories of knowledge, or the overlapping nature of those categories, were, for Marks, a source of difficulty. He claims that part of the difficulty is attributable to the different ways in which pedagogical content knowledge develops: in some cases, it develops from subject matter knowledge, while at other times it develops from general pedagogical knowledge. In his categorisation of knowledge, Marks refers to the broad category of subject matter knowledge, but it is not clear how he differentiates between this and one of his sub-categories of pedagogical content knowledge, namely subject matter for instructional purposes.

Grossman (1990), in a similar way to Marks (1990), has also categorised pedagogical content knowledge into four sub-categories. Her particular categories are: conceptions of the purposes for teaching subject matter; knowledge of students' understanding; curriculum knowledge; and knowledge of instructional strategies. These categories show a significant difference from Marks' categorisation through the absence of something equivalent to Marks' 'subject matter for instructional purposes'. However, it could be argued that this component would fit the broader category of subject matter knowledge, rather than being a sub-category of pedagogical content knowledge. Barnett and Hodson (2001) coin another variation of teacher knowledge, ‘pedagogical *context* knowledge’, since they recognise that teacher knowledge is situated in the detail and intricacies of everyday classroom life. They suggest that pedagogical context knowledge has four components: pedagogical content knowledge; professional knowledge; classroom knowledge; and academic and research knowledge. Through interviewing teachers, they have further categorised teacher knowledge into more sub-categories, and sub-sub-categories. They conclude “that pedagogical context knowledge provides a simple and effective way of examining teachers’ views and the knowledges on which they draw when they teach or talk about their teaching” (p. 448). Given that they recognise the importance of context in relation to knowledge and teaching, it is noted that their subsequent analysis of teacher knowledge is based only on interviews with the teachers away from the classroom.

Yet another variation for the classification of teacher knowledge is proposed by Cochran, DeRuiter, and King (1993). They suggest that teaching is concerned with developing ‘autonomous conceptual understanding’, and consequently to account for the dynamic nature of the teacher knowledge, they suggest that the term ‘pedagogical content knowing’ is more appropriate than a more static ‘pedagogical content knowledge’. As defined by Cochran, DeRuiter, and King, pedagogical content knowing is “a teacher’s integrated understanding of four components of pedagogy, subject matter content, student characteristics, and the environmental context of learning” (p. 4). They argue that the development of pedagogical content knowing is continual and strengthens over time. Also, the latter two characteristics of pedagogical content knowing (i.e., student characteristics and the environmental context of learning) are emphasised to a greater extent than in Shulman’s taxonomy, but all four components are necessary for a strongly integrated knowledge structure, even for beginning teachers. They suggest that the integration of the four components results in conceptual change sufficient for the resulting pedagogical content knowing to be “distinctly and qualitatively different from the types of understanding from which it was constructed” (Cochran et al., 1993, p. 5).

Differentiating between subject matter knowledge and pedagogical content knowledge has been a source of difficulty for a number of researchers. In acknowledging that difficulty, Sherin (2002) has formulated an alternative component of teacher content knowledge, namely ‘content knowledge complexes’. She argues that previous teaching enables teachers to make connections between the content they are teaching and the strategies for teaching that content, that is, between subject matter knowledge and pedagogical content knowledge. Consequently, teachers access both subject matter knowledge and pedagogical content knowledge simultaneously, and the connections between them make it impossible to distinguish the separate types of knowledge; such connected aspects of knowledge are categorised as ‘content knowledge complexes’. Sherin believes that teachers access these content knowledge complexes as a whole rather than as separate and distinct pieces of subject matter knowledge and pedagogical content knowledge.

Sherin (2002) found that for experienced mathematics teachers, when trying to deliver a teaching unit involving a teaching approach significantly different from what they usually used, one of three processes occurs: ‘transform’, ‘adapt’, or ‘negotiate’. Transform refers to the situation where the teacher recognises a similarity between the new approach and previously taught content, and instead of changing their existing pedagogical content knowledge to accommodate the new approach, the teacher implements his or her existing pedagogical content knowledge and consequently changes the intended outcome of the new approach to an outcome similar to the previous approach; the lesson reverts to a familiar, traditional one with different outcomes from what were intended. In the ‘adapt’ situation, a student’s question or response is responsible for the teacher realising that his or her current pedagogical content knowledge is inappropriate for the situation as presented, and the usual content knowledge complex associated with that particular subject knowledge is not suitable. The teacher therefore draws on his or her broader subject matter knowledge and develops an appropriate but new pedagogical response. So the teacher’s pedagogical content knowledge has been changed to ensure that the intended lesson outcome is maintained. In the third type of process, namely ‘negotiate’, again a student interaction forces the teacher to realise that the typical content knowledge complex is not appropriate, so the teacher develops new pedagogical content knowledge after drawing on his or her broader subject matter knowledge. In this case, the new instructional strategy changes the lesson structure or possibly the purpose of the lesson. This change has implications for the way that the teacher responds to students as the lesson progresses further. New connections begin to develop between the subject matter knowledge and the new pedagogical content knowledge, so essentially a new content knowledge complex begins to form while the existing content knowledge complex is refined.

As Sherin's (2002) research involved experienced secondary mathematics teachers, it is unknown whether primary teachers would develop content knowledge complexes in the way she has described, since it is well-accepted that primary teachers do not generally have the same level of subject matter knowledge as secondary teachers. Similarly, it is also unknown whether inexperienced teachers would develop content knowledge complexes in which pedagogical content knowledge is strongly connected to subject matter knowledge.

Other researchers have categorised teacher knowledge in ways significantly different from Shulman and his 'successors' as outlined above. Leinhardt and Greeno (1986) identify only two major categories of teacher knowledge: practical knowledge for teaching (such as of lesson structure) and subject matter knowledge. They claim that the structuring of a lesson takes priority over, yet is constrained by, the content knowledge of what is to be taught. They describe a lesson in terms of an 'agenda' (the overall dynamic plan for a lesson including its goals and actions), a 'script' (the outline of the content to be presented and the way of presenting it), explanations (what the teacher says, does or demonstrates), and representations (of the mathematics, whether physical, verbal, concrete, or numerical). From their research, Leinhardt and Greeno contend that lessons generally proceed as planned with only small deviations due to input from students. The teacher's content knowledge is used mainly in the planning of the lesson but also assists the teacher in dealing with deviations from what has been planned. There are limitations of this model for teacher knowledge, namely that it does not acknowledge or examine learning at the individual student level nor deal with the richness and depth of the content involved in the lesson. It is often at the level of working with individual students and dealing with their questions or problems that a teacher's content knowledge can be most challenged.

2.2.1 Subject matter knowledge

Within the domain of mathematics, a number of researchers have looked at the categorisation of knowledge related to mathematics. Some of these categorisations are: conceptual and procedural knowledge (e.g., Hiebert & Carpenter, 1992; Timmerman, 2002); substantive knowledge (knowledge of facts, concepts and algorithms) and syntactic knowledge (knowledge of methods of proof and argument) (Schwab, 1978); and knowledge of mathematics (meanings, and underlying procedures) and knowledge about mathematics (notions of mathematics as a discipline, where it comes from, how it changes, and how truth is established) (Ball, Lubienski, & Mewborn, 2001). Grossman, Wilson, and Shulman (1989), in discussing subject matter knowledge for teachers, suggest that it consists of four dimensions. These are: content knowledge for teaching (which includes the factual information, organising principles and central concepts), substantive knowledge (the explanatory frameworks or paradigms that are used both to guide inquiry in the field and to make sense of data – these can influence the curricular decisions that are made by a teacher), syntactic knowledge (that is, how new knowledge is introduced and accepted into the community – mathematics goes beyond just learning algorithms to solve problems), and teacher beliefs. Grossman, Wilson, and Shulman acknowledge that this subject matter knowledge must still be transformed into a form of knowledge that is appropriate for students. The classification of subject matter knowledge, as given by Grossman, Wilson, and Shulman, was developed through research conducted with novice secondary teachers and as such, it is questionable whether it would be helpful for examining the mathematical subject matter knowledge of primary teachers.

Some researchers on subject matter knowledge have categorised aspects into more than four dimensions. Even's (1990) classification of subject-matter knowledge for teaching the concept of mathematical function consists of seven categories. This framework for subject matter knowledge, which goes into significant detail, is relevant for secondary teaching. Although it is acknowledged that pre-service teachers will have developed some of their knowledge throughout their own schooling, the argument is put forward that teachers need the opportunity for courses in which they can learn mathematics for teaching. A limitation of Even's framework is that some of the seven categories of subject-matter knowledge could be classified as pedagogical content knowledge.

2.2.2 How does teacher knowledge develop?

Having defined and described various ways of categorising teacher knowledge, it is important to consider how this knowledge develops. Some of the research described above neglects this aspect. Shulman's (1986) premise is that teachers begin with some level of subject matter knowledge. In describing how the novice teacher becomes an expert teacher, Shulman concludes that pedagogical content knowledge develops through a process of 'transformation' from subject matter knowledge. However, since Shulman's research was conducted with secondary teachers, it is debatable as to whether primary teachers develop pedagogical content knowledge in the same way as secondary teachers. It is generally accepted that secondary teachers come to teaching with a higher level of subject matter knowledge than primary teachers.

As a result of conducting research with primary teachers, Marks (1990) contends that pedagogical content knowledge can develop in the same way as Shulman suggests, involving what he (Marks) termed 'interpretation'. However, as a point of difference from Shulman's idea, Marks also claims that pedagogical content knowledge can develop through a reverse process: "the application of general pedagogical principles to the particular subject matter contexts" (Marks, 1990, p. 7). He refers to this process as "specification, that is, the appropriate instantiation of a broadly applicable idea in a particular context" (Marks, 1990, p. 8). An example of specification would be the teacher applying his or her knowledge of questioning strategies (a general pedagogical skill) to a particular content area. For the knowledge that develops in this way, Marks coined the term 'content-specific pedagogical knowledge' which he suggests better describes the nature of that knowledge than the term 'pedagogical content knowledge'. Marks acknowledges, however, that there are situations in which the development of pedagogical content knowledge represents a synthesis of general pedagogical knowledge, subject matter knowledge, and previous pedagogical content knowledge.

Another to question the applicability to primary teachers of Shulman's model of teacher knowledge development is Poulson (2001). She suggests that knowledge of content appears to be pedagogically situated. Similarly, Grossman (1990) identifies four possible sources of pedagogical content knowledge, namely apprenticeship of observation, subject matter knowledge, teacher education, and classroom experience. However Grossman does not elaborate on the processes by which pedagogical content knowledge develops from these sources. Veal and MaKinster (1999), through a study of secondary science teachers, contend that for pedagogical content knowledge development, strong subject matter knowledge and knowledge of students (which includes understanding possible student errors and misconceptions) are both essential. They argue that there is a hierarchical structure to pedagogical content knowledge, although teachers may possess some aspects of pedagogical content knowledge prior to, for example, a full understanding of their students. Whatever the case, Veal and MaKinster claim that subject matter knowledge is a prerequisite. However, having developed a taxonomy for pedagogical content knowledge, Veal and MaKinster (1999) do not specifically address the development of pedagogical content knowledge, other than acknowledging that teaching experience plays an important part, and that pedagogical content knowledge develops throughout the teacher's career. The relevance to primary teachers, particularly inexperienced ones, of such a model of pedagogical content knowledge and its development, is questionable because of the stated importance of prior and strong subject matter knowledge.

2.2.3 Teacher knowledge and teaching

Research on teacher knowledge has been conducted in many ways. Historically, some researchers measured teacher knowledge in relation to indicators such as the number or types of courses that the teachers had passed (Ball et al., 2001). This approach essentially focused on the teacher as the unit of study. Another approach has been to focus on teacher knowledge through various categories (e.g., Shulman, 1986) and as such, teacher knowledge has been the unit of study. Subsequent approaches have tried to combine these two earlier approaches, thereby recognising the importance of the act of teaching.

Cobb and McClain (2001) advocate approaches for working with teachers that do not separate the pedagogical knowing from the activity of teaching. They argue that unless these two are considered simultaneously and as interdependent, knowledge becomes treated as a commodity that stands apart from practice. Their research focused on the moment-by-moment acts of knowing and judging. Similarly, Ball

(1991b) discusses how teachers' knowledge of mathematics and knowledge of students affect pedagogical decisions in the classroom. For instance, the subject matter knowledge of the teacher determines to a significant extent which questions from students should or should not be followed up. Similarly, subject matter knowledge enables the teacher to interpret and appraise students' ideas. Ball proposes four dimensions of subject matter knowledge: substantive knowledge of mathematics, knowledge of the nature and discourse of mathematics, knowledge about mathematics in culture and society, and the capacity for pedagogical reasoning about mathematics. In this categorisation, the dimension of pedagogical reasoning relates to how teachers draw on their knowledge of mathematics within the dynamics of teaching.

Questions concerning the mathematical knowledge used in teaching and how it is used should be the focus of research (Ball et al., 2001; Mewborn, 2001). Activities and tasks that teachers engage in include: "figuring out what students know; choosing and managing representations of mathematical ideas; appraising, selecting and modifying textbooks; deciding among alternative courses of action; and steering a productive discussion" (Ball et al., 2001, p. 453). They suggest that there is significant mathematical reasoning and thinking occurring as the teacher goes about these activities and tasks. However, these aspects of a teacher's role are really indicators of other knowledge categories of teachers, such as pedagogical content knowledge, or knowledge of students. The inference to be drawn from this is that Ball et al. recognise the relationships and interdependencies between the various components of teacher knowledge, particularly as it is situated in classroom teaching. This is complicated by the problem however that there will be times when the teacher is faced with a situation (such as an unexpected question or the need for a new representation or explanation) in which their current pedagogical content knowledge is inadequate (Ball & Bass, 2000). The ways in which teachers deal with these situations, especially through drawing on their mathematical knowledge, has been of particular interest to Ball and Bass.

A focus on the knowledge of content that is required to deliver high-quality instruction to students has led to another model of teacher knowledge, which involves a refinement of the categories of subject matter knowledge and pedagogical content knowledge. Hill, Schilling, and Ball (2004) developed assessment items relevant to number and algebra that could identify what and how content knowledge was used by teachers in instruction. The test items revealed whether a teacher's responses were related to the teacher's general mathematical ability (common knowledge of content) or indicated the existence of some specialised knowledge for teaching (specialised knowledge of content). Hill et al. (2004) claim that teacher knowledge is organised in a content-specific way, rather than being organised for the 'generic tasks of teaching', such as evaluating curriculum materials or interpreting students' work. Two sub-categories of content knowledge are further clarified by Ball, Thames, and Phelps (2005): *common knowledge of content* includes the ability to recognise wrong answers, spot inaccurate definitions in textbooks, use mathematical notation correctly, and do the work assigned to students. In comparison, *specialised knowledge of content* needed by teachers (and likely to be beyond that of other well-educated adults) includes the ability to analyse students' errors and evaluate their alternative ideas, and give mathematical explanations and use mathematical representations. Ball et al. (2005) also subdivide the category of pedagogical content knowledge into two components, namely *knowledge of content and students*, and *knowledge of content and teaching*. These two parts of teacher knowledge bring together aspects of content knowledge that are specifically linked to the work of the teacher, but are different from specialised content knowledge. *Knowledge of content and students* includes the ability to anticipate student errors and common misconceptions, interpret students' incomplete thinking, and predict what students are likely to do with specific tasks and what they will find interesting or challenging. *Knowledge of content and teaching* deals with the teacher's ability to sequence the content for instruction, recognise the instructional advantages and disadvantages of different representations, and weigh up the mathematical issues in responding to students' novel approaches.

Recognition of the need to examine teacher knowledge in relation to teaching has provided a basis for examination by other researchers of mathematical content for teaching. Kahan, Cooper, and Bethea (2003) categorise aspects of teaching into four processes (namely preparation, instruction, assessment, and reflection) in relation to six elements of teaching (setting of goals and objectives, selection of tasks and representations, motivation of content, development through connectivity and sequencing, allocation of time, points, and emphasis, and discourse). These six elements are, according to Kahan et al., the facets of teaching in which content knowledge will matter the most. For the resulting matrix of 24 cells, Kahan et al. describe features of teaching and knowledge, or give examples from classroom research, that are appropriate to each cell. The researchers acknowledge that in this two-dimensional matrix, some cells are better sources of aspects of mathematical content knowledge than others. They suggest that this framework is part of a larger, three-dimensional one, with the third dimension corresponding to another aspect of the teacher being explored, such as pedagogical content knowledge (Kahan et al., 2003). Such a three-dimensional framework would give a fuller description of teaching and teacher knowledge, although there would be limitations because of the complexity of clearly identifying and distinguishing between the various three-dimensional components of knowledge.

2.3 Researching teacher knowledge in the classroom

2.3.1 Introduction

Whether teacher knowledge can be adequately assessed outside of the classroom in which that knowledge is used has been increasingly debated. Sorto (2004) acknowledges that it is important to explore teachers' concepts within teaching contexts, but concedes that observation of teachers, in the real context of the classroom, was beyond the scope of her study of middle school teachers' knowledge of data analysis. Sorto's study was instead based around the development and administration of a written assessment instrument along with some follow up interviews with the teachers. The assessment items were developed with a focus on classroom contexts, in order to probe pedagogical content knowledge in addition to subject matter knowledge. Heaton and Mickelson (2002) concur with Friel and Bright (1998) that teachers' understanding of content can be examined by watching them teach, in particular observing how they deal with 'the teachable moments' that arise in a lesson. Heaton and Mickleson's study involved pre-service primary teachers while Friel and Bright's study arose from professional development workshops with primary teachers.

The context of any particular classroom influences the learning that occurs within that classroom. Since a classroom consists of not only individual students and the teacher, but also social practices that govern what happens within that classroom, Borko et al. (2000) insist that any study of knowledge and learning in the classroom must occur within the classroom. Likewise, Cobb and McClain (2001) argue that the tasks of teaching (such as planning for learning, interacting with students, and evaluating classroom incidents) provide the primary contexts for teachers' learning, and consequently research needs to be situated within these contexts. Such situated research must take into account and examine learning both at an individual level and the wider, social level (Cobb, 2000).

2.3.2 Impact of 'teacher talk' on classroom discourse

The importance of classroom discourse for mathematics learning is increasingly recognised in research literature. The teacher's role in, and responsibility for, developing students' appropriate use of mathematical language, thereby enabling the students to develop mathematical understanding, is explored by Anthony and Walshaw (2007). Forman (2003) argues that the continual use of appropriate language by the teacher enables students' concepts associated with such terms to become more refined. It has also been suggested (Boaler, 2000) that students' development of mathematical content knowledge cannot be separated from the practices of their classroom, such as the discourse-related practices.

Just as students' understanding and knowledge is linked to the practices of their classrooms, it can likewise be argued that teacher knowledge is also linked to, and impacts on, the practices of the classrooms in which the teachers teach. For example, Borko et al. (2000) found that in relation to classroom discourse, teachers with a high level of content knowledge (when compared with other teachers) asked fewer questions but the questions were of higher order; the teachers talked less and for shorter periods of time; and their students talked more, asked more questions, volunteered to speak more, and spoke in longer sequences. Mercer (1995), in considering the nature of teacher talk, has suggested that teachers elicit comments from students, respond to these comments, and at the 'highest' level, recap (including 'reconstructively' recap) so that the discursive experiences in the classroom contribute to the students' educational experience. Mercer identifies different ways in which teachers respond to students as part of the well-documented initiate-respond-feedback sequence of teacher talk. These responses, which enable the teacher to incorporate what students are saying into the general flow of the discussion, are: confirmation; repetition; paraphrasing or reformulation; elaboration; rejection; and ignoring. Although Mercer lists these as types of responses, 'ignoring' could be considered more of a non-response, and he does not suggest why such a non-response might occur. However, as responding must be preceded by listening to students, any type of response from a teacher must be dependent on what a teacher hears students say.

2.3.3 Impact of teacher listening on classroom discourse

Teacher practices in relation to listening to students have been found to have significant impact on classroom discourse. However listening to students is not a straightforward task for teachers. Research on teacher listening has examined types of listening (such as Anghileri, 2006), and levels of listening (such as Davis, 1997), but O'Connor (2001) claims that research has only rarely looked at how teachers deal with incorrect student responses. O'Connor goes on to suggest this is a topic that has serious consequences for classroom discussion. Davis (1997) identifies evaluative listening (whereby a teacher compares the response with a preconceived answer or standard, and is therefore not really interested in what the student is saying) as the lowest of three levels of listening with regard to developing classroom discourse. Davis refers to the other two types of listening as interpretive listening, in which there is a more active attempt at connecting with and sense-making, and hermeneutic listening, in which there is negotiated and participatory interaction between teacher and students. However in Davis's discussion of evaluative listening, there is no consideration given to the situation of a teacher not evaluating the student's comment (just as Mercer (1995) did not suggest reasons for 'ignoring' as a type of teacher response). Studies by Even and Wallach (Even & Wallach, 2003; Wallach & Even, 2005), one of which involved problem solving by fourth grade students, while the other included algebra with seventh graders, refer to different types of hearing on the part of teachers, and consider the reasons behind the types of hearing. They identify: over hearing – hearing more than the students actually say; under hearing – missing some of what the students say; compatible hearing – making sense of and connecting with what students say; non-hearing – missing the whole message of the student; and biased hearing – the amount heard depends on who is saying it. Even and Wallach claim these different types of hearing are brought about by teachers hearing ‘through’ a complex set of factors, such as teacher knowledge, dispositions, feelings about students, expectations, beliefs about mathematics learning and teaching, as well as the context in which the hearing takes place. O’Connor (2001) suggests that the complexity for a teacher in trying to understand what a student is saying, keeping track of the sequence of contributions from students, and thinking about how to respond, all within a ‘conversationally appropriate’ two to three second interval, impacts significantly on a teacher’s classroom decision making with regard to a response. Consequently, the type of teacher response to a student’s comment may not be the best and most appropriate.

Scaffolding of student learning involves making connections with students and finding a way to develop their understanding further, and is therefore dependent on teacher listening and teacher knowledge. Anghileri (2006) refers to one type of scaffolding as restructuring, which involves making contact with the student’s understanding and being able to move it forward. When this does not happen, as in the teacher not seeking further clarification from a student following the student’s contribution to a discussion, it can be considered a case of what is termed a ‘pseudo-interaction’ or ‘by-passing’ (Bliss, Askew, & Macrae, 1996). In this situation, the conditions for scaffolding are present, but not noticed by the teacher, and thus no real interaction occurs between the teacher and students. Bliss and colleagues do not account, however, for why the teacher does not help the students clarify their thinking. O’Connor (2001) refers to the possible lack of comfort experienced by the teacher, student, and other students, when attempting to make sense of students. This lack of comfort could therefore contribute to the teacher’s failure to seek clarification from the student.

Effective listening is important for the development of classroom discourse. A factor that may impact on teachers making sense of students is that classroom discourse involving interpretive or hermeneutic listening (Davis, 1997) takes time to evolve, and requires a shift in authority from the teacher to one shared between teacher and students (Doerr & English, 2006).

2.4 Summary of teacher knowledge research

Shulman's significant 1986 research on teacher knowledge provided the stimulus for an extensive range of similarly conceived research. Some of that research has been reviewed, and was shown to exhibit strong similarities, but also revealed some marked differences. The recognition that research in the general field of teacher knowledge may not account for some of the needs within particular domains, such as mathematics, indicated a need to examine the research particular to teacher knowledge for mathematics. The mathematics education field has been the focus of much research on teacher knowledge. It is only in more recent times that some of this research has been conducted in the classroom, even though much of the research has been directed towards and made links to the tasks of teaching. It is becoming apparent that more researchers are recognising that research on teacher knowledge will carry more weight if it is focused directly on what teachers do in the classroom and while they are doing it.

The next section in this literature review examines statistics, as a 'strand' of mathematics. The similarities and differences with regard to the domains of statistics learning and teaching, and mathematics learning and teaching, are explored. A framework is then proposed for studying teacher knowledge for teaching statistics.

2.5 Teaching and learning statistics

2.5.1 Statistics as a strand of mathematics: Tensions

Statistics is one 'strand' of the school mathematics curriculum in New Zealand. In many ways, New Zealand has led the world, as a number of countries have only relatively recently included Statistics (in some other countries, referred to as Data and Chance, or Stochastics) in the primary school curriculum. In comparison, New Zealand has had components of statistics in its primary school mathematics curriculum since 1969.

Although statistics is considered to be part of mathematics, there are some significant differences that have implications for the teaching and learning of statistics. In mathematics, students learn that mathematical reasoning provides a logical approach to solve problems, and that answers can be determined to be valid if the assumptions and reasoning are correct (Pereira-Mendoza, 2002), that the world can be viewed deterministically (Moore, 1990), and that mathematics uses numbers where context can obscure the structure of the subject (Cobb & Moore, 1997). In contrast, statistics involves reasoning under uncertainty; the conclusions that one draws, even if the assumptions and processes are correct, are 'uncertain' (Pereira-Mendoza, 2002); and statistics is reliant on context (delMas, 2004; Greer, 2000), where data are considered to be numbers with a context that is essential for providing a meaning to the analysis of the data. However, this reliance on context can lead to errors in reasoning which may be difficult to overcome (delMas, 2004).

It becomes necessary therefore when teaching statistics, to encourage students to not merely think of statistics as doing things with numbers but to come to understand that the data are being used to address a particular issue or question (Cobb, 1999; Gal & Garfield, 1997). While in mathematics the use of context may, but not always, be useful for developing conceptual understanding (Sullivan, Zevenbergen, & Mousley, 2002), in statistics context is essential for making sense of data.

2.5.2 Statistical literacy, reasoning, and thinking

Increasingly, it is recognised that statistics consists of more than a set of procedures and skills to be learned. Statistics education literature in recent years has introduced the terms of statistical literacy, reasoning, and thinking, and they are being used with increasing frequency, although in some cases interchangeably. Attempts have therefore been made to clarify and define the terms more carefully. Ben-Zvi and Garfield (2004, p. 7) provide some clarity for each of the terms.

> **Statistical literacy** includes basic and important skills that may be used in understanding statistical information or research results. These skills include being able to organize data, construct and display tables, and work with different representations of data. Statistical literacy also includes an understanding of concepts, vocabulary, and symbols, and includes an understanding of probability as a measure of uncertainty.
>
> **Statistical reasoning** may be defined as the way people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on sets of data, representations of data, or statistical summaries of data. Statistical reasoning may involve connecting one concept to another (e.g., center and spread), or it may combine ideas about data and chance. Reasoning means understanding and being able to explain statistical processes and being able to fully interpret statistical results.

> **Statistical thinking** involves an understanding of why and how statistical investigations are conducted and the "big ideas" that underlie statistical investigations. These ideas include the omnipresent nature of variation and when and how to use appropriate methods of data analysis such as numerical summaries and visual displays of data. Statistical thinking involves an understanding of the nature of sampling, how we make inferences from samples to populations, and why designed experiments are needed in order to establish causation. It includes an understanding of how models are used to simulate random phenomena, how data are produced to estimate probabilities, and how, when, and why existing inferential tools can be used to aid an investigative process. Statistical thinking also includes being able to understand and utilize the context of a problem in forming investigations and drawing conclusions, and recognizing and understanding the entire process (from question posing to data collection to choosing analyses to testing assumptions, etc.). Finally, statistical thinkers are able to critique and evaluate results of a problem solved or a statistical study.

Data analysis involves statistical thinking that is different from mathematical thinking. Wild and Pfannkuch (1999) describe five fundamental types of statistical thinking: (1) a recognition of the need for data (rather than relying on anecdotal evidence); (2) transnumeration – being able to capture appropriate data that represents the real situation, and change representations of the data in order to gain further meaning from the data; (3) consideration of variation – this influences the making of judgments from data, and involves looking for and describing patterns in the variation and trying to understand these in relation to the context; (4) reasoning with models – from the simple (such as graphs or tables) to the complex, as they enable the finding of patterns, and the summarising of data in multiple ways; and (5) the integrating of the statistical and contextual – making the link between the two is an essential component of statistical thinking. Along with these fundamental types of thinking are more general types that could be considered part of problem solving (but not exclusively to statistical problem solving). Wild and Pfannkuch's dimension of 'types of thinking' is one of four dimensions that explain statistical thinking in empirical enquiry. The other three dimensions are: the investigative cycle (problem, plan, data, analysis, and conclusions – these are the "procedures that a statistician works through and what the statistician thinks about in order to learn more from the context sphere" (Pfannkuch & Wild, 2004, p. 41)); the interrogative cycle (generate, seek, interpret, criticise, and judge) – this "is a generic thinking process that is in constant use by statisticians as they carry out a constant dialogue with the problem, the data, and themselves" (Pfannkuch & Wild, 2004, p. 41); and dispositions (including scepticism, imagination, curiosity and awareness, openness, a propensity to seek deeper meaning, being logical, engagement, and perseverance), which affect or propel the statistician into the other dimensions. All these dimensions constitute a model that encompasses the dynamic nature of thinking during statistical problem solving, and is non-hierarchical and non-linear. This framework for statistical thinking was developed through reference to the literature following interviews with statisticians and tertiary statistics students as they performed statistical tasks (Wild & Pfannkuch, 1999). Although it was developed as a model applicable to the statistical problem solving of statisticians and tertiary students, it has significant potential for examining content knowledge in statistics across wider groups. It has subsequently been used in a variety of other studies, such as an examination of the thinking of primary students (Pfannkuch & Rubick, 2002) and pre-service primary teacher education students (Burgess, 2001), through a professional development workshop with secondary teachers (Pfannkuch, Budgett, Parsonage, & Horring, 2004), and an investigation into how statistical thinking of learners can be encouraged through a teaching activity (Shaughnessy & Pfannkuch, 2002).

2.5.3 Teaching of statistics for conceptual understanding

To acknowledge that statistics differs from mathematics in some subtle yet distinctive and significant ways raises questions about the teaching of statistics in comparison with the teaching of mathematics. Greer (2000) suggests that such differences indicate a need for particular attention being given to statistics professional development for teachers. The argument provided in support of his claim is that mathematics professional development would not be sufficient to encompass some of the specific demands that statistics teaching entails. Greer further suggests that because of the changing emphasis in statistics education away from the development of statistical skills (or literacy) towards statistical reasoning and thinking, teachers will be required to develop ways of encouraging greater conceptual understanding of statistics in their students. For example, students' conceptual development in probability and randomness requires teaching strategies different from those used in other areas (Chance & Garfield, 2002) because of the nature and particular difficulties of the concepts in probability and randomness (Greer, 2001).

One of the 'big ideas' of statistics is that of variation. Moore (1990) suggests that recognition that data varies, and the measurement of that variability, is the essence of statistics. Wild and Pfannkuch (1999) consider variation to be one the cornerstones of their statistical thinking model. It appears that students, when confronted with data that exhibit variation, are more likely to notice the trends or patterns in the data than the variation (Ben-Zvi, 2004), or even the individual features in the data over and above the global features (e.g., Hancock, Kaput, & Goldsmith, 1992; Konold & Higgins, 2003). Understanding of variability, whether within one data set or between two data sets, takes time to evolve, and a hierarchical framework that captures this evolution has been developed by Watson, Kelly, Callingham, and Shaughnessy (2003).

Informal inference is recognised as an important stepping-stone to formal statistical inference (Burrill, 1998; Pfannkuch, 2005; Rubin, Hammerman, & Konold, 2006; Watson, 2001a; Watson & Moritz, 1999). A description of informal inference is given by Rubin, Hammerman, and Konold (2006), who suggest that it is reasoning involving four interrelated components. First, properties of 'aggregates' must be understood. These data aggregates or group propensities include: those related to signal and noise (Konold & Pollatsek, 2004), or trends/averages and variation; and types of variability, such as attributable to measurement errors, to multiple causes, or from one sample to another sample. Second, the effect of sample size must be taken into account, and knowing that 'bigger is better'. Third, controlling for bias in sampling helps ensure a representative sample, which is more reliable. Fourth, the property of 'tendency' enables one to distinguish between claims that are always true and those that are often or sometimes true. A thorough knowledge and understanding of all four properties is the basis of informal inferential reasoning.

Comparing groups of data, in order to make general statements about that data set, is considered an important prerequisite for the later development of formal inference (Konold & Higgins, 2003; Watson & Moritz, 1999). Pfannkuch (2005) describes how drawing inferences about the data set under examination is different from drawing inferences from the data set about a population, and as such requires a different level of understanding. Pfannkuch and Horring (2005), through a study of Year 11 secondary students undertaking investigations, acknowledge the challenges that students face when making statements from graphs. They state that approaching informal inference through making statements from data in this way involves informal language, which will lead eventually to the use of formal language and more sophisticated statistical understanding. Like Pfannkuch and Horring's study, Watson and Moritz's (1999) study also involved comparing groups through graphs but with younger students (Grades 3 to 9). They found that the youngest students were able to compare equal sized groups satisfactorily, using mainly visual strategies, while the older students used a combination of numerical and visual strategies, even with unequal sized groups.

2.5.4 Frameworks and models for knowledge of statistics

Significant research has been undertaken to explore the development of understanding, and the types and prevalence of students' misconceptions in particular domains of statistics, such as, the mean and other averages, graphing, variation, sampling, and probability. Much of the research has been directed at the secondary school level, although there are a growing number of studies being conducted at the primary level. General frameworks for statistics have also been developed (e.g., Jones, Thornton, Langrall, Mooney, Perry, & Putt, 2000). As a result of developing frameworks for students' understanding within particular components of statistics, some researchers have indicated the implications for teachers to consider when teaching those concepts. Consequently they have made suggestions pertinent to teacher knowledge but without making explicit the type of knowledge being addressed.

Only relatively recently have teachers become the focus of research concerning statistical knowledge. Watson (2001b) developed a profiling tool to examine pre-service teachers' knowledge of statistics (including probability) in relation to all of Shulman's (1987) seven knowledge bases. This tool, which involves a questionnaire, an interview, and a self-report on their teaching practice, can be used with both primary and secondary level pre-service teachers. Watson claims that the tool is an efficient alternative to the more expensive and time-consuming observations of practice. She argues that the tool may be just as valid in relation to teaching practice (because of what she claims to be the seriously considered reflection that teachers engage in while responding to the questionnaire and interview) as a single lesson demonstration specially prepared and performed for an external assessor. The question of validity of the tool is not dealt with further by Watson. However, although the tool is considered more useful for determining the knowledge of pre-service teachers than observation of one-off lessons that the pre-service teachers may plan and implement for an appraiser, the choice between the profiling tool and observation with regard to practising teachers may be open to argument. If the on-going work of practising teachers in the classroom is subject to observation, it may be possible to obtain a better 'picture' of the reality of teacher knowledge than was considered possible with observation of the one-off lesson of a pre-service student. Also, irrespective of the depth of reflection of teachers as they respond to a questionnaire and an interview for the knowledge profile, there is no certainty that the knowledge determined from that process would be used in practice in the classroom.

Teachers' knowledge (content knowledge as well pedagogical content knowledge) of particular statistical concepts has been investigated. The 'mean' has been the focus of some research, such as when secondary-level pre-service teachers' conceptions of the mean were studied (Gfeller, Niess, & Lederman, 1999) through a task involving the solving of problems and giving suggestions for more than one possible solution strategy. Gfeller, Niess, and Lederman found that the pre-service teachers did not possess knowledge of multiple and flexible representations that the researchers consider essential for teaching. This finding paralleled that of McDiarmid, Ball, and Anderson (1989), whose study was based on a particular domain of mathematics as opposed to statistics. In a study in which the knowledge of experienced and inexperienced teachers of 6th and 7th grade students was compared through interviews, the experienced teachers were found to have better knowledge of possible student strategies and errors, and of a broader range of possible solution strategies (Cai & Gorowara, 2002). However, the experienced teachers did not display their more extensive and diverse knowledge of students' representations within their lesson plans. Cai and Gorowara therefore propose the possibility of a "disconnectedness between teachers' knowledge and their planning" (2002, p. 6). An exploration of pre-service primary teachers' concepts of a sample was undertaken by Groth and Bergner (2005), in which teachers' written metaphors for a sample were analysed in relation to content knowledge as well as possible links to pedagogical content knowledge. Groth and Bergner express concern that those pre-service teachers who exhibited weak content knowledge through the metaphors could, in their future teaching, be constrained in their choice of tasks for students, since they claim that the use of metaphors in the classroom guides aspects of teaching.

Understanding graphs and being able to interpret them is an important part of a statistical investigation. Friel, Bright, Frierson, and Kader (1997) summarise what teachers and students should know and be able to do with respect to graphical representations in statistics. Although Friel et al. indicate that they have explored both students' and teachers' understandings to provide possible directions for assessment, they do not explicitly address what has been explored and found with regard to teachers' knowledge in general. They do, however, give some recommendations with regard to teachers' pedagogical content knowledge (although they do not refer to it as such). For instance,

> There is a need to understand what we don't understand about the ways in which the use and reading of graphs may be misunderstood or misinterpreted by students. ... There is a need to monitor learners' changes in thinking as they move among ungrouped and grouped data representations. Once we have some knowledge of learners' thinking, are we clear about what attributes of statistical thinking we want to promote and about ways to promote these attributes? ... We need to understand what students understand prior to and following instruction and be clearer about how we will judge their responses in light of what we think reflects sound statistical thinking. (Friel et al., 1997, p. 62)

Probability concepts have been well researched in connection with students' misconceptions. However, Kvatinsky and Even (2002) developed a framework unique to teachers' knowledge rather than relying on a framework related to student understanding (or misunderstanding) to describe teachers' subject matter knowledge related to probability. The framework was developed following an examination of texts, other curriculum materials, and research on understanding and learning of probability, and interviews with mathematicians and mathematics educators. Kvatinsky and Even's framework consists of descriptions of some components of teacher subject matter knowledge and the connections between the components, rather than a list of topics and concepts to be learned.

Some indication has been given above of research that is relevant to both particular concepts in statistics and teacher knowledge. In comparison with that type of research, Heaton and Mickelson (2002) focus on an aspect of a process of statistics, namely investigations, which involve a broader range of statistical concepts. Their goal was to obtain information on pre-service teachers' statistical knowledge (presumably subject matter knowledge) as well as their pedagogical content knowledge as applied to the process of statistical investigation. Initially, the pre-service teachers were required to undertake a statistical investigation themselves (related to teaching some other curriculum area) and following that, to help children develop a statistical investigation. Heaton and Mickelson (2002) found (similar to Burgess, 2002) that the pre-service teachers, while working on their own investigation, lost sight of the goal of the investigation and instead focused on the production of a graph as the end result. The pre-service teachers had insufficient understanding and knowledge of the process of statistical investigations to be able to carry their investigation through to completion. Heaton and Mickleson claim that this affected their teaching effectiveness. A study by Makar and Confrey (2002), involving secondary teachers, focused on how teachers' understanding of data analysis emerged through a context of investigating data related to their students. By being immersed in genuine investigations about their students, the teachers gained statistical content knowledge, particularly in relation to data analysis. The research was, however, situated away from the classroom and there was no further investigation of subsequent changes to the teachers' teaching practices.

2.5.5 Summary of teaching and learning statistics

Statistics provides challenges for teachers that are different from the challenges of teaching and learning mathematics. Various frameworks have been summarised, from the broad and general statistical literacy, reasoning, and thinking, down to specific and in some cases very detailed frameworks for particular concepts in statistics. Most frameworks have been developed from work with students and only some are specific to teachers' knowledge. In the next section, a framework is proposed for investigating teacher knowledge in statistics. It draws on aspects of teacher knowledge from the mathematics education literature, in particular that of Hill et al. (2004) and Ball, Thames, and Phelps (2005), and the statistical thinking framework of Wild and Pfannkuch (1999).

2.6 A framework for investigating teacher knowledge in statistics

Teacher knowledge frameworks from the mathematics education domain are inadequate for examining teacher knowledge for statistics because of the differences between statistics and mathematics, as discussed earlier. The development of a teacher knowledge framework that takes into account the particular needs of statistics teaching and learning is therefore required. Such a framework must be specific to statistics, since teacher knowledge is organised in content-specific ways (Hill et al., 2004). Consequently the proposed framework draws heavily on the statistical thinking model of Wild and Pfannkuch (1999). Since teacher knowledge is acknowledged to be important in relation to what and how students learn and is dependent on the context in which it is used (Barnett & Hodson, 2001; Borko et al., 2000; Cobb & McClain, 2001; Friel & Bright, 1998; Heaton & Mickelson, 2002), it is argued that research should therefore take place in the classroom. The categories of teacher knowledge that are described by Hill, Schilling, and Ball (2004) and Ball, Thames, and Phelps (2005), namely mathematical content knowledge and pedagogical content knowledge, and each of these with two sub-categories, appear to provide a good starting point for examining statistics content knowledge as enacted in classroom teaching.

A matrix for a conceptual framework, against which statistical knowledge for teaching can be examined, is proposed and shown in Table 1 below. This framework, based on various frameworks discussed earlier, is described in more detail in the following sections.

Table 2-1: Components of teacher knowledge in relation to statistical thinking and investigating.

<table>
<tr><td colspan="2" rowspan="3"></td><td colspan="4">Statistical knowledge for teaching</td></tr>
<tr><td colspan="2">Content knowledge</td><td colspan="2">Pedagogical content knowledge</td></tr>
<tr><td>Common knowledge of content (ckc)</td><td>Specialised knowledge of content (skc)</td><td>Knowledge of content and students (kcs)</td><td>Knowledge of content and teaching (kct)</td></tr>
<tr><td rowspan="5">Thinking</td><td>Need for data</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Transnumeration</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Variation</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Reasoning with models</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Integration of statistical and contextual</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Investigative cycle</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Interrogative cycle</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Dispositions</td><td></td><td></td><td></td><td></td></tr>
</table>

The columns of the matrix refer to the types of knowledge that are important in teaching. These four types are: common knowledge of content (ckc); specialised knowledge of content (skc); knowledge of content and students (kcs); and knowledge of content and teaching (kct). Hill, Schilling and Ball (2004) and Ball, Thames, and Phelps (2005) describe the features of these four categories of teacher knowledge in relation to number and algebra. These descriptions arise from a consideration of the question, “What are the tasks that teachers engage in during their work in the classroom, and how does the teachers’ mathematical knowledge impact on these tasks?” From the researchers’ close examination of teachers’ work, it is apparent that much of what teachers do throughout their teaching is essentially mathematical. The features of the four categories of teacher knowledge with regard to mathematics are:

- Common knowledge of content: ability to identify incorrect answers or inaccurate definitions, and the ability to successfully complete the students' problems;

- Specialised knowledge of content: ability to analyse mathematically whether a student's unconventional answer or explanation is reasonable or mathematically correct, or to give a mathematical explanation for why a process (such as a particular algorithm) works;

- Knowledge of content and students: ability to anticipate student errors and misconceptions, to interpret incomplete student thinking, to predict how students will handle specific tasks, and what students will find interesting and challenging;

- Knowledge of content and teaching: ability to appropriately sequence the content for teaching, to recognise the instructional advantages and disadvantages of particular representations, and weigh up the mathematical issues in responding to students' unexpected approaches.

Just as Ball et al. (2001) claim that many of the everyday tasks of the teacher of mathematics are essentially mathematical, it is suggested that much of what a teacher engages in during the teaching of statistical investigations essentially involves statistical thinking and reasoning. Consequently, the four teacher knowledge categories will be examined in relation to statistical thinking. Even though the mathematics education literature has described the categories in relation to number and algebra, some of the above descriptions may be inappropriate for the statistics field. For example, it is widely acknowledged that reasoning under uncertainty and the concept of variation lead to an understanding of statistics as more subjective than mathematics. Therefore descriptions of teacher knowledge pertaining to 'correct answers' or 'explaining particular algorithms' may not feature in the descriptors of the matrix cells for statistical teacher knowledge.

The main feature that sets this proposed framework apart from those offered for the mathematics domain is the inclusion of the elements of statistical thinking and empirical enquiry (Wild & Pfannkuch, 1999). These are listed as the rows of the matrix, and they are: the recognition of a need for data with which to reason; the ability to transnumerate the data, in order to put the data into a form that is more useful for investigating and seeking patterns and relationships; recognition and understanding of variation in data; being able to use a variety of models for reasoning; being able to continually integrate and move between contextual knowledge of where the data has come from and is relevant to, and statistical knowledge of what can be done with that data; an understanding and use of the investigative cycle within problem solving; thinking within the interrogative cycle – generate, seek, interpret, criticise and judge; and dispositions such as imagination, scepticism, curiosity, awareness, openness, propensity to seek deeper meaning, being logical, engagement, and perseverance. The ways in which each of the aspects of statistical thinking interact with and exist as evidence for the various categories of teacher knowledge is examined in this thesis.

Chapter 3
Methodology in Theory

3.1 Introduction

This chapter initially examines the important questions of knowledge that relate to its nature and its development. Popper's conceptualisation of knowledge is discussed, along with the logic of learning approach to research in classrooms that is derived from Popper's ideas. This philosophical approach to research appears appropriate for examining the complexity of teacher knowledge through being able to account for the dynamic nature of knowledge.

Having examined knowledge from an epistemological perspective, three potential research approaches are compared for their utility in relation to this study. These approaches are an empirical-analytic paradigm, an interpretive or symbolic research paradigm, and a post-positivist realist paradigm. Through consideration of factors such as how knowledge is conceived, its dynamic nature, and the diversity of contexts in which it is used, a post-positivist realism paradigm is argued as being the most appropriate of the three approaches for this study.

The consequences of adopting a post-positivist realist paradigm for this study are considered, in relation to a number of methodological issues or factors. These include the types of questions on which such research can be based, the generalisation of findings from the research, some of the potential difficulties of situating the research in the classroom, appropriate data collection tools (in particular, video and stimulated recall interviews), and some ethical issues regarding the effect of participation in the research on the teachers' practices and knowledge.

3.2 What is knowledge and how does it grow?

Research on teacher knowledge needs a framework that will support an understanding of how we come to know. A variety of research was reviewed in the previous chapter, many of which had described various categorisations of teacher knowledge. However, few of the research studies have addressed the combination of what teachers know, how that knowledge develops, how it is used, and for what it is used. Fennema and Franke's (1992) conceptualisation of teacher knowledge (as it relates to mathematics teaching) bears strong similarities to that of other studies reviewed. They identify four categories of teacher knowledge, namely knowledge of the subject of mathematics, knowledge of how to best represent mathematical concepts to students, knowledge of students (such as how they learn and think), and general knowledge of teaching and decision-making. What is important, however, is that Fennema and Franke argue that teacher knowledge evolves through teaching. They also claim: "Within a given context, teachers' knowledge of content interacts with knowledge of pedagogy and students' cognitions and combines with beliefs to create a unique set of knowledge that drives classroom behavior" (p. 162). They recognise that while much research has focused on identifying categories of teacher knowledge, research in the area of teacher knowledge must acknowledge and accommodate the dynamic aspects of teacher knowledge (Manouchehri, 1997), and be based on an understanding of how knowledge evolves. Consequently, it is important to explore first how knowledge develops.

3.3 Popper and how knowledge develops

Popper's epistemology gets to the essence of knowledge. His theorising is able to explain where knowledge comes from, and how it grows in a dynamic fashion. He first differentiates between three 'worlds' (Popper, 1985b): 'world 1' consists of physical objects and physical states; 'world 2' includes states of consciousness, mental states, and dispositions to act; and 'world 3' consists of objective knowledge, such as problems, descriptions, hypotheses, theorems, and arguments, many (although not all) of which are included in the contents of journals, books, musical scores, paintings, films and libraries (Swann, 2003b). Subjective experience or knowledge (world 2), although susceptible to criticism, is not provable or "susceptible to refutation by reference to empirical evidence" (Swann, 2003b, p. 14). In contrast, objective knowledge (world 3) is testable, falsifiable, and open to refutation, because it exists in the public domain. As an example, a student might claim, on the basis of given data, "I think that boys have a faster reaction time than girls." This corresponds to subjective knowledge – the student has given expression to her thoughts, which, as it stands, cannot be disputed. It is not until the student gives a justification for her thinking that her 'world 2' 'product' (the 'I think' statement) moves into the 'world 3' of objective knowledge, and her justification then becomes open to refutation.

Popper proposes that objective knowledge grows in a logically defensible manner, through a process of trial and error elimination. This process (see Figure 3-1) involves the recognition of a problem (P_1); the development of a trial solution (TS) that is subject to the elimination of error (EE) – possibly through critical discussion or experimental tests of competing conjectures; and the possible emergence of new problems (P_2).

$$P_1 \rightarrow TS \rightarrow EE \rightarrow P_2$$

(Popper, 1979 p. 243)

Figure 3-1: Popper's schema of the growth of public knowledge

New problems are also claimed to lead to creations of "new unintended facts; new unexpected problems; and often also new refutations" (Popper, 1985b, p. 70). These new problems "are not in general intentionally created by us, they emerge autonomously from the field of new relationships which we cannot help bringing into existence with every action, however little we intend to do so" (p. 71).

An alternative view that claims new knowledge develops through induction (generalising from a finite number of observations) has been discounted by Popper (1985c) as neither logical nor a valid process:

> I hold that neither animals nor men use any procedure like induction or any argument based on the repetition of instances.... What we do use is a method of trial and the elimination of error; however misleadingly this method may look like induction, its logical structure, if we examine it closely, totally differs from that of induction. (Popper, 1985c, p. 103)

From this perspective, Popper argued that whereas induction relies on a human *belief* in regularity as it pertains to people's reality, a human *need* for regularity becomes the basis for observations leading to learning and a growth of knowledge. His rejection of induction helps explain the intransigence of people's ideas (Popper, 1979). He argued that people have a need for regularity and therefore seek it, often in situations where it does not exist. This 'need for regularity' also explains why people's ideas can remain fixed in the face of disconfirming evidence. He suggests that ideas arise, not from repetition (i.e., inductively), but prior to repetition since "repetition presupposes similarity, and similarity presupposes a point of view – a theory, or an expectation" (p. 24). He proposes that "beliefs are partly inborn, partly modifications of inborn beliefs resulting from the method of trial and error-elimination" (p. 27). Popper argued against observation, itself, as a source of knowledge (1985a); he asserted that knowledge is:

> largely based on action and on thought: on problem-solving. Admittedly observations do play a role, but this role is that of posing problems to us and of helping us to try out, and weed out, our conjectures. (Popper, 1985e, p. 278)

However, all observation is dependent not only on the thing to be observed:

> but also on the prior expectations (implicit assumptions or explicit theories) of the observer: there is always something prior to an observation. All observation is theory-laden. This is no less true for observations of the physical world than it is for the social world. (Swann, 1999b, p. 22)

If observation leads to discovery of incorrect or inadequate expectations, then the mismatch between the experience and the expectation may lead to learning through the resolution of that mismatch, as long as there is a desire on the part of the learner to resolve the mismatch. This is contrasted with learning through induction, which relies on observations to confirm prior expectations. This latter situation has logical inadequacies in that it is impossible for a finite number of observations to be able to logically and fully confirm the truth of a theory. However the 'mismatch between experience and expectation' approach to learning can be logically verified – one disconfirming observation is sufficient to disprove a theory and therefore leads to the development of a tentative new theory, that is, leads to learning.

The power of disconfirming evidence for a theory has much greater 'strength' than evidence that confirms a current expectation – the former indicates the inadequacy of the theory, whereas the latter does not provide absolute confirmation of the truth of the theory. Searching for and eliminating false theory is a way of heading towards truth, even though there can be no such thing as absolute truth; and 'bold' theories "which entail a large number of consequences are preferable to those theories which predict or imply little" (Swann, 1999b, p.25). The bolder the theory that is put up for refutation, the greater the risk of refutation occurring; it is likely that more learning will have occurred through the process of refutation of the bold theory than if a lesser theory was refuted.

With Popper's description of and argument for such a theory of learning, a logical problem is potentially indicated when analysing the stage where a new trial solution has been formulated. The process of using previous experiences and expectations to develop a new trial solution for the current problem (i.e., for new learning to occur) could suggest that an inductive approach has been used to formulate the trial solution. However, Popper (1979) discounts this by arguing instead that creativity and criticism have a role to play in the development of trial solutions and therefore in the generation of knowledge.

3.4 Catering for the dynamic nature of knowledge

The dynamic nature of knowledge is dealt with effectively by both Fennema and Franke (1992) and Popper (1979). Building on Popper's ideas that knowledge develops through trial and elimination of error, Burgess (1977) proposed the logic of learning model for examining learning in classroom settings (Swann, 1999c). In this section, similarities between the two models, Fennema and Franke's and the logic of learning approach, will be discussed.

In this comparison, FF denotes a step from the Fennema and Franke (1992) model, while LL denotes an aspect from the logic of learning model (Swann, 1999c). First, the realisation by the teacher that 'the knowledge specific to the given context is unavailable' (FF) matches the 'recognition of a problem or a mismatch between experience and expectation' (LL). Second, the decision by the teacher to 'use more general knowledge relevant to a variety of solutions or use knowledge that closely matches the situation' (FF) could be considered to parallel the 'development of a trial solution (or tentative theory)' (LL). The final stage of 'knowledge being brought to a new situation, so adapted and stored as new knowledge' (FF) is the same as the 'successful elimination of error and consequently an improvement in knowledge' (LL). This stage then exists until a new problem is recognised. Thus there is general agreement between Fennema and Franke's (1992) description of the stages of knowledge generation and the logic of learning approach.

In the teaching situation, the length of the phase from the recognition of a problem to error elimination could occur over a very short period of time. Therefore, there could be a large number of iterations of the process during a teacher's day, with each iteration potentially contributing to learning by the teacher. However, the question arises as to whether this process of learning can be differentiated from the teacher's normal decision-making (such as, 'what should I do here, should I intervene, how should I respond, what question should I ask, or how should I help?') that occurs almost endlessly? When the decision-making process is examined closely, it is logically the same as the learning process (from problem to tentative solution to error elimination). However, unless the decision-making is dealing with the totally routine and mundane decisions of the classroom, it can, in fact, be considered to be part of the process of learning for the classroom teacher. Some of the decision-making questions listed above have much wider implications for teacher knowledge, and consequently students' learning opportunities, than what might be considered merely routine and mundane.

If the task of learners is to discover and eliminate mistaken ideas, or to modify and develop those which are inadequate (Swann, 1999b), how can learning take place when the learner does not perceive, or is not aware, that their current ideas are inadequate or mistaken? How do we account for the situation when no problem is recognised? With respect to teacher knowledge, awareness of a problem through a mismatch between current ideas and experience could occur through an interaction with students. By considering a student's question or comment, the teacher may become aware of a problem. Such recognition is similar to the idea of 'cognitive conflict' (Eade, 1988), in which a learner is faced with a mismatch between what they expect (maybe from intuition) and the current experience.

From such cognitive conflict, and from this situation, new understanding may develop, but not necessarily so. Cognitive conflict does not always lead to new learning, sometimes because the existing knowledge (from prior experiences) is too firmly entrenched. Although a problem has been recognised by the teacher (as a learner) through a mismatch occurring, the required change to accommodate the new experience and eliminate the error is too great. In terms of the logic of learning approach, the trial solution to the problem in this case is the same as the existing theory; the error has not been eliminated, so the status quo exists. In this situation, a decision was required between two possible trial solutions (the status quo or a new trial solution), or alternatively between a greater number of possible trial solutions. When one choice for a trial solution is to retain the status quo, the subsequent outcome is that the existing theory (although shown to be inadequate) could become even more entrenched and more strongly held. For the teacher, the evidence that supports the recognition of a problem with the existing theory is discarded in favour of the original, but flawed theory. Popper (1985d) would suggest that in this type of situation, the learner (in this case the teacher) is not acting rationally; the learner has fixed views that are resistant to change, which indicates a mentality "akin to that of the madman" (p. 364)! Although Popper argues that a rational choice is used on which to base a preference for one conjecture over another (Popper, 1985c), and that this preference is governed by the idea of truth, the reality of learners sometimes making a choice in spite of contradictory or disconfirming evidence is problematic. The logic of learning approach to the growth of knowledge somewhat addresses this situation. If the mismatch that the learner has perceived is resolved by deciding that the mismatch is insufficient to develop a new, alternative theory, the logic of learning approach suggests that the choice (which is a creative act on the part of the learner) to retain the status quo is a deliberate and conscious act (Swann, 1999d).

3.5 Researching knowledge

Research methods that fail to recognise the dynamic aspect of knowledge provide an inadequate approach for furthering our understanding of teacher knowledge (Fennema & Franke, 1992). Hiebert and Carpenter (1992) recognise the necessity of developing research that not only deals with the dynamic nature of teacher knowledge, but also acknowledges and accommodates the *context* of thinking, and knowledge of both teachers and students as it occurs in the classroom and as it changes over the course of instruction.

Since knowledge is dynamic rather than static, Fennema and Franke (1992) suggest that the change in required knowledge, which corresponds with a change in context (such as different content to be covered, or different students to be taught), will depend on various factors in relation to the teacher's knowledge base. However, Fennema and Franke do not address the circumstances surrounding the teacher's choice of which knowledge is subsequently used for that new context. So although they have advocated that research should be capable of accounting for the dynamic nature of knowledge, Fennema and Franke's explanation of knowledge growth has limitations. A research approach is therefore needed that can account for some of the contextual factors that affect teacher knowledge and its dynamic nature.

Three possible approaches to researching teacher knowledge are considered for this current study. These include an empirical-analytic paradigm, an interpretive or symbolic research paradigm, and a post-positivist realist paradigm. Each approach is examined in relation to the conceptions of knowledge, knowledge growth, the dynamic nature of knowledge, and the contextual nature of knowledge, which have been discussed earlier in this chapter. From these considerations, one approach is selected as being the most appropriate and useful methodological basis for this study.

3.5.1 Empirical-analytic research paradigm?

An empirical-analytic approach to research derives from logical positivism and is based on the idea that the goal of research is to explain the relationship of humans to the natural world, and those explanations are used to gain technical or intellectual control of the world (Romberg, 1992). According to Romberg, some of the assumptions of this approach are that: theory is to be universal and not confined to a specific context or circumstances that gave rise to the generalisations; theories only describe the facts of relationships, and that people's goals and values are irrelevant to those theories; within the social world, systems of variables exist and that a cause is a relationship between the variables which can be explained or manipulated to produce conditionally predictable outcomes; and there is a belief in formalised knowledge, which in turn creates a reliance on mathematics to quantify variables, test hypotheses, and improve theories.

Adopting such an empirical-analytic approach for research in mathematics education would indicate a number of assumptions about knowledge and learning. This approach suggests that the knowledge to be learned:

> can be specified in terms of facts, concepts, procedures, and so forth; that the job of students is to master that knowledge; that the job of the teacher is to present that knowledge to pupils in an organized manner and to monitor their progress towards mastery; and that the organization and technology of the classroom and school are arranged to make the teaching and mastery of that knowledge as efficient as possible. (Romberg, 1992, p.55)

There is, however, a difficulty with such an approach in that it does not take into account or adequately explain why not all students gain that knowledge efficiently. Changing the context (e.g., the situation or classroom), or some of the participants (either learners or teachers) will produce different effects on learning. The empirical-analytic paradigm does not have an explanation for this phenomenon. Consequently, this paradigm is not considered sufficient or appropriate for researching teacher knowledge.

3.5.2 Interpretive or symbolic research paradigm?

In order to take account of the context in which learning occurs, one alternative to the empirical-analytic paradigm is the interpretive or symbolic paradigm. The goal of research conducted within this paradigm is to understand how humans relate to the social world that they have created. Theories are developed about:

> the social rules that underlie and govern social actions. Such theories are about the nature of discourse rather than behaviour.... This perspective translates into the belief that knowledge is situational and personal, that pupils learn by construction as a consequence of experiences, that the job of teaching is to create instructional experiences for students and negotiate with them intersubjective understandings gained from those experiences, and that the organization and technology of the classroom and school are arranged so that all of the experiences can be rich and meaningful. (Romberg, 1992, p. 55)

A similar viewpoint is expressed by Donmoyer (1996), who claims that all knowledge is subjective, whether it is derived from research or from the viewpoints of ‘ordinary’ people. Research results, and conclusions are all claimed to be dependent on “a priori assumptions and metaphors employed by the researcher” (p. 101) just as much as on the empirical data.

Recognition of the importance of the context for learning suggests that the interpretive paradigm could be useful for examining aspects of teaching and learning. However, there are inadequacies in following an interpretive paradigm with its emphasis on researcher subjectivity (which opens itself to criticism for not recognising the ‘evidence’), in that some objectivity should be possible and, in fact, is desirable. Hence, another research paradigm is needed that will address the inadequacies of both the empirical-analytic and the interpretive or symbolic paradigms.

3.5.3 Post-positivist realist research paradigm?

Even within the social world, aspects of empiricism would be useful to the examination of teacher knowledge. A useful position to take would be one that acknowledges both empiricism and rationalism (which considers that reason is most important in the development of knowledge): post-positivist realism is one such position, and is advocated by Popper:

> Popper is a 'rationalist of sorts' in that he believes in a priori knowledge; however he departs from classical rationalism in that he regards such knowledge as conjectural and fallible rather than absolute. He is also 'an empiricist of sorts' in that he argues that 'we learn from experience – that is, from our mistakes – how to correct them', but nonetheless rejects the idea of pure observation and pure sense-experience. In Popper's account, experience includes our internal subjective state, the external physical and social environments, and the world of public ideas. (Swann, 1999b, p. 20)

Such a combination of empiricism and rationalism is not considered to be contradictory, in that some aspects of the human world are considered to be important with regard to the research process, even if not easily measurable or directly observable. As such, and as it relates to mathematics education, it appears to be a position which could provide a useful way of examining some of the phenomena related to teacher knowledge.

Types of research questions appropriate to a logic of learning approach

A logic of learning approach to research, based on Popper's ideas, is more rigorous than an interpretive approach in that it opens itself to possible refutation. Research questions can be developed in a manner consistent with a problem-based enquiry, as advocated by Swann (1999a), for investigating the complexity of practice in the classroom. Swann identifies two types of problem: practical problems, which deal with how to get from one state of affairs to another; and theoretical problems, which focus on what is the case, what is of value, what ought to be done, what was done in the past, what is logically valid, and what is aesthetically pleasing. Researching a practical problem has the explicit aim of improving practice. For instance, "How can we improve students' understanding of inference?" is a practical problem on which research could be developed. The desired outcome of such research, which "requires a new state of affairs that arises as a consequence of something having been done" (Swann, 2003a, p. 28), could include practical suggestions to address the problem, with those suggestions having been evaluated as part of the research. In contrast, a theoretical problem could ask in general terms, “Is this theory true?” or “Is this argument valid?” In line with the example given above, a theoretical problem might be, “Does a teacher’s knowledge impact on students’ understanding of inference?”

For researching teacher knowledge as proposed for this study, a ‘theoretical problem’ approach provides the focus of finding out about the state of teacher knowledge in the context and situation of classroom interactions. As improving teacher knowledge is not the main desired outcome for this research, the problem underpinning this research would not be formulated as a practical problem. It must be recognised that, although the anticipated interaction between each teacher and the researcher, as well as the usual interactions between teacher and students, quite likely will result in a change of teacher knowledge, such an outcome would, with respect to the research, be unintended although positive; the specific aim of the research is not to change teacher knowledge per se.

For this research, the major research hypothesis (see Chapter 1, *Introduction*, p. 6) has been developed as a bold, theoretical problem. As such, it is posed in such a way that it opens itself to possible refutation.

Case study research within the logic of learning approach

The generalisation of research findings that derive from studies carried out in a specific context, such as case studies, can be problematic. The logic of learning approach (Burgess, 1977), based on Popper’s ideas, supports the provisional and tentative nature of generalisations since a finite amount of available evidence cannot logically support a generalisation, whereas a single refuting instance is sufficient to disprove a generalisation.

As this study is based in the specific contexts of a number of classrooms, it utilises a case study design. Although case study research can take a number of different forms (Bassey, 1999), it is based on a ‘singularity’ (a study of a particular, bounded event), and consequently there is a difficulty in generalising from a particular event to anything broader. To overcome this difficulty, Bassey advocates the use of ‘fuzzy generalisations’ from such research. These are recommended as giving greater credence to case study research in education that attempts to 'theory-seek' or 'theory-test'. He argues that the study of a singularity (i.e., a case study) may not be considered by many people to be able to yield a valid generalisation; but when that generalisation is expressed in a way to indicate possibilities rather than something absolute, a convincing and worthwhile argument for that generalisation can be mounted. Any fuzzy generalisation arising from a case study extends an invitation to others, if it seems appropriate for their context, to act on the generalisation within their own context. A fuzzy generalisation suggests that, given a context similar to the one studied (although recognising that no two educational situations are identical), similar results may be found. In other words, 'This may be the outcome if the circumstances are similar, so try it.' An alternative outcome of case study research may be the generation of refuting evidence of a tentative theory. A fuzzy generalisation could, in this case, be a new tentative theory and trial solution to replace the previous and now discredited theory.

Bassey's (1999) fuzzy generalisations fit satisfactorily with the logic of learning approach to research. They (fuzzy generalisations) may be thought of as part of the trial solution that leads to error elimination. At this point they become part of the future expectations, hence open to refutation.

Educational research such as this that involves a case study approach, and when based on Popper's realism paradigm, can be developed in such a way as to be sufficiently rigorous to stand up to scrutiny. Pratt and Swann (1999) argue that this is achievable, even for educational phenomena, through the logic and validity of the chosen research methods and analysis, and the rational basis for the choice of certain knowledge statements over others. Reason and evidence provide, according to Pratt and Swann, the basis for justifying the chosen methods.

3.6 Some challenges with classroom-based research

Hiebert and Carpenter (1992) compare the development and use of theories by students as they learn mathematics with the development and use of theories by researchers in their investigations. The way that teachers require students to give reasons for what they do, and to give evidence of the connections they make between their knowledge and performance, is seen as no different from part of the research process in which the theories of researchers must be open to testing and possible refutation.

In considering teacher knowledge and student learning (particularly in relation to mathematics education), as was argued earlier, how knowledge is used, and how it develops, is dependent on both context and cognition. To be able to carry out research on these aspects means that the knowledge and understanding of the teachers needs to be explored in the context in which it occurs, since it is quite common for these aspects to be enacted differently depending on that context (Ball & Bass, 2000; Barnett & Hodson, 2001; Borko et al., 2000; Cobb, 2000; Cobb & McClain, 2001; Fennema & Franke, 1992; Foss & Kleinsasser, 1996; Friel & Bright, 1998; Marks, 1990; Sorto, 2004; Vacc & Bright, 1999). However, with such classroom-based research, the interpretation by the researcher of what is happening in the context of the classroom and why it is happening can be fraught with difficulties (Phillips & Burbules, 2000). Although the researcher adopts one interpretation of what is happening in the classroom, the teacher's interpretation should be obtained, as the teacher's reasons and intentions guide the actions; likewise the students' interpretations should be obtained, as they are the 'recipients' who interpret the situation that is intended for them. All the while, the researcher must be focused on uncovering the 'truth of the matter' in a competent manner. Consequently taking heed of the comments from Phillips and Burbules means that methods must be developed that allow for verification of the researcher's interpretations of what is viewed. Data from another perspective (such as obtained from stimulated recall interviews with the teacher) may mitigate the limitations of a researcher's subjective interpretation of the classroom events. Stimulated recall interviews are further discussed in a later section of this chapter.

3.7 Summary

Teacher knowledge, and the way it impacts on student learning in mathematics, involves a complex array of relationships based in the social world. Research on such phenomena must be based on an understanding of what knowledge is and how it grows. In this chapter, it has been argued that a post-positivist, realist approach based on Popper's ideas provides a particularly useful way of approaching the research. Framing the research, including its findings, in such a way that criticism and possible refutation are invited, indicates an openness and desire for truth.

3.8 Research data

3.8.1 Introduction

Issues and problems surrounding the gathering of research data are considered in this section. Situating the research in the classroom raises the question as to the most appropriate way to gather data in order to explore teacher knowledge. The choices of video methods for data are discussed. Additionally, as noted earlier, the researcher's interpretations of what is going on can be problematic (Phillips & Burbules, 2000). Consequently data gathering methods need to be considered that may alleviate some of this concern. The use of stimulated recall interviews with the teacher, as a way to reduce the limitations of the researcher's interpretations, is advocated. With the use of such multiple sources of data, triangulation is possible and some aspects of this process are considered. Finally in this section, ethical considerations are discussed with regard to the conduct of research in classrooms.

3.8.2 Video

This research focuses on teacher knowledge in the classroom. However the teacher is not the only participant in a classroom – students are an integral part of that classroom. Consequently, if videoing the teacher is to provide a way of examining the teacher's knowledge, then should it be required to obtain and examine the students' perspectives in addition to those of the teacher? It has been argued (Schoenfeld, 1998) that a focus on the teacher is often sufficient, as it explains a significant proportion of what takes place in the classroom, particularly in relation to teacher knowledge. Consequently, videoing of the teacher, along with her or his interactions with students, is considered an appropriate data collection tool for this research.

Why use videotape instead of other methods of recording what is going on in the classroom? One reason is the recognition that videotape captures so much more information than could be obtained by other methods (Erickson, 2006). This means that, at the stage of analysis, repeated viewing can yield data that was not noticed in earlier viewings. Swann (2003a) describes such an example, also as an illustration of observations being expectation-laden, and therefore not a source of knowledge in their own right. Her example relates to scientists' examination of the mummified remains of a 5300-year-old Stone Age corpse, found in the early 1990s. In spite of extensive scientific examination of the corpse, it was almost 10 years before scientists discovered evidence of an arrowhead in an x-ray of the corpse. Although this was discernible in the original x-rays, it had not been noticed at that time, and therefore the scientists' original conjecture of how the man died had to be revised. The value of being able to re-examine the sources, and thereby obtain more data that had not been noticed earlier, supports the use of videotape.

A question arises, however, as to whether or not the videotape per se constitutes the data. It is considered by some (e.g., Erickson, 2006; Pirie, 1996) that the videotape itself is not the data, especially for research in which categories for coding are predetermined (such as the knowledge categories in this study). If, however, the research was to study videotapes for grounded categories which may emerge, then the videotape does constitute the data (Pirie, 1996). In this study, categories of teacher knowledge have been proposed, and evidence is being sought for their existence, so the videotape is being used to notice occurrences of the categories. Consequently, data are obtained from the videotape, rather than the videotape itself constituting the data.

As was argued earlier, observation by a researcher is theory-laden rather than neutral. Erickson (2006, p. 178) claims that "the theory–driven nature of observation is a fundamental problem in all empirical research but it manifests in especially tricky ways in attempts to analyze and report on information derived from video footage." He uses the metaphor of 'grain size' in relation to the analysis and reporting of data. Using a large grain size avoids the labour intensive and time-consuming work of microanalysis and the detailed transcription of small pieces of tape to be used as examples of instances. In the process of microanalysis, how to choose and justify small pieces as representative, either typical or atypical, is problematic. Erickson alerts us to the fact that at either extreme – 'molar coding' or microanalysis – it is difficult to adequately represent the extent of variation in what is being reported.

Issues for this study in relation to extracting data from the videotape, and to grain size for analysis, are discussed in Chapter 4, *Methodology in Practice*.

3.8.3 Stimulated recall

There are many ways in which stimulated recall is used in research. Lyle (2003) claims the two most common ways involve structured time-sampling of the videotaped period, or identification of critical incidents. Such identification can be by the researcher, the participant, or both. As discussed earlier, for this research the videotape is considered to contain data about categories of teacher knowledge rather than constitute the data, and as such, relevant parts of the videotape need to be selected to use in a stimulated recall situation. Consequently Lyle's category of the use of critical incidents corresponds to the type of stimulated recall that suits this study.

The methodology surrounding stimulated recall needs to be considered. Lyle (2003) claims that few studies that have used stimulated recall have indicated that the use of stimulated recall is potentially problematic. One of the major issues regarding its use is the question as to whether the participant recalls the thinking that occurred in the original event, or is reacting, in a reflective way, to the viewing of the original episode (Gass, 2001; Yinger, 1986). The participant will not know whether their thinking is recalled, or constructed as a result of viewing, and consequently neither will the researcher. Also, the participant is able to provide elaboration for their interpretation of the videotaped incident. Yinger (1986) describes how teachers are able to report on things that they did not notice in the original event. This is because there are more cues available in the videotape to the teacher than in the original event, such as expressions, words, or mannerisms. All this provides for the teacher a "luxury of meta-analysis and reflection that was most likely absent in the original event" (p. 271). Another problem with stimulated recall is its dependence on the ability of the teacher to verbalise thinking (Yinger, 1986). Because of stimulated recall's reliance on memory, Gass (2001) points to the importance of minimising the delay between the original event and the stimulated recall interview; greater recall 'decay' occurs with consecutive, delayed or non-recent stimulated recall events.

In spite of the recognised limitations, the benefits of stimulated recall interviews significantly outweigh the potential problem areas (Gass & Mackey, 2000; Lyle, 2003), as long as the possible negative effects have been considered in the design of the research. The design considerations that were taken into account in this study are discussed in the next chapter.

3.8.4 Triangulation of data

In order to overcome some perceived inadequacies in qualitative research, a process of 'triangulation' is used. The term derives from a geometrical source, but now refers to approaches to improve the validity and reliability of data and the derived analysis. A number of types of triangulation exist in research, namely data, investigator, theoretical, and methodological triangulation (Denzin, 1989), and time and space triangulation (Cohen, Manion, & Morrison, 2000). For this study, both data and time triangulation are considered appropriate. The justifications for such a choice are outlined.

Many of the methods are considered inappropriate for this research. First, investigator triangulation uses more than one researcher to collect and analyse data. Second, theoretical triangulation involves a variety of theoretical approaches to interpret findings. However this research is based on a realist paradigm, and other theoretical positions were argued as inappropriate for the focus of this study. Third, with regard to methodological triangulation (either within method or between methods), Denzin (1989) claims that it allows a more complete picture to emerge than by single methods alone. Methodological triangulation uses either different data collection methods within one paradigm (as a type of replication for theory confirmation), or a combination of qualitative and quantitative data collection approaches and subsequent analysis. Finally, space triangulation is appropriate when a study might face significant limitations if conducted within one culture (Cohen et al., 2000). Consequently, none of the above approaches are deemed appropriate for this study.

In this study, with its basis in a realist paradigm, such objective reality *is* the goal due to its potential fallibility. Consequently, data triangulation, which involves comparing data from a number of different collection sources, appropriately fits the needs of this research. The stimulated recall interviews provide the extra perspective to supplement the researcher's interpretation of the videotape data. Time triangulation, of which one type relates to cross-sectional studies, can also be considered as useful for this study. Cross-sectional studies collect data from different groups at one point in time and thereby can take into account the effects of social change and process (Cohen et al., 2000). The involvement of a number of teachers in different schools represents a cross-sectional study that allows for the comparison of data to help 'test' the theory about teacher knowledge. As such, a combination of data and time triangulation ensures a greater confidence in the reliability and validity of results.

3.8.5 Ethical issues

Ethical issues related to qualitative research need to be considered. Newkirk (1996) indicates a potential problem with regard to whether participants should be informed if the researcher becomes aware, during the research, of aspects related to potential harm. He questions whether the researcher should be obliged to intervene, or make the teacher-participant aware of the researcher's concerns. Newkirk also discusses the vulnerability of the teacher participant, such as in research that 'studies down' and creates descriptions of teachers where the teacher may feel some professional discomfort. As such descriptions of teacher knowledge may indicate areas of deficit, Newkirk questions whether this possibility should be conveyed to the teachers prior to their consent to participate in the research.

Negative descriptions cannot be avoided if that is what the data shows – an integrity is required to represent it as it is (Newkirk, 1996). Steps to deal with possible negative findings are suggested by Newkirk. They include: offering the teacher, at the time of consent, the chance to reconsider participation if issues, problems or questions arise during the research; giving the teacher a chance to give their interpretation of the researcher's questions and interpretations, prior to final writing of the research; and working with the teacher in order to overcome problems. Although some could see this last suggestion as contamination of the study, Newkirk argues that this is not the case. In Chapter 4, *Methodology in Practice*, these issues are discussed in relation to this particular study.

3.9 Summary

In this chapter, it has been argued that a post-positivist realist research paradigm, based on the ideas of Popper, is most appropriate on which to base the research on teacher knowledge in the classroom. The logic of learning approach, as a refinement within the realist paradigm and as appropriate for classroom based research, is useful for considering the growth of knowledge. Because the research is situated in the classroom, methodological issues around the collection of data, specifically video and stimulated recall interview data, and ethical issues, have been examined.

The next chapter outlines the conduct of the research, from a practical viewpoint. Descriptions are given of the ethics procedures, the selection of and consent from participants, the data collection procedures, and the coding and analysis of data.

Chapter 4
Methodology in Practice

4.1 Introduction

This chapter describes the research methods that were adopted for this study. The ethics approval process for the research is described, as is the subsequent process of selecting possible teacher participants and the seeking of approval through the principals of the schools. The consent process for students, which occurred subsequent to gaining the approval of each principal and the agreement of a teacher within the school, is outlined.

The data was obtained from lesson videos and teacher stimulated recall interviews. The methods for collecting these, and for selecting incidents from the video to use in the stimulated recall interviews, are described. The coding of the data for the analysis, and consideration of 'grain size' in relation to that analysis, is explained.

4.2 Ethics

'Low risk notification' was provided to the Massey University Human Ethics Committee in 2004 regarding the conduct of this research. The notification was in accordance with Massey University's *Code of Ethical Conduct for Research, Teaching and Evaluations involving Human Participants*. It provided assurance that consideration had been given to aspects such as risk of harm, consent procedures for the school, the teacher, the students, and their parents/caregivers, and concerns regarding the safety and privacy of all participants. Copies of all information sheets and consent forms are attached in Appendix 1.

The issues raised by Newkirk (1996), as discussed in the previous chapter, with regard to possible need for intervention should the researcher become aware of aspects of potential harm, or to the effect on the teacher of negative findings, are addressed by the stimulated recall process. Through the reflection on incidents in the video, and discussion with the researcher, both of which are part of the stimulated recall process, the teacher has the opportunity to make changes to her or his subsequent practice. The researcher therefore has the chance to raise issues if, by doing so, potential 'harmful' effects for learners, or the teachers themselves, might be reduced or avoided. Consequently, the teacher, following such input from the researcher during the interview/discussion, has the opportunity to change her or his subsequent practice. It is considered that, because the research focuses on teacher knowledge and the possible growth of that knowledge, for the teacher to make changes to their practice as a result of the research process is not significantly different from the usual reflection and evaluation that teachers engage in as part of their usual practice. The stimulated recall interview can therefore be considered a type of research debriefing that Sieber (1992) advocates as being a worthwhile strategy to address such ethical concerns.

Newkirk (1996) suggests possible strategies to deal with potentially negative findings from research. These were listed in Chapter 3, *Methodology in Theory*, and include, first, the chance for the teacher to reconsider participation in the research should such issues arise. Although reconsideration on these grounds was not explicit in the teacher's consent form, the form included a statement covering the opportunity to withdraw at any stage of the research. Newkirk's second suggested strategy involved giving the teacher an opportunity to provide their interpretation of the researcher's interpretations, prior to the final writing of the research. The stimulated recall interviews provided that opportunity for the teachers to give their interpretations, through the viewing of episodes and the subsequent discussion, clarification, and justification of questions and/or ideas raised by the researcher or the teacher. Also, the researcher's interpretations and findings are expressed in such a way as to be open to refutation, as per the methodological approach for the study. Newkirk's third suggestion involves working with the teacher to overcome problems. Although it could be suggested that such an approach would constitute contamination of the research, Newkirk (1996) argues that this is not the case. As discussed earlier, the stimulated recall interview process involved working with the teacher, through sharing and discussing ideas. These discussions were not solely focused on interpreting the events shown in the video, but on occasions addressed ideas for the next lessons. In the results and discussion chapters, examples are given of situations that were discussed in the interviews, from which the teachers had the opportunity to make improvements to their practice.

4.3 Teacher Participants

Once ethics approval had been received, a number of school principals were contacted about the possibility of being involved with the research. The schools were identified as possibilities because of a number of factors. These factors included having a teacher early in their career (either year one or two) with a class at the upper primary level (from Years 5 to 8). The first factor was based on the desire to work with inexperienced teachers, as the components of teacher knowledge that were to be investigated were to reflect what teachers had available following their initial teacher education programme. If the research involved experienced teachers, distinguishing between knowledge that had developed as a result of their experience from that that they had obtained from initial teacher education would have been impossible. The second factor, namely a class from the upper primary level, was linked to the potential for students to be involved in statistical investigations at a more in-depth way than would be possible with younger students. It was anticipated that the statistical knowledge needed for teaching at this level would be of significant interest and value to the research field. The two factors linked to the researcher's work as a teacher educator at the primary level, and therefore the research could enable connections to what would be appropriate for inclusion in an initial teacher education programme as preparation for teaching statistics.

Four principals gave approval for a teacher from each of their schools to be involved. The four schools included two intermediate schools (one class consisted of Year 7 students, and the other Years 7 and 8 students), one primary school (a class of Years 5 and 6 students), and one 'full' primary school (a class of Years 6 and 7 students). The teachers (by their pseudonyms) were Linda (School 1), John (School 2), Rob (School 3), and Louise (School 4), and were all in their second year of teaching.

An initial meeting was held with each teacher to discuss the research and to begin the planning for the unit to be taught, based on a unit plan obtained from the nzmaths web site (Ministry of Education, 2006). This web site, funded by the Ministry of Education, has available for teachers a large range of information and resources. The downloaded unit plan was given to the teachers to use for planning their units. The teachers were given the opportunity to directly use the unit plan, or to adapt it to suit their teaching, and the students' learning needs.

The discussion between the teacher and the researcher covered various aspects of teaching statistics through data based investigations, and some of the 'big ideas' of statistics relevant to the teaching unit. The teachers were also given a poster illustrating the investigation process, the 'Data detective poster' (see Appendix 3), which could be used as part of their teaching.

4.4 Student participants and informed consent

Arrangements were made for the researcher to meet and talk with the class about the research, and to distribute the information sheets and consent forms. The students were given the opportunity to ask questions about the research. This meeting occurred about one week prior to the first lesson, so that the students had the opportunity to return the consent forms indicating whether or not consent had been granted. Consent was required from both the student and parent/caregiver. The teacher took the responsibility to collect the consent forms. Those students for whom consent to be part of the research was not received (a maximum of three students per class), were relocated in the classroom by the teacher so as to not be filmed, and also were identified to the researcher so that it could be ensured that those students would not appear on tape, particularly when the camera was moved to follow the teacher working with small groups.

4.5 Data Collection

4.5.1 Videoing of lessons

For each teacher, four consecutive lessons were videoed. For one teacher, Rob, the videoed lessons did not include the first lesson of the unit, whereas for the other three teachers, the videoing started with the first lesson of the unit.

Following the videoing, the researcher edited the video to obtain a 'movie' of between 7 and 29 minutes duration. The edited videos included episodes from the classroom in which teacher knowledge in relation to teaching statistics appeared to be a feature, and were selected as interesting and worthwhile to follow up in a stimulated recall interview. Such episodes showed teachers' explanations, responses to students' questions or answers, and discussions with individual students or the whole class, and were potentially worthwhile in relation to their links to the teacher knowledge framework. During the editing process, the researcher made notes about each episode in relation to what might be discussed in the follow-up interview.

4.5.2 Follow up Interviews

The follow up interview occurred as soon as possible after the lesson (as recommended by Gass, 2001), and generally was the same day, and after school. In most cases, it was arranged that the follow up interview be held prior to the next lesson, so that the teacher could make changes to a subsequent lesson as a result of what was discussed in the interview. Each episode from the edited video was shown to the teacher (anything from less than a minute to a few minutes duration), and the subsequent discussion based on the researcher's question or comment about the episode was digitally audio-recorded.

Table 4-1 below shows the lengths of videos and interviews from which the data were obtained.

Table 4-1: Lengths of edited videos and interviews, by school and lesson numbers

School number	*Lesson number*	*Video length (minutes:seconds)*	*Interview length (minutes:seconds)*	*Notes*
1	1	14:11	15:54	
	2	19:38	32:45	
	4	29:22	35:16	Lesson 3 was not used as it involved the students in collecting data, and there were no episodes relevant to teacher knowledge worth following up.
2	1	20:46	41:27	
	2 and 3	11:44 and 7:18	37:06	One interview covered two lesson videos, due to difficulty scheduling separate interviews for each lesson.
	4	9:18	35:14	
3	1	10:36	35:21	This lesson was the second lesson in the unit, because the first lesson was not able to be videoed due to scheduling difficulties.
	2	13:32	33:47	Third lesson of unit
	3	10:33	34:43	Fourth lesson of unit
	4	14:38	39:43	Fifth lesson of unit
4	1	17:50	25:35	
	2	14:00	10:26	
	3	7:38	7:19	
	4	7:21	24:19	

4.6 Coding and analysis

The video and interview data were imported as 'records' into AnnoTape software version 2.0.6 (Rosehill Software Limited 1999-2002). AnnoTape enables coding of segments of audio or video 'records', and allows the attachment of notes and/or transcriptions to those segments. Every segment was given: a code based on the framework of teacher knowledge and statistical thinking; index marks for time codes (identifying start and finish times); and notes (either field type notes, or transcriptions if it was decided that a transcription would be useful, or both). A search facility in AnnoTape allows searching for segments by codes, or terms/phrases within notes. By carrying out a search, all segments from all records that contain the search term are listed, and for each result, the video could be viewed or the audio listened to.

Codes were used based on the cells of the framework. There were 32 'dual codes', based on the four categories of teacher knowledge (namely, *common knowledge of content*, *specialised knowledge of content*, *knowledge of content and teaching*, and *knowledge of content and students*) by the eight components of statistical thinking (namely, need for data, transnumeration, variation, reasoning with models, dispositions, investigative cycle, interrogative cycle, and integration of contextual with statistical). An example of one of these 32 dual codes is *KCT: InvestCycle*, which represents knowledge of content and teaching in relation to the investigative cycle. As well as these 32 dual codes, codes representing one of the four categories of teacher knowledge or one of the eight components of statistical thinking, were used and resulted in 12 'single' codes.

On the first coding cycle, segments were identified in each record, and were coded with either a single code or a dual code. During subsequent coding cycles, the single codes were eventually eliminated, being replaced by dual codes, as the differences between the coding categories became clearer. In addition, many segments in the video and audio data were coded with multiple dual codes. For instance, one segment was coded *SKC: InvestCycle*, *KCT: ReasonModels*, and *KCT: InterrogCycle* as there were aspects in that segment that were pertinent to all three framework components. During coding iterations, the original coding was sometimes changed, as categories became more clearly defined and refined.

The notes and/or transcriptions that were attached to each segment indicated different 'grain sizes' for the analysis (Erickson, 2006). In some cases it was determined that a full transcription of that segment was not necessary, and brief notes were sufficient for the analysis. This corresponded to use of molar coding (Erickson, 2006), which was used when it was determined that transcriptions of the segment would not have yielded further evidence. Full transcriptions enabled small grain size analysis to take place, for example when analysing a teacher's response to a student's question.

Viewing of the video segments was, at times, insufficient for thorough analysis and insight into the teacher's knowledge. At such times, extra clarification was sought through the stimulated recall interviews. Often (although not always), such data triangulation (Denzin, 1989) provided that clarification.

From the coding and analysis of the coded segments, other themes began to emerge. These themes were added to the relevant notes attached to the segments, so that searching for these themes (using AnnoTape's search facility) enabled the incidence of such themes to be identified, and addressed in the subsequent discussion of results.

The comparison of data between the different teachers, which resulted in the emergence of themes based on similarities and differences, constituted the use of time triangulation (Cohen et al., 2000). 'Cross-sectional' views across the four teachers (as one type of time triangulation) help provide evidence in relation to the existence, or otherwise, of the categories of knowledge represented in the framework. Such evidence would not have been gained from restricting the study to only one teacher.

Chapter 5
Results

5.1 Introduction

In this chapter, the framework for examining teacher knowledge is explained and examples are given to support the descriptions of the cells of the framework. A statistical knowledge profile for each teacher is then described in relation to the framework. As part of each profile, 'missed opportunities' are discussed, where these missed opportunities have been interpreted as classroom incidents in which a lack of teacher knowledge resulted the teacher not taking advantage of a chance to enhance student learning.

Each teacher developed a sequence of lessons using a given unit plan that included a number of multivariate data sets (Appendix 2, *Unit plan*). The research data, from classroom videos and teacher interviews, were analysed in relation to the framework. Parts of the framework did not emerge from the data analysis, and these aspects are discussed in the next two sections. Following this, the remaining cells of the framework that did emerge from the data are defined and described, with supporting examples from the video and/or the interview data. The supporting examples are referenced to the data through a code, for example [S1L4 V 21:58]. This code refers to **S**chool **1** (and therefore Teacher 1), **L**esson number **4**, the example is drawn from the lesson **V**ideo (or alternatively the related interview **Int)**, and the starting time reference is **21**mins**58**secs within that video (or interview).

In each lesson, the students investigated multivariate data sets, which were in the form of a set of data cards (or sometimes referred to as data squares). An example of a data card from three different data sets is shown in Figure 5-1.

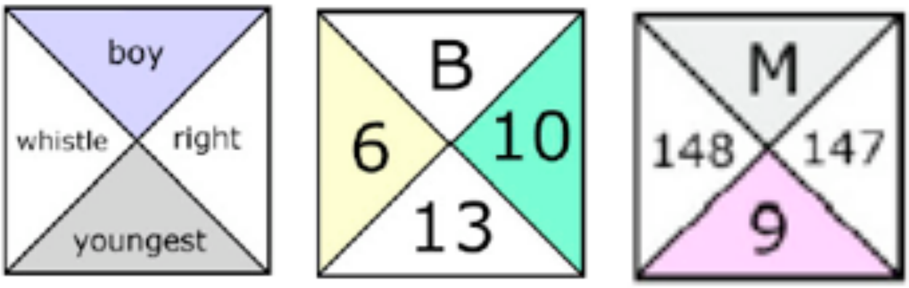

Figure 5-1: Example of data card from three data sets

By moving the set of cards around (generally 24 cards in a set), the students were able to group and sort the cards to help them with investigating the data.

5.2 Framework description

A framework (see Table 5-1) for examining teacher knowledge for teaching statistics was proposed in Chapter 2, and is based on two significant strands of research. First, in the statistics education field, Wild and Pfannkuch's (1999) model for statistical thinking in empirical enquiry, and subsequent related work by a number of researchers, has contributed significantly to the proposed framework. The dimensions of statistical thinking include types of thinking (recognition of a need for data, transnumeration, consideration of variation, reasoning with models, and integrating contextual with statistical ideas), the investigative cycle, the interrogative cycle, and dispositions (including scepticism, imagination, curiosity and awareness, openness, a propensity to seek deeper meaning, being logical, engagement, and perseverance). Second, the work of Hill, Schilling, and Ball (2004), and Ball, Hill, and Bass (2005), which focus on identifying the mathematical work that teachers engage in during the everyday work of teaching, is also strongly represented in the framework. These researchers describe four categories of teacher knowledge for teaching mathematics (specifically in relation to number and algebra), namely *common knowledge of content (ckc)*, *specialised knowledge of content (skc)*, *knowledge of content and students (kcs)*, and *knowledge of content and teaching (kct)*.

Through combining these two different models, the resulting framework consists of a matrix of cells, in which each cell describes a category of teacher knowledge for teaching statistics in relation to an aspect of statistical thinking.

Table 5-1: Components of teacher knowledge in relation to statistical thinking and investigating.

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (ckc)	Specialised knowledge of content (skc)	Knowledge of content and students (kcs)	Knowledge of content and teaching (kct)
Thinking	Need for data				
	Transnumeration				
	Variation				
	Reasoning with models				
	Integration of statistical and contextual				
Investigative cycle					
Interrogative cycle					
Dispositions					

5.2.1 The need for data

An understanding of the *need for data* on which to base sound statistical reasoning, instead of relying on and being satisfied with anecdotal evidence, is important in the development of statistical thinking. Classroom investigations can be conducted through two different approaches. First, an investigation can start with a question or problem to be solved and move onto data collection, which requires an understanding that data needs to be collected in order to solve the question or problem. The second approach is to start with a data set and generate questions for investigation from that data. By adopting this second approach for this study, teachers and students were not faced with the issues pertinent to establishing the need for data to help solve their questions. Consequently the *need for data* did not feature in this research. As such, the *need for data* is not described in relation to the four categories of teacher statistical knowledge for the framework.

5.2.2 Dispositions

Dispositions, as another component of statistical thinking, did not emerge specifically in relation to the individual components of teacher knowledge but in a more general way. Teachers' statistical dispositions were apparent in the classroom. For example, inquisitiveness and readiness to think in relation to data along with an anticipation of what was to come was evident when Linda asked the students what they had started to notice when filling in their own data cards. She justified this question in the subsequent interview [S1L1 Int 14:17] by saying that it was "to give them a hint of what was to come … to see if the students had the inclination to start making their own conclusions already." At other times, each teacher encouraged openness and inquisitiveness by asking what interesting things the data revealed, or through responding to students' answers with comments such as, "Wow, that's really interesting." An indication of dispositions that are a feature of statistical thinking came through in another way, not so much in what was said by the teacher, but more in the way that it was said. A teacher's tone of voice conveyed a positive and interested disposition with regard to the investigation, which was conducive to developing a similar outlook and interest in the students.

The framework component of dispositions is not analysed further in relation to the four categories of teacher knowledge. The next sections in this chapter give descriptions of the remaining cells of the framework with supporting examples from the data.

5.2.3 Transnumeration

Wild and Pfannkuch (1999) describe transnumeration as the ability to: sort data appropriately; create tables or graphs of the data; and find measures to represent the data set (such as a mean, median, mode, and range). In general, transnumeration involves changing the representation of data in order to make more sense of it.

Common knowledge of content: Transnumeration

For teaching, *common knowledge of content: transnumeration* includes the knowledge and skills described above, along with the ability to recognise whether, for instance, a student gave the correct process or rule for finding a measure, had created a table correctly, or had sorted the data cards appropriately. Evidence of this category was not often observed because the teachers generally used other types of teacher knowledge in relation to *transnumeration*. For instance, if a teacher asked questions that led the students towards sorting the data in a particular way, it was assumed that the teacher also had the *common knowledge of content* of how to do this for him or herself. There were instances where the researcher verified that this was indeed the case by asking the teacher during the interview to sort the cards, calculate a measure, or something similar. Consequently, *common knowledge of content: transnumeration* was subsumed within other categories of knowledge.

Specialised knowledge of content: Transnumeration

A teacher requires *specialised knowledge of content: transnumeration* to analyse whether a student's sorting, measure, or representation was valid and correct for the data, particularly if the student has done something in a non-standard and unexpected way. It includes the ability to justify a choice of which measure is more appropriate for a given data set, or to explain when and why a particular measure, table, or graph would be more appropriate than another. Some of these skills, although considered part of statistical literacy (Ben-Zvi & Garfield, 2004), are still currently beyond what many educated adults can undertake. As such they are considered to be part of *specialised knowledge of content: transnumeration*.

Specialised knowledge of content: transnumeration was identified for all the teachers in the study. For example, Linda attempted to follow a student's description of how she had sorted the data and converted it into an unconventional table involving all four variables [S1L4 V 28:06]. The table consisted of: four columns labelled G, B, G, B; four rows with labels on the left to account for two more variables; labels on the right for three rows to account for the fourth variable; but no numbers or tally marks in the cells of the table to represent the sorted data. To determine the statistical appropriateness of that particular representation, Linda had to call on her *specialised knowledge of content: transnumeration* as she tried to make sense of the table. In another example in relation to some students deciding which measure or measures they should calculate for the data set (out of the mode, median and mean), Rob recognised that the mode would not be the most appropriate measure to use for the numerical data in question, and was able to give some justification regarding the inappropriateness of the mode.

Knowledge of content and students: Transnumeration

The *knowledge of content and students: transnumeration* component includes: knowledge of the common errors and misconceptions that students develop in relation to the skills of transnumeration (including sorting data, changing data representations such as into tables or graphs, and finding measures to summarise the data); the ability to interpret students' incomplete or 'jumbled' descriptions of how they sorted, represented, and used measures to summarise the data; an understanding of how well students would handle the tasks of transnumeration; and an awareness of what students' views may be regarding the challenge, difficulty, or interest in the tasks of transnumeration.

There were situations in which students, when handling the data cards and sorting them, tried to consider too many variables at once and could not manage the complexity in the sorting of the cards and in making sense of what the cards showed. Linda was aware of this difficulty and guided the students to sort the cards 'more slowly' [S1L4 V 17:39].[1] She suggested sorting by one variable, and then splitting the groups by a second variable; she knew how many groups of data there would be from sorting by three variables and therefore that it needed to be simplified for the students [S1L4 Int 18:32]. In general, the teachers did not realise how much the students would struggle with sorting the data cards, especially when the students were looking at numeric data such as arm spans, heights, and so forth. The teachers were surprised that the students did not naturally order the numeric data but simply grouped the data cards into piles. Furthermore, sorting data cards to check for and show relationships between two data sets was difficult for students, and most of the teachers underestimated the level of challenge that students would therefore face with sorting to show relationships in the data.

Knowledge of content and teaching: Transnumeration

The ability to plan an appropriate teaching sequence related to transnumerating data, to understand which representations are likely to help or hinder students'

[1] The transcript and/or notes from the data analysis software Annotape, for the lesson that this code identifies, is included for reference in Appendix 4. The associated interview for this lesson is also included in the appendix. This appendix also includes other selected examples of lesson transcripts and/or notes rather than the full set from all the lessons/interviews.

development of the skills of transnumeration, and to decide from a statistical point of view how to respond to a student's answer, are all aspects of *knowledge of content and teaching: transnumeration*.

All the teachers displayed this component of knowledge. Some examples of its use included suggestions: for how the data cards might be arranged on the desk when sorting; to spread the data cards within each group so that all the data cards could be seen, which helped with noticing patterns or irregularities within the data and then making statements about what had been found; and for creating a two-way table of frequencies as another useful representation of the sorted data cards.

5.2.4 Variation

Consideration of variation in data is an important aspect of statistical thinking (Wild & Pfannkuch, 1999). It affects the making of judgments based on data, as without an understanding that data varies in spite of patterns and trends that may exist, people are likely to express generalisations based on a particular data set as certainties rather than possibilities.

Common knowledge of content: Variation

The knowledge category of *common knowledge of content: variation* manifests itself in the classroom when the teacher gives examples of statements about data that acknowledge variation through the language used. Some of the more common situations that were observed related to inferential statements. Such statements were either about the actual data set and based on it, or generalisations about a larger group (population) from the smaller data set (sample). Such language included words and phrases such as "maybe …", "it is quite likely that …", and "there is a high probability that …". In addition, when the teacher talked about another sample being similar, but not identical, to the first sample, *common knowledge of content: variation* was evidenced.

Specialised knowledge of content: Variation

Making sense of and evaluating students' explanations around whether it is possible to generalise from the data at hand to a larger group involves *specialised knowledge of content: variation*. For instance, when Linda asked whether there would be many boys who watched a particular programme on TV based on the class data that showed only a small proportion of such boys, a student answered, “Don’t know; she hasn’t asked all the classes yet.” The teacher had to evaluate whether that was a reasonable response in relation to understanding of variation; Linda explained [S1L4 Int 21:58] that there are factors that might affect the validity of this generalisation, but that the student’s justification (about not having the data from the population so therefore it was not possible to make such a generalisation) was not a good reason for not generalising from the class data.

Knowledge of content and students: Variation

Knowledge of content and students: variation includes knowing what students may struggle with in relation to understanding variation, and to predict how students will handle tasks linked to variation. Whether students can appreciate and think about variation in data while looking for patterns and trends in the data is something that a teacher needs to listen for in students’ explanations and generalisations. Although all the teachers posed questions as to whether it was possible to generalise from the class data to a wider group, there was no significant evidence of *knowledge of content and students: variation* being used by the teachers. It may be that for the investigations being conducted, such teacher knowledge of *variation* was not called on because the students were not ready for this inferential-type thinking. Since it was something new for the teachers to teach, they had not considered the statistical implications relevant to the students’ readiness for thinking in relation to *variation*.

Knowledge of content and teaching: Variation

How to structure teaching for understanding variation is the main component of *knowledge of content and teaching: variation*. Teachers intentionally modelled appropriate explanations and generalisations, through the use of language that acknowledged the existence of variation, and their questioning encouraged the students to consider whether various generalisations were appropriate. Students were challenged to consider the presence of variation in the data and therefore how it would affect statements that could be made about the data.

An example of a teacher using *knowledge of content and teaching: variation* arose when Linda challenged a student who claimed that, although all boys in the class could whistle, not all boys could whistle. She asked: “Why not? We have just found that all boys in this class can whistle. Why wouldn’t it be the same everywhere else?" [S1L2 V 9:50]. Linda justified this as encouraging the students to think about "the bigger picture … This was data for our class. It was just a sample of maybe everyone in our school" [S1L2 Int 10:22]. Another teacher, Rob, posed a question for the students to consider: "Will the things that we found out from the data squares yesterday be similar or different to our class?" [S3L1 V 2:16]. This question was designed to encourage the students to consider variation; the challenge for students was to consider and account for similarities along with differences at the same time. Louise also posed a question that encouraged students to consider variation in data between samples; she asked how many boys in the school might have the same data square (i.e., respond identically to the four data questions), given that there were four boys in the class with that particular data square [S4L1 V 7:43]. When one student answered, "I don't know the right answer but there could be four in every class," Louise pushed the students' variation thinking further by asking whether there were other possible answers. By using her *knowledge of content and teaching: variation* in this way, she was encouraging the students to develop their conceptual understanding of variation.

Beyond asking questions such as in the examples above, the teachers did not know how to further develop the students' thinking about variation. Teaching the relatively sophisticated and complex concept of variation and inference was new for these teachers. Therefore it is not surprising that evidence of *knowledge of content and teaching: variation* was relatively limited.

5.2.5 Reasoning with models

For people to be able to make sense of data, statistical thinking requires the use of models. At the school level, appropriate models with which students could reason include graphs, tables, summary measures (such as median, mean, and range), and as used in this research, sorted data cards.

Common knowledge of content: Reasoning with models

If teachers demonstrated evidence of *common knowledge of content: reasoning with models*, it would be through making valid statements for the data, based on an appropriate use of a model. As with some of the other categories of knowledge, there was no direct evidence of this type of knowledge, but it was indirectly seen through other categories as discussed below.

Specialised knowledge of content: Reasoning with models

Specialised knowledge of content: reasoning with models is needed to interpret students' statements to determine the validity or otherwise of those statements. Students often struggled with making sensible and valid statements about the data based on a particular model they were using, and as a consequence it was not always straightforward for the teachers to make sense of the students' statements. Consequently, this category is seen as being quite distinct from *common knowledge of content: reasoning with models*.

Specialised knowledge of content: reasoning with models was a very commonly occurring component of teacher knowledge, especially as the focus of the unit was on finding interesting things in multivariate data sets, and making statements about these data sets. In many cases, students justified their statements through reference back to the model and as such, the teachers needed *specialised knowledge of content: reasoning with models* to help check the veracity of the students' statements. For example, the following interaction [S1L2 V 12:16], initially between Linda and one student but later extended to the whole class, exemplifies the challenge for teachers to listen to and make sense of students' statements:

Student: That most girls can write with their right hand, … most girls write with their right hand ... [inaudible].

Teacher: Sorry, I didn't catch what you said. Can you say that again for me? Slower this time.

Student: Most girls can write with their right hand are the youngest in …

Teacher: Hang on. Most … what are you saying? Most girls who produce their neatest handwriting with their right hand can whistle. … [pause]. Okay … [pause]. How many girls who produce their neatest handwriting with their right hand can whistle? … [pause] Is that what you have got in front of you? [pointing at the cards on the desk] ... How many is that? [Student can be seen nodding as he counts cards] … Is that these ones?

Teacher: So there are 5? … These ones can whistle as well? But are they right handed? Okay. So what are you comparing that with? You said "most." So most compared with what? [No response from student.] In comparison with the right handed boys or in comparison with the left handed girls?

Student: Left handed girls.

Teacher: Okay… [pause] So R and J have taken that a step further and they have got … [teacher moves to the whiteboard and starts drawing a type of two-way table – see Figure 5-2] … here right-handed girls and right-handed boys and they have taken just this square [lower right] and sorted those people [the right handed girls] into different piles, into whistlers and non-whistlers. And they have found that there are more whistlers who are girls who are right handed than non-whistlers who are girls who are right handed. I think that is what they are trying to say.

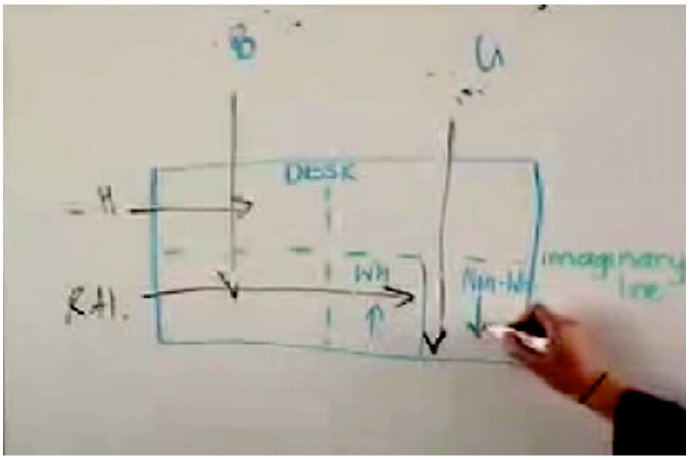

Figure 5-2: Diagram drawn by Linda to help students make sense of the statement from R and J.

The interaction indicates the use of *specialised knowledge of content: reasoning with models* by the teacher, involving initially the model of sorted data cards on the student's desk, followed by the model on the board that she created from transnumeration of the data cards.

This interaction, involving Linda and one student illustrates a number of iterations of searching and eliminating mistaken ideas, as per the logic of learning model (Swann, 1999c), which was examined in Chapter 3, *Methodology in Theory*. The teacher is seeking to make sense of the student's statement, and does so in stages, where each stage can be considered as involving the discovery a lack of clarity, a desire to eliminate that lack of clarity through obtaining a better understanding of what the student is saying. This occurs a number of times, as the understanding is more and more clarified and refined. Because of the number of iterations occurred over a relatively short period of time, this situation also links to a discussion in Chapter 3 about the time frames over which learning can occur. In this case, the learning occurred in a number of relatively short 'bursts'. There were other examples of such a refining of understanding taking more time, as longer explanations were required from students.

Knowledge of content and students: Reasoning with models

If a teacher can anticipate the difficulties that students might have with reasoning using models, or can make some sense of students' incomplete descriptions, then the teacher would be showing evidence of *knowledge of content and students: reasoning with models*. In one example of such knowledge, Rob described how he worked with a group of students who had made a statement from the data cards comparing the number of boys with the number of girls who were right or left handed [S3L1 Int 1:45]. Rob knew that the students were capable of proportional thinking so he encouraged them to consider proportions. He did so because the numbers of boys and girls in the data cards were different, and therefore using proportions for the comparison would be more appropriate than using frequencies. Rob knew these students sufficiently to encourage them to reason with a proportional model, which two of the students handled particularly well. Likewise, Louise encouraged a group of students, who she knew were capable and ready, to make statements using proportions instead of frequencies [S4L3 V 2:44]. Both teachers used their *knowledge of content and students: reasoning with models* to extend their students thinking using proportional reasoning.

Knowledge of content and teaching: Reasoning with models

How should a teacher structure the teaching to encourage students' statistical thinking in relation to reasoning with models? This question is at the heart of the teacher knowledge category of *knowledge of content and teaching: reasoning with models*. A teacher with sound knowledge in this category would have considered various approaches to teaching this aspect, could justify a particular approach that was taken and maybe why other approaches were rejected, and could consider any statistical issues that might arise from students' statements or explanations.

John commented [S2L1 IntB 24:32] that because the students had tended to focus on only one variable at a time and make frequency-based statements for comparisons, he would structure the next lesson differently. He intended to encourage the students to consider two variables simultaneously, and would do this by posing some questions to focus the students, as well as suggest to them ways of sorting the data cards to enable the questions to be investigated. John's *knowledge of content and teaching: reasoning with models* and *transnumeration* developed as a result of becoming aware of a difficulty that the students had with reasoning with models, that is, as a result of a development of his *knowledge of content and students: reasoning with models*.

5.2.6 Integration of the statistical and the contextual

Wild and Pfannkuch (1999) describe the importance of continually linking contextual knowledge of a situation under investigation with statistical knowledge related to the data of that situation. The interplay between these two enables a greater level of data sense and a deeper understanding of the data, and is therefore indicative of a higher level of statistical thinking.

Common knowledge of content: Integration of statistical and contextual

The component of *common knowledge of content: integration of statistical and contextual* is characterised by the ability to make sense of graphs or measures, and by an acknowledgement of the relevance and interpretation of these statistical tools to the real world from which the data was derived. For example, John gave some possible reasons to support the finding that all the youngest students could whistle [S2L2 Int 14:55][2]. He suggested that the older siblings could have taught the younger ones to whistle. This shows thinking of the real-life context in association with what the statistical investigation had revealed; such integration of the two aspects can sometimes enable the answering of 'why might this be so' that is being illustrated by the data.

Specialised knowledge of content: Integration of statistical and contextual

Being able to evaluate a student's explanation based on both statistical data and a knowledge of the context under investigation is one aspect of the category of *specialised knowledge of content: integration of statistical and contextual knowledge*. There were a number of situations in which the teacher prepared the

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[2] The transcripts and notes for this lesson, as well as its associated interview, are included in Appendix 4, as a sample.

students to gather data. Data collection questions had been suggested, such as, "What position are you in the family, youngest, middle or eldest?" When the students were considering the question prior to the actual data gathering, Linda was asked:

> Does it count if you have half brothers or sisters?
> What if your sister or brother has died?
> What if your brother or sister is not living at home?
> What would you put if you were an only child?

[S1L1 V 10:45]

Each of these questions and others, involving the definition of family, were unexpected by Linda. She had to decide 'on the spot' how to respond to each question from students. She was required to weigh up the statistical issues related to answering such a data gathering question with the contextual issue of interpretation of 'family'. Her answers indicated that she was able to do so satisfactorily and therefore were evidence of her having *specialised knowledge of content: integration of statistical and contextual*.

Knowledge of content and students: Integration of statistical and contextual

Can a teacher anticipate that students may have difficulty with linking contextual knowledge with statistical knowledge? Are students, through focusing on statistical knowledge and skills, likely to ignore knowledge of the real world, that is, contextual knowledge, or vice versa? Such aspects would give an indication of a teacher's *knowledge of content and students: integration of statistical and contextual*.

Whereas Linda's students' questions (which related to the data question of position in the family, as discussed above) were unexpected, John anticipated such possible difficulties for his students and pre-empted their questions by asking the class how each child from a four-child family might answer the question, "Are you youngest, middle, or eldest in the family?" John's question encouraged the students to think about the data question (the statistical) in association with their knowledge of particular families (the contextual). This helped the students understand that statistics is not performed 'in a vacuum', removed from real issues, but deals with numbers that have a context (delMas, 2004).

Knowledge of content and teaching: Integration of statistical and contextual

Knowing how to encourage students to consider the relevance of contextual knowledge in relation to the statistical investigation being undertaken is part of a teacher's *knowledge of content and teaching: integration of statistical and contextual*. The situations described above for *specialised knowledge of content: integration of statistical and contextual* (in relation to the definition of family and unusual cases) required the teacher to weigh up, prior to answering each student's query, the extent to which such interpretations of 'family' might affect the reliability of the data obtained. Linda commented:

> Everyone has their own definition of what a family is … so I decided that the children could, if they wanted to, include their half brothers and sisters.
>
> [S1L1 Int 11:58]

Also John decided on an approach to teaching that involved asking students a question based on a 'what if …' scenario, as he had anticipated a possible difficulty that students might have with interpretation of the question for a particular family. Louise encouraged her students to integrate the statistical and the contextual when she asked them to think of situations involving various aspects of statistics (such as graphs and summary measures of data), and what these are used for [S4L1 V 0:00]. These examples show that each teacher demonstrated some *knowledge of content and teaching: integration of statistical and contextual*.

5.2.7 Investigative Cycle

One of the four dimensions of statistical thinking, as defined by Wild and Pfannkuch (1999), is the investigative cycle. This cycle, characterised by the phases of 'problem, plan, data, analysis, and conclusions', is what someone works through and thinks about when immersed in problem solving using data.

Common knowledge of content: Investigative Cycle

If a teacher can fully undertake and engage with an investigation, then that teacher would be demonstrating *common knowledge of content: investigative cycle.* The teacher would be able to: pose an appropriate question or hypothesis, or set a problem to solve; plan for and gather data; analyse that data; and use the analysis to answer the question, prove the hypothesis, or solve the problem.

For example, Linda discussed [S1L1 Int 4:20] how data might be handled with an open-response type of question in a survey or census. Linda had considered, at the question posing phase of the investigation, how the responses from such an open-response type question would present a challenge at the analysis stage. This clearly indicated that Linda had some knowledge of the phases of the investigative cycle. She was able to maintain an awareness of a later stage of the cycle (analysis) while dealing with an early stage (planning data collection), and consider how decisions at that early stage could impact on the later stages.

Specialised knowledge of content: Investigative Cycle

A teacher needs *specialised knowledge of content: investigative cycle* when dealing with students' questions or answers in relation to phases of the investigative cycle, or when discussing or explaining various phases of the cycle and how they might interact. When thinking about suggestions for what could be investigated in a data set, the teacher needs to be able to evaluate the suitability of the problem/question, and whether it needs to be refined to be usable and suitable, in relation to the subsequent analysis.

The teaching unit used in the study involved introducing the students to an example of a data card that contained four data about an individual, and asking students to suggest some data collection questions that could have been used to generate the data on the data card. There were a number of instances when students suggested inappropriate data questions. For example, to generate the response of 'right', one suggestion was, "What is the opposite of left?" [S1L1 V 5:32]; and for the response of 'whistle', a suggestion was, "What does a referee use to control a game?" [S1L1 V 6:49]. In both of these situations, Linda had an awareness of the investigative cycle to realise that such questions would be inappropriate for gathering data. One of Louise's students suggested that for a data card that contained 'B, 6, 10, 13', the numbers could have been the ages of the person's siblings. Louise, like Linda in the previous examples, had to use some knowledge to quickly evaluate the response and recognise that it was inappropriate as a data collection question. Such recognition of the inappropriateness of the questions was an indication of each teacher using *specialised knowledge of content: investigative cycle*.

Knowledge of content and students: Investigative Cycle

Knowledge of where students might encounter problems or particular challenges in an investigation, and whether students will find an investigation interesting or difficult, are aspects of *knowledge of content and students: investigative cycle*.

One teacher knew that students might have difficulty when developing data collection questions for their own investigation. Another teacher predicted that students could have a problem with knowing how to interpret a data collection question so had to consider how he would deal with this potential problem within an early phase of the investigative cycle. The analysis phase of an investigation was predicted to present challenges for students in relation to them deciding on the form to present the data.

Some teachers were aware that students would be challenged within the investigative cycle with moving from the analysis stage to the drawing of conclusions or the answering of questions that had formed the basis of the investigation. Such awareness meant that those teachers had thought about how to address the students' difficulties.

Knowledge of content and teaching: Investigative Cycle

Being able to encourage students to think about each phase of the investigation and to consider how these phases link to one another (i.e., to deal with the parts without losing sight of the whole) are components of the *knowledge of content and students: investigative cycle*.

In the previous section, an example was given of a teacher predicting that students may have problems interpreting some data questions. John, based on this knowledge of students, considered how to approach his teaching so as to prevent the students from having such problems. He handled it in two ways: on one occasion, he discussed an example with the students about their experience of having students from another class gather data from them, and how they had found some of the data questions difficult to answer; on another occasion John asked the students about how they would answer a particular data question, knowing that different interpretations were possible. By structuring the teaching in this way, based on *knowledge of* *content and students: investigative cycle,* the teacher successfully utilised *knowledge of content and teaching: investigative cycle*.

5.2.8 Interrogative Cycle

A statistical thinker engages in the interrogative cycle when working with data through such activities as: generating possibilities; seeking or recalling of information (from within the data or from a wider context); interpreting the results of seeking (by linking with the results obtained from analysis; by comparing and contrasting; and by making connections); criticising the information and ideas as they evolve (with both internal and external reference points; a form of metacognition, monitoring one's own thinking); and judging what to ignore and what we now believe or know (Wild & Pfannkuch, 1999). The interrogative cycle can be considered a thinking process in which information and ideas are considered and refined, to finally emerge in a more useful and improved form.

Common knowledge of content: Interrogative Cycle

A teacher would have *common knowledge of content: interrogative cycle* if it was evident that possibilities in relation to the data were considered and weighed up, with some possibilities being subsequently discarded but others accepted as useful. Engaging with data and being involved in 'debating' with it would be evidence of such knowledge. Likewise, developing questions that the data may potentially be able to answer is an aspect of *common knowledge of content: interrogative cycle*.

Teachers who had immersed themselves with a data set prior to using it in teaching, so that they were aware of some of the things that might be found from the data, would be showing *common knowledge of content: interrogative cycle*. Such teachers would be prepared for knowing what their students might find in the data and what conclusions might be drawn from that data.

Specialised knowledge of content: Interrogative Cycle

So what does *specialised knowledge of content: interrogative cycle* look like, as distinguished from *common knowledge of content: interrogative cycle*? When a teacher has to consider whether a suggestion from a student is viable for investigating within that data, the teacher requires *specialised knowledge of content: interrogative cycle*. Also, it involves determining whether a student's suggested way of handling and sorting the data would be useful to enable the later interpretation of results in relation to the question at hand. For instance, when a student proposed to investigate whether boys or girls had bigger feet and suggested to Rob how they were going to sort the data cards to help answer the question, Rob had to consider whether this was likely to be a worthwhile approach [S3L4 V 4:58]. In this situation, Rob employed *specialised knowledge of content: interrogative cycle*.

Knowledge of content and students: Interrogative Cycle

Knowledge of how students would handle the development of appropriate questions for investigating the data, and the extent to which they might engage with the data and be prepared to question and consider various possibilities, are elements of *knowledge of content and students: interrogative cycle*.

There were a number of instances when teachers became aware that students, rather than fully engaging with the data and seeking possibilities, were focusing on a narrow aspect of the data, such as individual data points. The students then used this narrow focus to argue for or justify a particular position. Teachers who had *knowledge of content and students: interrogative cycle* were able to consider ways in which this tendency amongst students could be mitigated. Such considerations led to the next component of knowledge that is described below.

Knowledge of content and teaching: Interrogative Cycle

The strategies a teacher might use to address students' tendency to ignore a wide range of possibilities and, instead, be content with a narrow, restricted focus in their investigation of data, constitutes a part of *knowledge of content and teaching: interrogative cycle*. Being able to consider, from a statistical point of view, how such limited views of the data might impact on an investigation is another component of this category of teacher knowledge.

One teacher, Linda, decided that to assist the students to examine possible relationships in the data, it was important to spend some time discussing with the students what are relationships. Following this, Linda brainstormed with the class some possible relationships that might be investigated in the data [S1L2 Int 4:04]. She considered that this was time well spent, as it enabled the students to focus quite quickly on the data and engage with it meaningfully from the outset. It was quite common for teachers to ask the students to think about what might be found in the data, once the students had an idea of what the data set contained (in terms of the variables), but prior to seeing the complete data set. Again, this teaching strategy helped the students to engage quickly with the data as they had already started to think about the data and had developed an interest in it. These examples are evidence of the teacher having *knowledge of content and teaching: interrogative cycle*.

5.2.9 Framework Summary

The components of teacher knowledge for teaching statistics have been described in relation to the cells of the framework. Some components were more frequently observed in the classroom than others. Examples from the classrooms have supported the descriptions of the cells. The subsequent sections in this chapter profile each teacher's knowledge as enacted (or not enacted) in the observed lessons. Included with each profile is a description and discussion of missed opportunities within the lessons.

5.3 Profiles of teachers' statistical knowledge

5.3.1 Introduction

In this section, each teacher's statistical knowledge is profiled in relation to the categories of teacher knowledge that are described in the first part of this chapter. These profiles are derived from the coding of teacher knowledge in the lessons and the interviews.

For each teacher, a framework profile shows shading of cells. A shaded cell indicates an 'incident' or 'incidents' that arose within the teacher's lessons or associated interviews for which that aspect of teacher knowledge was identified. If the incident revealed a lack of knowledge, and therefore potentially a missed opportunity to positively influence student learning, the cell shows an 'M'. The presence of an 'M' in a cell does not necessarily preclude other positive aspects of teacher knowledge for that cell. Consequently a cell can indicate presence of an aspect of that particular knowledge (through being shaded), while simultaneously showing a missed opportunity (through an 'M') with regard to that knowledge.

As discussed earlier in the chapter, the statistical thinking dimensions of dispositions and recognising the need for data are not included in the following descriptions, as they were not evidenced in the observed lessons. However, they are still aspects of teacher knowledge that could be found in relation to teaching investigations using an approach different from that used in this study. Consequently, both *Need for data* and *Dispositions* remain part of the framework profiles for the teachers, but are blank.

5.3.2 Linda

Lesson Sequence

Lesson 1 consisted of the students being introduced to a data square, a 'tool' for recording data and subsequent sorting of data, in order to find patterns and relationships in the data. The students suggested data questions that could have generated the data on the data square. They were required to put their own data on a data square (using the same data questions that had been used for the sample data square) so that in the subsequent lesson, the students would be able to sort the set of data squares for their class and thereby discover interesting findings about their class. Lesson 2 mainly focused on using the data squares to find relationships between the variables represented on the data square. Linda posed some questions as prompts to encourage the students to examine two variables at a time. One such question was, "Do you find anything interesting when you compare place in the family and whistling?" From their investigation of the data, the students were expected to record their findings and share these with the class. Following the investigation of the class data, it was intended that individual students would develop their own four data questions (all based on category data). The students were asked to predict what they might investigate with their data once they had conducted their surveys in Lesson 3 (which was not videoed for the research). Lesson 4 involved the students in investigating their data and recording interesting findings using a diagram or table, and writing a conclusion or conclusions based on these.

Profile for Linda

For Linda, the data collated in Table 5-2 below was obtained from three lesson videos (for Lessons 1, 2, and 4) and their associated interviews. Lesson 3 did not yield an edited video because that lesson mainly consisted of data gathering by the students and Linda was not involved at a level that would give an insight into her teacher knowledge. In this lesson, she was primarily engaged in management of the students' data gathering.

Table 5-2: Summary of Linda's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				M
	Reasoning with models			M	
	Integration of statistical and contextual				M
Investigative cycle					M
Interrogative cycle					
Dispositions					

Key: = direct evidence of that knowledge used; = indirect evidence of that knowledge; M = missed opportunity related to that knowledge.

General Description of Teacher Knowledge Profile

An examination of Table 5-2 reveals a noticeable absence of *common knowledge of content* in all the lessons, other than *common knowledge of content: reasoning with models*. However, as discussed earlier, it can be assumed that Linda had *common knowledge of content* in a particular category if there was *specialised knowledge of content* in that same category. For instance, following a student's suggestion that the question, "What is the opposite of left?" could have generated the data 'right', Linda considered how to respond to that suggestion. It was clear that she knew that such a question was not suitable for generating data and responded accordingly. This incident indicates *specialised knowledge of content: investigative cycle*, and it subsumes the teacher's *common knowledge of content: investigative cycle* as it relates to designing survey questions. Consequently in the profile, the shading for *common knowledge of content: investigative cycle* indicates that this knowledge category was indirectly evidenced through another category. The same procedure has been followed throughout the teacher profiles for aspects of *common knowledge of content* being indirectly evidenced.

It must be noted that shading of a cell does not necessarily indicate knowledge of all relevant aspects. For some of the statistical thinking components (as, for example, the *investigative cycle*), a wide range of concepts would be involved. The evidence that results in a cell being shaded might, however, only relate to a small portion of all relevant concepts. Nonetheless, the cell would be shaded to show verification of some knowledge in relation to the statistical thinking component.

For the three other main categories of teacher knowledge, namely *specialised knowledge of content*, *knowledge of content and students*, and *knowledge of content and teaching*, Linda demonstrated links with most of the dimensions of statistical thinking (as shown by almost complete shading of the cells in Table 5-2). She displayed *specialised knowledge of content* in all aspects of statistical thinking other than the *interrogative cycle*. The exceptions for *knowledge of content and students* were with regard to *variation* and the *integration of the statistical with the contextual*, while for *knowledge of content and teaching*, all links with the components of statistical thinking were observed.

Missed Opportunities Within the Lessons

The question arises as to whether the 'missed opportunities', which were mainly confined to the *knowledge of content and teaching* component, had significant impact on the students' learning opportunities. By analysing the four incidents that were identified as missed opportunities, the relative importance of these missed opportunities can be considered. The situation in relation to *knowledge of content and students: reasoning with models* involved Linda listening, along with the rest of the class, to a student's finding from the data [S1L4 V 12:36]. Linda had to ask the student to repeat the finding because it had not been explained clearly and she had not made sense of the explanation. Once the student had repeated the finding, Linda accepted it even though it was not supported by the data. Linda had previously drawn a two-way table for that data on the whiteboard and could have referred to it in order to check the validity of the student's finding. However this did not happen, nor was there any questioning from other students. Also, Linda did not suggest, for example, that the other students could refer to the two-way table to check the statement. Both aspects indicate that this was a relatively significant missed opportunity in relation to encouraging students' reasoning with models.

There were three missed opportunities in relation to *knowledge of content and teaching*. The first, with *knowledge of content and teaching: variation,* followed a discussion as to whether an inferential statement could be made, based on the class data showing that all boys could whistle. Linda explained to the students that, "We can only say that all boys in our class can whistle, not all boys in our school; we are only a sample of the school" [S1L2 V 10:49]. The opportunity to make an inferential statement about the whole school using qualifiers such as 'most' or 'it is likely that' was missed. The relative significance of this is dependent on the students' readiness for engaging in inferential thinking. Such inferential thinking is further discussed in the next chapter.

The other two situations involving *knowledge of content and teaching*, one in connection with *integration of the contextual with the statistical*, and the other in relation to the *investigative cycle,* were both relatively insignificant; with the former, when comparing students' predictions of what they might find in the data (prior to actually investigating the data) with their actual findings, the chance to discuss the 'real world' relative incidence of right-handedness with left-handedness was not considered; and with the latter, how to word a data question in relation to how the response might be recorded was not considered. The potential data question, "Are you right handed?" would be answered with either 'yes' or 'no', which then could be recorded however as 'right' for 'yes' and 'left' for 'no'. However Linda and the students were more concerned to ensure that if the response was to be recorded as 'right' or 'left', then the data question should be worded so that one of those responses would be given in answer to the question. As already postulated, neither of these potential missed opportunities could be considered significant in relation to the development of students' statistical thinking.

Overall, Linda demonstrated a reasonably comprehensive knowledge base for teaching statistics, both through her planned lessons and as the lessons 'unfolded' in the reality of the classroom. The lessons progressed in such a way that the learning opportunities for students were generally ably supported by Linda's knowledge in relation to the statistical thinking dimensions.

5.3.3 John

Lesson Sequence

The following summary of the four observed lessons covers what was planned and delivered by John. Lesson 1 started in a similar way to Linda's first lesson, with the students being introduced to the data squares and considering the data questions that may have generated that data. Some consideration was given to how a data question might be answered for 'unusual' cases; for example, if you were a twin, how would you answer the data question, "What is your position in the family – oldest, middle or youngest?" Following this discussion, John explained to the students the aim of the investigation, which was to make statements about what the data showed, draw conclusions from the data, and maybe make some generalisations about a bigger group. The students, in groups, undertook an investigation of the data, and John discussed with various groups what they were doing in relation to sorting the data cards, what they were looking for, and some of their initial findings.

Lesson 2 commenced with a class discussion about interesting things that were found from the previous lesson's investigation. John posed questions to focus the students' subsequent investigations, and following the investigations, an extended discussion was held on the answers to these questions,. Lesson 3 continued with discussion of findings from the data, involving initially the whole class and then groups. Lesson 4 moved into using a multivariate data set that consisted of, rather than solely category data, two sets of category data with two sets of measurement data. Discussion took place around transnumerating some of the data into measures to represent the data; a question was posed about which measure would be best but the discussion mainly related to how to calculate the measures. The students then investigated the data set, again looking for interesting things within the data.

Profile for John

Table 5-3 below gives a profile for John of the knowledge categories in relation to the dimensions of statistical thinking. The data for this table came from four lessons and three interviews; the videos based on Lessons 2 and 3 were relatively short (about 7 mins and 11 mins respectively) and only one interview was conducted to cover both lessons following Lesson 3, due to difficulty scheduling an interview between Lessons 2 and 3.

Table 5-3: Summary of John's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				
	Reasoning with models	M	M	M	M
	Integration of statistical and contextual				
Investigative cycle		M	M		M
Interrogative cycle			M		M
Dispositions					

Key: = direct evidence of that knowledge used; = indirect evidence of that knowledge; M = missed opportunity related to that knowledge.

General Description of Teacher Knowledge Profile

A similarity between John's and Linda's profiles is that there was evidence of most aspects of teacher knowledge in conjunction with statistical thinking. However, there were a number of cells of the framework that were not present in John's profile. Consideration of *variation* in data did not feature in any of the forms of teacher knowledge and this aspect will be discussed in more detail in the next chapter.

There was little evidence of knowledge in relation to the *interrogative cycle* other than *knowledge of content and teaching: interrogative cycle*. Similar to the discussion of Linda's profile, the evidence for John's *common knowledge of content: interrogative cycle* is indirect but came from some examples that he gave the students as possible investigations. The suggestions included: whether there is a relationship between being left handed and being able to whistle (in Lesson 1); "Could we make a statement that girls who are youngest in family and right handed can all whistle? (Lesson 4); and “Does being a boy or a girl affect how quickly you react?” (Lesson 4). These suggestions indicate that John could pose suitable questions as a basis for an investigation.

Although it has been argued that *common knowledge of content* can often be evidenced indirectly from *specialised knowledge of content*, it is not possible to find evidence of *specialised knowledge of content* within other categories. Knowledge of how to evaluate and respond to a student’s suggestion for interrogating data (i.e., *specialised knowledge of content: interrogative cycle*) cannot be determined from the presence of another category of knowledge in relation to the *interrogative cycle*. Similarly, it is not possible to make assumptions about the presence of *knowledge of content and students: interrogative cycle* because of the presence of another category in relation to the same component of statistical thinking.

Missed Opportunities Within the Lessons

A significant difference between the profiles of John and Linda is the prevalence of ‘missed opportunities’. Analysis of John’s ‘missed opportunities’ reveals some interesting patterns.

A number of these missed opportunities can be attributed to different interpretations of a question that John posed the class. In relation to the data on ‘handedness’ (right or left) and ‘whistling’ (can or can’t), John’s question was, “Are there more whistling right handers or whistling left handers proportionally?” (Table 5-4 shows a representation of the data being addressed by this question.)

Table 5-4: Two-way table showing the data related to the question posed by John about proportions of whistling right handers and whistling left handers

	Right handers	Left handers	Total
Whistlers	15	2	17
Non-whistlers	6	1	7
Total	21	3	24

The somewhat ambiguous wording of the question may have contributed to John’s misinterpretation of it. The question was presumably intended to focus on the proportion *of right handers* who could whistle (15/21) compared with the proportion *of left handers* who could whistle (2/3). However, John interpreted the question as comparing the proportions *of the class* who were whistling right handers (15/24) and whistling left handers (2/24).

During some of the class discussion concerning this question, John inadvertently changed the wording from whistling right handers to right handed whistlers, and similarly for left handers. At the time, John was not aware that this significantly changed the focus of the question, and therefore the answer. In relation to the table above, the change in wording resulted in the original question's focus on columns and their totals being changed to a focus on one row (i.e., the whistlers). Only one student in the class questioned this, but John misunderstood his question. Consequently John's explanation to the student was inappropriate in relation to the student's question. If John had transnumerated the data from the sorted cards into another form, such as a two-way table similar to that shown in Table 5-4, the explanations from either the teacher or students could have been linked to that data representation to assist the making sense of what was being said.

The missed opportunities resulting from the misinterpretation of this question relate to a number of different teacher knowledge categories. John had not attempted to answer the question prior to the lesson nor check the data, so did not exhibit either *common knowledge of content: reasoning with models* or *common knowledge of content: transnumeration*; also *specialised knowledge of content: reasoning with models* was absent because of John's inability to analyse whether the student's answer and explanation were valid and appropriate. Not being able to interpret, 'on the spot', the student's incomplete explanation indicates a lack of *knowledge of content and students: reasoning with models* for this particular situation.

There were other instances where a similar misunderstanding of a student's response led John to miss taking advantage of potentially valuable learning opportunities. Misinterpretation of students' responses will be further discussed in the next chapter in relation to the 'theme' of 'teacher listening'.

A number of missed opportunities resulted from John's lack of guidance as to what students could look for within the investigation, and lack of suggestions for ways of sorting and arranging the data cards to assist the noticing of things within the data. For instance, John posed some questions to focus the students' investigations. These were taken directly from the unit plan. They were: Question 1: "Are there more whistling right handers or whistling left handers proportionally?"; Question 2: "Is there anything interesting when comparing place in the family and whistling?"; and Question 3: "All the boys in this group who are youngest can whistle. Does this mean every boy who is the youngest in their [sic] family can whistle?" Each of these three questions involved two variables from the data set. However, John suggested to one group to check, "Are there any relationships between girls being the oldest, right handed and whistling?" This suggestion involved all four variables, which complicated it significantly. The suggestion also did not provide clear guidance to the students on either how to proceed with the sorting, or with what type of relationship the teacher might expect that students could find from the data. Another overly complicated suggestion was:

> Put the data squares into some groups and see if there are some relationships between middle, oldest, youngest with left handed/right handed, boy/girl, left handed people can whistle.
>
> [S2L1 V 13:27]

Both these examples illustrate a problem with John's *knowledge of content and teaching* in relation to the *investigative cycle, transnumeration,* and *interrogative cycle*.

5.3.4 Rob

Lesson Sequence

The first lesson of Rob's unit, referred to as Lesson 0 as it was not observed as part of the study, introduced data squares to the students. The data were distributed to and investigated by the students. Lesson 1 (the first observed lesson) started with the class sharing and discussing findings from the previous day's investigation. This activity was designed to provide a platform from which the class could investigate their own class data and look for comparisons with the previous data. Rob asked for predictions about what might be found from the data. Then the students, in groups, investigated their class data and shared their findings with the rest of the class.

Lesson 2 used another data set from the unit plan, this time involving both categorical data (gender and class level) and numerical data (age and average reaction time). The initial discussion was around the possible data questions that might have generated the data. Because one of the variables was average reaction time, the discussion included something on how an average is obtained. Small groups of students investigated the data set, motivated by the instruction to find something interesting in the data. A class discussion on some findings occurred at the end of the lesson.

Another data set, consisting of one categorical variable (gender) and three numeric variables (height, arm span, and age), was used for Lesson 3. Parts of the lesson involved small groups while other parts involved the whole class sharing and discussing relevant aspects of the data (including measures such as median and mean) and findings from the data.

Lesson 4 used a data set from the class, again consisting of three numeric variables (hand span, foot length, and age) and one categorical variable (gender). The aim for this lesson was for the students to write summaries of their findings, as Rob had found in the previous lessons that the students had had difficulty with making statements about the data, and so needed more practice with this.

Profile for Rob

Table 5-5 shows the summary of teacher knowledge in relation to the dimensions of statistical thinking for Rob. The data for Rob were derived from four lessons, and interviews linked to each of those lessons.

Table 5-5: Summary of Rob's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				M
	Reasoning with models		M	M	M
	Integration of statistical and contextual			M	M
Investigative cycle				M	M
Interrogative cycle		M	M	M	
Dispositions					

Key: = direct evidence of that knowledge used; = indirect evidence of that knowledge; M = missed opportunity related to that knowledge.

General Description of Teacher Knowledge Profile

Rob's profile illustrates some noticeable differences from the previous teachers' profiles. While a significant number of cells of the framework indicate evidence of an aspect of teacher knowledge, there are a number of cells, particularly with regard to *knowledge of content and teaching,* that indicate an absence of teacher knowledge. The most noticeable difference from the other teachers' profiles is the prevalence of missed opportunities, which is discussed in the next section. Rob used all types of knowledge in connection with *transnumeration*. In each of the other dimensions of statistical thinking, he exhibits some, but not all, the components of teacher knowledge.

A pertinent question, similar to that asked in relation to the knowledge of the previous teachers, is whether absence of *common knowledge of content* in some aspect of statistical thinking indicates cause for concern, or does another component of teacher knowledge subsume that *common knowledge of content*. In each case of an absence of direct evidence of *common knowledge of content*, evidence was available from at least one other category that *common knowledge of content* was available for Rob to call on. As one example, Rob talked to the students about what is known about the number of right handed people compared with left handed people. The context for this discussion in Lesson 1 was that the question had been posed as to whether another class might have the same or a similar number of left handed girls as this class. He was drawing on his *knowledge of content and teaching: integration of statistical and contextual* when talking with the students. Consequently it is reasonable to conclude that Rob must have had *common knowledge of content: integration of statistical and contextual* in order to realise the value of such a discussion with students. Because Rob was able to deal with student responses appropriately, and had knowledge of how to teach some aspect in an appropriate way, it is reasonable to assume that he must have *common knowledge of content*. Consequently in Table 5-5, the remaining cells of the framework profile that are linked to *common knowledge of content* have been shaded according to the key.

The absence of other categories of knowledge for Rob may be of more concern however. Evidence of *knowledge of content and teaching* cannot be found within different categories, in the same way that *common knowledge of content* was accounted for. Absence of knowledge categories was problematic as there were situations that arose in which those absent categories of knowledge were needed.

Missed Opportunities Within the Lessons

Fifteen of the 24 cells in the framework indicate missed opportunities throughout Rob's four lessons. A number of these stem from situations that were similar and applicable to John.

The extent to which a teacher does not hear what students are saying or does not seek clarification from them as to what they are talking about can create such missed opportunities. This has relevance to *specialised knowledge of content*, and the profile indicates that there were a number of missed opportunities related to this category of knowledge. In Lesson 2, the following interaction took place as Rob and two students discussed some possible questions that could be investigated:

> Student: Put them [the data cards] into year and age and gender.
>
> Teacher: What are you trying to find out?
>
> Student: We will figure that out once we have sorted the cards. ... Like if there are more girls who are Year 6 than boys who are Year 6.
>
> Student: We can add them together and do averages.
>
> [S3L2 V 3:24] [3]

At this point, Rob just accepted the student's suggestion without seeking any further clarification as to what the student was meaning or intending, or how an average would have helped make sense of the data and helped find something interesting. Shortly afterward, a further discussion took place between Rob and the same two students in regard to setting up an investigation:

> Student: I don't get it.
>
> Teacher: What don't you get?
>
> Student: What we are doing.
>
> Teacher: We're trying to see if there is anything interesting about this class. Like: Does it affect your reaction time to how old you are?
>
> Student: The older you are the slower you are. [Student's hypothesis, rather than a data based finding.]
>
> Teacher: Is that true for this class?
>
> Student: No idea.
>
> Teacher: Well let's see if we can find that out.
>
> Student: We could see which one was the highest and sort them out oldest to youngest and see who has the highest.
>
> Student: Do a median for this group.
>
> Teacher: So we get it sorted so we can see it, can't we, and then we can see some sort of thing.
>
> [S3L2 V 3:24]

As with the first exchange, Rob did not seek clarification from the student as to how a median might be used to help answer the question. Both of these situations indicate a missed opportunity in relation to *specialised knowledge of content:*

[3] The transcripts and notes for this lesson, as well as its associated interview, are included in Appendix 4, as a sample.

transnumeration. Rob did not engage with the students' comments from a statistical point of view, to determine whether their suggestions regarding use of, first, an average and, second, a median, had statistical merit.

Two different situations related to reasoning with models also arose because Rob either did not hear the students' suggestions or decided to ignore them. A class discussion followed Rob's posing of a question: "Can we take that statement about our class and generalise to all children in the school?" After some worthwhile discussion on this, one student commented, "No you can't generalise … because of genetics." Rob disregarded the response so gave no opportunity for the student to justify the relevance of this from a contextual point of view (if any) to the important statistical concept of inference. This missed opportunity related to *knowledge of content and teaching: integration of statistical and contextual* along with *knowledge of content and teaching: reasoning with models* because Rob did not weigh up any statistical issues with regard to the student's unexpected response.

In one incident, some students suggested a way of sorting the data cards in order to answer a question that they had posed. Rob did not notice that the suggested way of sorting the cards would not be useful for making sense of the data and answering the question. He merely accepted what was suggested and encouraged the students to continue. This indicated that *specialised knowledge of content: reasoning with models* was not being used. He did not connect the students' explanation of how they would sort the data and the focus question, thus did not realise that this sorting method would not be useful for that particular question.

Just as John had missed opportunities because of not having examined the data prior to the lesson, so did Rob. In the Lesson 2 interview, Rob discussed the situations in which he became aware that he could not help the students pose questions relevant to the numerical data because he had not examined the data himself. Rob did not know what might be worthwhile to investigate because he did not know the data. This significantly limited the support that he could give the students with 'getting inside' such numeric data. This missed opportunity relates primarily to *common knowledge of content: interrogative cycle*, but consequently impacts on other categories as well.

5.3.5 Louise

Lesson Sequence

Lesson 1 started with Louise asking the students about what they could remember about statistics, particularly from a unit that the students had completed earlier in the year with another teacher. From there she introduced the students to the data square and, just as with other teachers in this study, sought possible data questions that could have generated the data. The students, in groups, sorted and investigated the data, before returning to a class discussion on what they had found. Louise used the same three questions from the unit plan that the other teachers used, so as to help the students focus on considering two variables simultaneously.

Lesson 2 focused on similar data to that used in Lesson 1, the difference being that it was collected from this class rather than from a hypothetical class. The students investigated this data, again in small groups. Following this, the students shared their findings, which Louise recorded on the board.

Lesson 3 was based on a data set from the unit plan. This data set consisted of two categorical variables (gender and class level) and two numeric variables (age and average reaction time). A similar pattern occurred in this lesson to the previous lessons. For this new data set, the students suggested possible data questions that could have generated the data, and also considered why averages might be used for the reaction times. Groups of students investigated their own questions, and then shared their findings with the whole class. In this discussion, some students mentioned averages but there were some inconsistent or contradictory statements regarding these. Consequently the teacher decided, for Lesson 4, to focus on the use of measures for reducing and representing the data; the three averages (mode, median, and mean) and the range were the focus for this final lesson.

Profile for Louise

Table 5-6 shows a summary of the components of teacher knowledge in relation to the statistical thinking dimensions as exhibited by Louise within four lessons and associated interviews.

Table 5-6: Summary of Louise's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M		M
	Variation	M	M		
	Reasoning with models		M	M	M
	Integration of statistical and contextual		M		
Investigative cycle			M		
Interrogative cycle			M		
Dispositions					

Key: [grey box] = direct evidence of that knowledge used; [patterned box] = indirect evidence of that knowledge; M = missed opportunity related to that knowledge.

General Description of Teacher Knowledge Profile

Most of the cells of the framework were evident with Louise from the four lessons and interviews. As with each of the other teachers, the absence of *common knowledge of content* for three dimensions was examined. Evidence of all three components was found elsewhere. For example, Louise guided the students in Lesson 2 to suggest suitable questions for investigating. This corresponded to *knowledge of content and teaching: interrogative cycle* but also indicated that *common knowledge of content: interrogative cycle* was part of her knowledge base.

It can be found that Louise demonstrated evidence of *common knowledge of content: integration of statistical and contextual* through other categories. When she encouraged the students to think about the contextual use of statistics, such as when group points are averaged over the three 'sessions' in a day, Louise was calling on her *knowledge of content and teaching: integration of statistical and contextual*; she had decided on a teaching approach that would be appropriate for dealing with these measures and which made links with the use of these measures in the 'everyday classroom'. Louise must therefore also have had *common knowledge of content: integration of statistical and contextual*.

In a similar way, examination of the situations in which Louise engaged in the *interrogative cycle* shows that *common knowledge of content: interrogative cycle* can be assumed to be part of her knowledge base. Louise questioned the students about the statistics unit from earlier in the year. The students were encouraged to talk about the data they had collected, how they had analysed it, and the conclusions they had drawn. As Louise specifically asked about each of these different aspects of an investigation, *specialised knowledge of content: investigative cycle* was evident, and so it can be assumed that Louise also had *common knowledge of content: investigative cycle.*

Missed Opportunities Within the Lessons

Louise had a number of situations that could be considered missed opportunities in relation to the students' learning. The prevalence of types of missed opportunities that are apparent in Louise's framework profile is different from each of the other three teachers.

Two components of knowledge not engaged by Louise were in relation to measuring the range of a data set.

> Teacher: The range: What is it?
>
> Student 1: Starts from one and goes up ...
>
> Teacher: I know what you are trying to say.
>
> Student 1: ... bar graph from 7 up to 11.
>
> Student 2: My book says difference between the highest and the lowest numbers.
>
> Teacher: Right,... eg range 10 goes to 13.

[S4L4 V 0:00]

Here Louise, rather than asking Student 1 to further explain and clarify what she meant, acknowledged her response by saying, "I know what you are trying to say." Student 2, in referring back to previous notes in her notebook, gave a correct definition. However Louise did not really hear this and continued with the incorrect answer based on a common usage of the term range. This episode illustrates that she did not call on *specialised knowledge of content: transnumeration* when listening to Student 1, and *common knowledge of content: transnumeration* (in relation to range) was missing when she gave the answer as "10 to 13". Interestingly, while viewing the lesson video and without any prompt from the researcher, Louise recognised the error and commented that she had not noticed it at the time. A possible reason she gave was that she had been confused by something that another student had said with regard to the range. So although in the classroom situation this category of knowledge was not used, it was however part of her repertoire.

Most of the missed opportunities with regard to *specialised knowledge of content* occurred as a result of Louise not hearing what students were saying or not responding to students' answers. In one class discussion, students shared their findings. Two groups had reached contradictory conclusions about whether boys as a group or girls as a group had faster reaction times. One group of students had compared means and concluded that boys had faster reaction times, whereas the other group compared medians and found that there was no difference in the reaction times between boys and girls. The teacher did not pursue this discrepancy, which indicates *specialised knowledge of content: reasoning with models* was not used in assessing the validity of the two groups' conclusions. Furthermore, during the same discussion, two groups had calculated different values for both the boys' mean reaction time, and the girls' mean reaction time. Even though one student commented that he may have made an error by putting an extra value into the calculation, Louise did not follow this up to verify which group was correct. This indicated a problem with her use of *specialised knowledge of content: transnumeration*.

There were further examples relating to *specialised knowledge of content*. When students made statements comparing frequencies of two groups of unequal size, Louise did not consider suggesting that fractions would be more appropriate for the comparison than frequencies. This missed opportunity relates to *specialised knowledge of content: reasoning with models*. Another example for the same category of knowledge was when, in response to a student's answer, Louise asked, "Everyone agree?" She sought no further clarification or justification from the student for the validity of the answer, nor did she have the answer available to know whether the student's answer was correct.

One of Louise's missed opportunities related to *specialised knowledge of content: investigative cycle*. When a student suggested a possible data question that might have generated particular data, Louise did not evaluate the question nor recognise that it was not a suitable data question. For example, in relation to a data card that included B and 10, a student suggested that the data generating questions could have been, "Does your name start with a B? Are you younger than 10?" [S4L3 V 0:40]. Louise made no response to those suggestions at any stage. Had she used *specialised knowledge of content: investigative cycle*, Louise would have been able to consider how to respond suitably to these inappropriate and unworkable suggestions.

5.4 Summary

In this chapter, detailed descriptions have been given for the framework of knowledge for teaching statistics through investigations. Framework profiles have been outlined for each of the teachers. These profiles have indicated the types of teacher knowledge that were used in lessons, along with some aspects of teacher knowledge that were not in evidence. Also, missed opportunities have been described in relation to each teacher's knowledge profile.

Some missed opportunities have, however, not been discussed in this chapter. From the general profiles and the discussion of missed opportunities, some significant themes and critical incidents have been identified from similarities and differences between the teachers. In the next chapter, these recurring themes, which cover either aspects of a pedagogical nature or important statistical ideas, or both of these, are discussed.

Chapter 6
Discussion: Significant Themes

6.1 Introduction

In the analysis and profiling of the teachers' knowledge, some important and recurring themes arose. These themes are either relevant to general pedagogical practices as encompassed by the categories of teacher knowledge, or directly statistical in nature and hence related to one of the statistical thinking dimensions, and in some cases they are relevant to both. They impacted on the trajectory of the statistics lessons and/or on the subsequent learning opportunities for students. The significant themes to be discussed, with links to relevant literature (where available), are: teachers listening to and interpreting students' statements; teacher familiarity with the data; posing questions for investigation; students' handling of category and numeric data; sorting data – moving from noticing individual data to group features; students' difficulty with data-based statements; and understanding variation, and the development of inference.

6.2 Teacher listening to and interpreting students' statements

As discussed in the previous chapter, many missed opportunities arose in relation to the teachers' listening to, or interpreting, students' statements or questions. There were a number of different types of such incidents. First, the teacher, having listened to a student's statement or question, did not appear to correctly interpret it, and consequently responded in a way that was different from what the student intended. Second, there were situations in which the teacher did not evaluate an incorrect or 'off-track' response, and consequently did nothing to correct or question the student further so as to help the student's understanding. A third type of situation arose when a student's explanation was incomplete, and yet the teacher did not seek further clarification or explanation from the student in order to make sense of what the student was saying. Each of these three types of 'listening problems' is expanded on and linked to the knowledge for teaching statistics framework. The discussion is supported by examples of classroom exchanges, and where appropriate, by references to relevant literature.

6.2.1 Teacher responds to a different question or statement from the one asked

The teacher did not always respond appropriately to the questions or statements of the student. Because of misinterpretation, the teacher's response did not address the intent of the student's statement or question. For example, following a discussion of the possible data questions that could have generated the responses, 'youngest' and 'whistle' (from the sample data card), John indicated that the correct data question for 'whistle' was, "Can you whistle?" A student questioned this:

Student 1: How do you know that that question is the right one? Like, all these questions could be different, because, like, … he could be the youngest in his class or something, or his favourite toy could be his whistle. How do you know that it's not going to be ...

Teacher: So you think we might need more for "Can you whistle?" So how could we extend that question to make it more specific, then, if you see a problem with that question? What could we add to it? Because we are trying to find out if the person can whistle. Not get a whistle off the teacher or off a toy and whistle. Can you whistle ... what do we need to add?

Student 2: Can you whistle using only your mouth?

Teacher: So that means you can't use your fingers as well then? Okay then, "Can you whistle using only your mouth?"

[S2L1 V 7:51]

The student's original question appeared to be suggesting that there were possible data questions for the 'whistle' response other than the one given by the teacher. For instance, the student indirectly suggested that another possible question could be: "What is your favourite toy?" However, John did not interpret the student's suggestion in this way, but appeared, from his subsequent response, to think that the student was suggesting that someone might be able to whistle by using one from the teacher or by using a toy whistle. For whatever reason, the student did not seek to get this clarified. Although generating a data question from data is not a typical process in a statistical investigation, the discussion that could have leveraged from the student's comment would have had the potential to help students understand the need to have clear data collection questions. In this particular case, misinterpreting a student's intended meaning resulted in a missed opportunity to engage in a discussion that would have had the potential to develop students' understanding of the investigative cycle. This would have been relevant to address, as it was intended that the students would gather their own data for an investigation later in the unit.

In this classroom interaction, it was clear to the researcher, but not to John, that the student was suggesting an alternative data collection question. This situation can be explained in terms of the logic of learning approach, which was discussed in Chapter 3, *Methodology in Theory*. In spite of disconfirming evidence from the student in the form of another possible data collection question, the teacher's multiple expectations with regard to what the student was meaning, the 'correct' data collection question that the teacher had obtained from the unit plan, and the situation that the teacher had not considered the possibility of alternative answers, all 'overrode' the evidence, and consequently the teacher was not aware of a problem. The 'fixed views' (Popper, 1985) of the teacher were resistant to change, a situation brought about by not listening carefully enough to and interpreting what the student was saying.

A different type of example of misinterpreting a student's response arose in a discussion about the *range* that took place between Louise and her class. Although this incident has already been discussed in the previous chapter, there are aspects about it that are worthwhile considering in terms of Louise's listening. She recorded the range on the board (Figure 6-1) as "10 – 13", following the discussion on finding the range, and despite questioning from one student who had found some notes about the range.

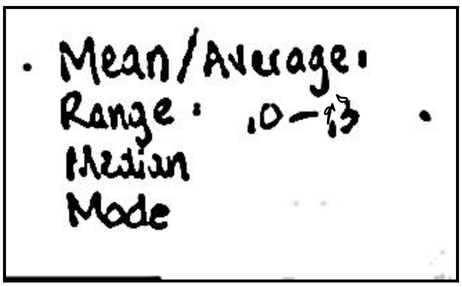

Figure 6-1: Record on whiteboard following discussion of 'range'.

[S4L4 V 0:00]

Interestingly, while watching the lesson movie during the follow up interview, Louise noticed this discrepancy in how to find the range (without prompting from the researcher):

> I don't know why I didn't mention that the range was 3. I don't know why. I didn't even click that I had done it until afterwards when one of the kids was saying to me, no, Miss J said that you have to get something in the middle. I was getting confused that she was thinking of averages or something. Because of the way she worded it, I never realised. And it wasn't until afterwards, I went, oh yes, I didn't explain it was how many. So I had given them the range as [from 10 to 13].
>
> [S4L4 Int 0:44]

From the interview, it was clear that Louise knew how to find the range; she had *common knowledge of content: transnumeration* (in relation to range). However, in the context of the classroom and the 'real-time' interactions, she did not employ *specialised knowledge of content: transnumeration* or *knowledge of content and students: transnumeration*. She misled the students with her common use of range. The student's jumbled description had moved Louise's thinking away from the correct notion of range towards an incorrect one. Such teacher 'errors' cannot be easily explained in terms of knowledge; in some way, the demands of the classroom overrode her knowledge and thus she used an erroneous way of representing the range.

This situation exemplifies the relevance of context for teacher knowledge, as discussed in Chapter 3, *Methodology*. Louise's knowledge of range was enacted differently in the classroom compared with the interview situation. Although she knew how to find the range, this knowledge was not used in the context that it was needed, namely the classroom. This example supports the need to conduct teacher research on teacher knowledge in the classroom; if the research had been conducted elsewhere through another means (such as, through a test or questionnaire), it may have appeared that the teacher had the appropriate knowledge for enacting in the classroom, yet this incident shows that although a teacher may have the knowledge, it cannot be assumed that it will be enacted when needed.

The value of using stimulated recall interviews for data triangulation, as discussed in Chapter 3, *Methodology in Theory*, is illustrated by this incident. It appeared from the classroom video that the teacher did not have a particular aspect of knowledge. Disconfirming evidence of this initial conjecture became available through the data from the interview; the application of data triangulation revealed a different understanding of the situation. Also, it showed that the researcher recognised a problem (Swann, 1999) as a result of the interview. His expectation (that the teacher did not have correct knowledge of how to find the range of a set of data, which had developed as a trial solution from viewing the video) and the current experience (of listening to the teacher in the interview) indicated a mismatch. The new trial solution, to eliminate the error in the researcher's understanding of the teacher's knowledge, was that the teacher knew how to find the range of a data set, but had not used that knowledge in the classroom lesson when it was needed.

The two examples discussed above (from John and Louise) have a similarity; their misinterpretations were not because of a lack of *common knowledge of content*. At the same time, there are significant differences between the examples. With John, the misinterpretation came about from not listening carefully, and hearing the student saying something that was not being said. This is an example of 'under hearing' (Wallach & Even, 2005), a situation in which the teacher has heard something, in this case about the whistling question, but has not heard the 'correct' message. It resulted, not in misleading the students' understanding of statistics, but in a response that did not make sense in relation to what the student had raised. John's student did not query the response that was not appropriate for the original comment. If John had heard correctly, he would have been faced with a decision as to whether to open up a discussion that could have been worthwhile for developing understanding of the need for clarity with data collection questions, an aspect of the investigative cycle. So, John's under hearing meant that a potential opportunity for discussion was not realised. All teachers have to make countless decisions as to whether to take such an opportunity to discuss something that was unplanned. Had John heard the student, there is no guarantee that a discussion would have eventuated, as it is known (O'Connor, 2001) that decision making by teachers happens in the pressure of the moment, and with teachers considering a number of factors simultaneously. Missing an opportunity for discussion was not as significant however as not responding appropriately to the student's comment.

In contrast to the outcome from John's lack of listening, Louise's misinterpretation meant that she led the students towards a potential misconception. Despite the 'classroom' error that Louise subsequently recognised in the interview, she felt that it was not significant in relation to her students' understanding. In the interview, she commented:

> [A range of] 10 to 13, for the base of the class today, would have made a lot more sense, using lowest to highest, than just having that number 3. I think that they would have got quite lost, especially when looking at all 4 of these things [measures] as well, muddled between which was which, all these numbers. So for today, I think that it didn't hurt that we looked at range, not in the statistical way, but as the whole range of what was there. I don't think it hurt too much for the base of the class.
>
> [S4L4 Int 7:12]

The relative significance of this incident to the students' subsequent understanding of range (as one measure of spread) is unknown. Range is the measure of spread that is simplest to calculate, but its common use (such as the way that Louise used it with her students) is different from the statistical measure. Students encounter more sophisticated measures of spread later in their schooling. These measures of spread, in addition to the range, contribute to an understanding of distribution (Bakker & Gravemeijer, 2004), an important cornerstone concept of statistical thinking and reasoning. The role that range itself plays in the development of understanding of spread, and subsequently therefore of distribution, has not been researched.

6.2.2 Teacher does not evaluate a student's answer

A teacher's non-evaluation of a student's response indicates a missed opportunity to analyse whether a student's answer or explanation is correct or reasonable. The aspects of teacher knowledge not being drawn on are *specialised knowledge of content: reasoning with models* and *specialised knowledge of content: transnumeration*. In many instances, explicit reference back to the data in one form or another, could have enabled the teacher to check the validity of the student's statement. For example, the following exchange between John and his class was based on the question, which John had written on the board, about the proportion of whistling right handers compared with whistling left handers. The data is represented in Table 6-1, although John did not use such a representation during the discussion.

Table 6-1: Two-way table showing the data related to the question posed by John about proportions of whistling right handers and whistling left handers

	Left handed	Right handed	Total
Whistlers	2	15	17
Non-whistlers	1	6	7
Total	3	21	24

The actual question was, "Are there more whistling right handers or whistling left handers proportionally?"

Student 1: There are more right handed people that can whistle.

Teacher: Can we extend our answer at all?

Student 2: There are 15 right handed whistlers out of 24 and 2 left handed whistlers out of 24.

Teacher: Anybody disagree with that statement – there are 15 out of 24 right-handed whistlers and 2 out of 24 left handed whistlers? Our biggest proportion is right handed whistlers, because 15 out of 24. Does anyone disagree with those numbers?

Student 3: We put 8 out of 21 can whistle for right handed. And 2 out of 3 can [whistle] because there were 3 left handed.

Teacher: There were 3 left handed people? Where did you get your 21 from?

[S2L2 V 4:56]

Here, John questioned one aspect of Student 3's answer, namely the 21. However, neither the '8' out of 21 (the number should have been 6) nor the 'can whistle' (it should have been 'can't' in relation to the 6) was questioned or evaluated. The discussion continued:

Student 3: um ... out of those …

Student 4: It is supposed to be 24.

Teacher: Okay.

Student 4: Was it 6 or 8 out of right handed who could whistle?

Teacher: Let's see, we'll check with another few groups.

Student 4: There's 8 so it's 3x8 = 24, which is 1/3 of them. And 1 out of 3 left handed ... no 2 out of 3 left handed can whistle.

Teacher: Say that again please.

Student 4: There's 8 right handed people who cannot whistle out of 24 which is 1/3. And there's 1 out of left handed people that cannot whistle which is 1/3.

Teacher: So you've looked at the ones that can't whistle in the results as well, out of the group of right handed people. Is that what you've done?

[S2L2 V 4:56]

Again, the comment about 6 or 8 right handed who could whistle (from Student 4) should have been 'could not whistle', but this was not noticed. Subsequently, Student 4 corrected himself. John also did not notice that the student, when talking about the right handed people, took the number who cannot whistle out of the complete group, that is, 8 out of 24, whereas for the left handed people, the student took the number who cannot whistle out of the group of left handers, that is, 1 out of 3. So in the first case, it was 1/3 **of the class** who are right handed and cannot whistle, and in the second case, it was 1/3 **of the left-handers** who cannot whistle. The teacher did not notice this 'incorrect' comparison. The discussion continued:

Student 5: The rest of them from our group, which was 15 whistlers in total.

Teacher: 15 right handed?

Student 5: No there was 9 right handers and 6 left handers and then we ... that means that the rest of the numbers ... so that means that the rest of the numbers ... they can't whistle.

Teacher: Okay. So proportionally are there more right handed whistlers or left handed whistlers?

Student 6: Most of the oldest cannot whistle.

Teacher: So you've moved on a little bit from where we are at the moment.

[S2L2 V 4:56]

Again John did not evaluate the student's comment about the 9 right handed and 6 left handed students. It was not clear where these numbers came from, yet he did not notice this or question the student about it.

Another example of a teacher overlooking or ignoring a student's answer came from a discussion comparing the heights of boys with those of girls. In the particular data set being used, the group sizes (i.e., number of boys and number of girls) were not equal. Rob paraphrased a student's answer:

Teacher: So you added all girls' heights and added all boys' heights, and found that the girls are taller?

Student: Even if there are 4 more boys at 150cm, the girls are still taller.

Teacher: So what is the problem making our statement from that, adding the heights?

[S3L3 V 7:50]

Rob had recognised a problem with comparing the heights of the two unequal-sized groups by using the sums of the heights. However, he made no attempt to evaluate the student's response that suggested a way to ameliorate the problem by adding on four extra boys; instead he only addressed the problem of comparing unequal groups through the use of totals. The next response from the same student was in two parts, the first part relevant to the teacher's question, but the second not relevant and yet not evaluated by Rob:

Student: Because there aren't an even number of boys and girls. Sometimes they're random heights so they could be taller or shorter.

[S3L3 V 7:50]

A further example of a teacher not evaluating students' comments arose in a discussion part way through an investigation of handedness (right or left) and gender. At that point, John had four piles of cards (left handed boys, left handed girls, right handed boys, right handed girls) and then asked the students how the cards might be further sorted. (Table 6-2 shows the data in relation to the suggested sorting, as referred to in the transcript below; this representation of the data was not used by Rob, but is included here to support the discussion.)

Table 6-2: Numbers of students by gender and whistling vs. handedness (and position in family for the right handed students)

	Left handed	Right handed
Boys whistlers	0	9 (2 oldest/4 middle/3 youngest)
Boys non-whistlers	0	3 (3 oldest/0 middle/0 youngest)
Girls whistlers	2	6
Girls non-whistlers	1	3

Student: All the ones that cannot whistle.

Teacher: From whole lot or just boys or girls? ... boys ... let's see if there's a pattern between who cannot whistle. ...

[S2L3 V 2:15]

At this point, as there are no left handed boys, John took out the three cards of non-whistlers from the right handed boy pile and left eight cards for right handed boys who could whistle, although there should have been nine of these cards; somehow one card was unaccounted for.

Teacher: What can we say about the boys who cannot whistle?

Student 1: They are exactly the same.

Teacher: So they are all exactly the same as well. What could we say about that group within our boys group?

Student 2: That the oldest can't whistle.

Teacher: Yes, from this it looks like the boys that are the oldest can't whistle.

[S2L3 V 2:15]

John did not notice that the statement from Student 2 was not correct for the data and the way it had been sorted. The data had been sorted to look at boys, followed by non-whistlers; and with the non-whistling boys, it was noticed that all of them were the oldest in their families. However, John agreed with Student 2's incorrect statement that all the oldest boys couldn't whistle. Another student, who had listened carefully to the statement, disagreed:

Student 3: Not all of them. There are 2 oldest in there though [pointing the whistling boy group].

Teacher: Sorry ... we've got 2 boys are who the oldest that can whistle. Does that change what we would say then? We've got 2 oldest boys that can whistle, and 3 oldest boys that can't.

[S2L3 V 2:15]

Rather than returning to Student 2's statement and evaluating why the statement had been incorrect, John went on to seek suggestions for how the original statement from Student 2 could be modified to account now for the two oldest boys who can whistle.

Conditional statements, such as from considering the proportion of non-whistlers who are boys, or proportion of boys who are non-whistlers, were problematic for the students, and the teachers did not necessarily notice the difference. In the above example, the proportion of non-whistling boys who are oldest is different from the proportion of oldest boys who are non-whistlers – one was dealing essentially with data from a row in the tabular representation of the data whereas the other related to data from a column. The same problem occurred in relation to the question, "Are there more whistling right handers or whistling left handers proportionally?" In some of the class discussion, the question was inadvertently 'turned around' to focus on right-handed whistlers or left-handed whistlers. This suggests that the teachers who did not notice the change in the problems brought about by changing the order of the wording, were lacking in *specialised knowledge of content: reasoning with models*. When clarification was sought by the researcher in the interviews, it was apparent that the teachers, other than John, did not have experience with such conditional statements, and therefore that their *common knowledge of content: reasoning with models* was deficient. The use of a representation such as a two-way table would have assisted such a discussion by making the data available in a more permanent form than groups of data cards; such a table would have enabled the teachers and students to refer to the relevant rows or columns for the particular conditional statements.

The examples given above of teachers in this research not evaluating students' comments appear to be cases of under-hearing or non-hearing. As Wallach and Even (2005) suggest, teacher knowledge is a significant factor in the breakdown of really listening to students. To be able to listen effectively in order to evaluate a student's response requires *specialised knowledge of content*. The lack of *specialised knowledge of content: transnumeration* and *reasoning with models* could be attributed to the teachers' lack of familiarity with teaching statistics through investigations. Consequently, they may not have had much opportunity to develop the appropriate *specialised knowledge of content* that was needed. Davis (1997) concedes that effective discourse involving either interpretive or hermeneutic listening on the part of the teacher takes time to evolve. Doerr and English (2006) also acknowledge the significant shift in authority that is required to achieve effective classroom discourse, from authority held by the teacher to it being shared between teacher and students. So, for inexperienced teachers who have not significantly used an investigative approach to teaching in which students are encouraged to communicate their thinking, this type of discourse community in the classroom would not have had time to develop.

Interpretive or hermeneutic listening is obviously dependent on *specialised knowledge of content*. It was argued earlier that *specialised knowledge of content* could provide evidence of *common knowledge of content*; absence of *specialised* *knowledge of content* could indicate, although not necessarily so, a problem with the associated *common knowledge of content*. Further evidence would be needed to establish whether the teachers had *common knowledge of content* about these conditional statements, and the difference that the order of the wording can make to the problem being considered.

6.2.3 Teacher seeks no further clarification

There were numerous situations in which the teacher made no attempt to seek clarification from a student in regard to an answer that seemed incomplete or unclear as to what was meant. In one situation [S3L4 V 1:36] where a student suggested finding an average by "add[ing] them together and divide by two," Rob did not ask the student to explain what was to be added together, what would it be the average of, or why finding such an average might have helped with answering the question about whether having bigger hand spans means that they have bigger feet. The student's final comment that it was for the purpose of finding out "who has the biggest feet ... on average" went unchallenged. Within the two statements from the student in that episode, there were a number of aspects that should have been probed further by Rob in order to get the student to think about what he was saying, and for Rob to make sense of what the student was saying. It appears that Rob was satisfied that the students were supposedly engaged and responding, irrespective of whether the responses were clear or even valid.

Other examples follow in which the teacher sought no further clarification from the student. In one situation Rob was working with a small group of students. When discussing what had been found from sorting the cards in relation to whistling, handedness, and position in the family, one student commented:

> Student: It's weird, there are no left handed girls in the class.
>
> [S3L1 V 2:58]

Rather than ask the student why it seems 'weird', Rob moved on with another question. Rob had been presented with an opportunity to probe the student's thinking in relation to integration of the contextual with the statistical; it may be that the student had expectations of the data in relation to what he knew of the real incidence of left-handedness. This incident constituted a missed opportunity to make links with other aspects of statistical thinking.

The next example involved Rob and two students discussing potential questions for investigating the data and suggesting how to sort the data for that investigation.

Student 1: Put them [the data cards] into year and age and gender.

Teacher: What are you trying to find out?

Student 1: We will figure that out once we have sorted the cards.

Student 2: Like if there are more girls who are Year 6 than boys who are Year 6.

Student 1: We can add them together and do averages.

[S3L2 V 3:24]

Rob did not challenge any of these statements in order to seek further clarification. For instance, Student 1 could have been asked to propose a question that could potentially be answered by sorting the cards in the way suggested. Such a move may have clarified for the student that an investigation needs a purpose on which to base the 'interrogation' of data. Rob's question to Student 1, namely, "What are you trying to find out?" may have been interpreted by the student to have been asking what the actual findings from the data might be, rather than the intended, "What are you wanting to investigate in the data?" Student 1 was not questioned about what averages might be found, and for what purpose. There are different approaches to conducting investigations: one involves giving students data so they can 'get inside' the data to find something that the data tells us; and the other involves posing a question or hypothesis, and from there collecting data and engaging in the other parts of the investigative cycle in order to answer the question or prove (or disprove) the hypothesis (Ministry of Education, 2006). Even if the first approach is adopted (as was the case for the teaching unit in this study), students must engage with the interrogative cycle from the outset, so that they have some idea of what might be worth looking for in the data. Consequently teachers need *knowledge of content and teaching: interrogative cycle* in order to know how to help students focus and start to get inside the data, even though they do not necessarily have a fixed question to investigate. Also teachers need *specialised knowledge of content: interrogative cycle* and *investigative cycle* to evaluate the students' suggestions and be able to respond appropriately.

The same pair of students considered, along with Rob, the question, "Does it affect your reaction time how old you are?" Student 1 suggested, prior to examining the data and therefore possibly from the utilisation of real world knowledge and experience:

Student 1: The older you are the slower you are.

Teacher: Is that true for this class?

Student 2: No.

Student 1: No idea.

Teacher: Well let's see if we can find that out. How are we going to do that?

Student 2: We could see which one was the highest and sort them out oldest to youngest and see who has the highest.

Student 1: Do a median for this group.

Teacher: So we can get it sorted so we can see it, can't we, and then we can see some sort of thing.

[S3L2 V 3:24]

This time, Rob took up Student 1's hypothesis and asked how the cards could be sorted in order to investigate this claim. However, Student 2's suggestion was not clear as to how useful it would be for addressing the hypothesis. In spite of the lack of clarity with the suggestion, Rob did not follow up on this. Again this created a missed opportunity in relation to linking the sorting of the cards (i.e., transnumerating the data) with the interrogation of the data, and as part of the investigative process. Student 1's suggestion, which was quite clear in one sense (calculate a median), was not pursued further as to how a median might help. Possible follow-up questions could have been: What data would you find the median of? How would that help with proving or disproving the hypothesis that the older you are the slower you are? Such clarification could have helped the students understand more about reasoning with statistical models.

The components of knowledge that most directly relate to not seeking further clarification relate to both *knowledge of content and students* and *specialised knowledge of content* across a number of the statistical thinking dimensions. The broad category of *knowledge of content and students* covers, among other things, the ability to interpret incomplete student thinking. It can be seen that some examples for which no further clarification was sought would indicate a lack of connection with the students' comments, and therefore be a part of the category of 'not listening'. As Anghileri (2006) suggests, making contact with a student's understanding is a requisite for scaffolding the student's understanding. Without that contact, a pseudo-interaction or by-passing (Bliss, Askew, & Macrae, 1996) has occurred. Although Bliss and colleagues do not suggest possible reasons for such an event, and O'Connor (2001) proposes that a lack of comfort on the part of the student, teacher, and other students, when attempting to make more sense of the student, works against the seeking of extra clarification, the evidence here suggests that teacher knowledge is a factor in the lack of connection. This concurs with Wallach and Even (2005), who propose that instances of under-hearing and non-hearing, which apply to the examples involving Rob, can be attributed to a lack of teacher knowledge; this study has identified that particular knowledge as *knowledge of content and students*, and *specialised knowledge of content*. It seems clear that if Rob had stronger and more robust teacher knowledge, the incidence of hearing problems would have been lower, and effective scaffolding of the students' learning would have occurred. Conversely, the absence of non-seeking of clarification of students' responses, with Linda for example, and the corresponding presence for her of *knowledge of content and students* and *specialised knowledge of content* creates a stronger argument that such knowledge is necessary to make good connections with students' talk, and to avoid hearing problems of the types mentioned by Wallach and Even.

6.2.4 Summary of teacher listening and interpreting

Three different situations have been described, and supported by relevant examples from the classrooms, of teachers not listening to and not really hearing what students were saying. These situations were identified as: misinterpreting, and therefore responding to something other than what the student actually asked or said; not interpreting and evaluating a student's response and thereby missing the opportunity to challenge or correct a misunderstanding; and not seeking any further clarification from a student, even though it was unclear what was being said. There are a number of factors that may contribute to the listening problems (as suggested by Wallach & Even, 2005), and these factors include teacher knowledge, dispositions, feelings about students, expectations, beliefs about mathematics learning and teaching, as well as the context in which the hearing takes place. It is apparent from this study that teacher knowledge has a significant role in avoiding listening problems, and that various types of teacher knowledge are necessary (although possibly not sufficient) to avoid such problems.

6.3 Teacher familiarity with the data

Situations arose within the classroom in which it appeared that the teacher was not familiar with the data that were being investigated by the students. This lack of familiarity manifested itself in various ways. There were instances when the teacher could not readily help the students formulate a question to investigate, as they did not know what would be achievable with the given data. Other instances arose when the teacher did not know whether a student's finding was valid for the data. For example:

> Students: Oh, oh, all the boys that cannot whistle are the oldest.
>
> Teacher: Is that right? Have you checked that?
>
> Students: Yes.

[S2L1 V 16:41]

Although it is not absolutely clear from this response alone, it was apparent during the classroom observation that John had not previously investigated the data. There were other instances where a similar type of response was made by the teacher but in the form of, "that's an interesting result" – a type of encouragement of the student, or a divesting of authority from solely the teacher towards the students, thereby encouraging mathematical practices of explanations and justifications. It is accepted that it would be impossible for a teacher to know everything about a data set or be able to evaluate every student statement 'on the spot', without reference to the data. However, to have a general familiarity with the data and what students might find, is an important aspect to being prepared for the learning opportunities that might arise.

Not being familiar with the data was more of an issue in the situation where John had posed a question (obtained from the unit plan) to the class, but did not know the answer. It quickly became apparent that John had misinterpreted the question and dealing with students' responses in the reality of the classroom interactions became problematic. This misinterpretation has already been discussed earlier in this chapter (see page 112). Had John investigated the data prior to using it in the classroom and found an answer to the question prior to the lesson, this difficulty may not have occurred. Alternatively, having a greater level of teacher knowledge in a number of dimensions could have ameliorated the effect from not being sufficiently familiar with the data.

On occasions the teacher was aware of not being able to make helpful suggestions as to what to investigate, due to not knowing what the data might show. For example, John commented:

> I hadn't spent any time with the data myself, and seeing what I would do and what I'd notice. I need to do that.
>
> [S2L4 Int 6:44]

and:

> The teacher needs to have used data squares to make sense of data before getting students to do it. This would also help with deciding whether to tell students or whether you leave it.
>
> [S2L4 Int 19:09]

In relation to being able to evaluate a student's response, John commented:

> If your knowledge of the material is really good, then okay, but it can be quite tough to guide [the students] when there are no absolute answers.
>
> [S2L4 Int 19:09]

This comment, as well as indicating the need to know the data, also reflects something of the difference between statistics (with its inherent variation in data) and the deterministic nature of mathematics (Moore, 1990; Pereira-Mendoza, 2002).

Rob commented about the new approach (for him) of teaching statistics through investigations:

> Teaching this type of investigation with using data to compare groups is different from what I have taught before, but is interesting. I have enjoyed looking at the data myself, going through the process for myself.
>
> [S3L1 Int 11:05]

Rob's enjoyment from looking at and working with data might well have had a subsequent and positive impact on students. In contrast, in the interview following the next lesson and in relation to the mainly numerical multivariate data set that was used in the lesson, Rob commented:

> I found it probably ... probably with the confidence of myself with it, I hadn't experimented with it [the data] and looked at all the different patterns, or comparing all the different information myself. Which probably led me to ... I wasn't able to give them scaffolding to find something interesting.
>
> [S3L2 Int 0:00]

and:

> I hadn't played with the cards myself, so I wasn't sure what they would find. I couldn't lead them. After the lesson I thought that I should go back to using numeric data, instead of going on to something else that we collect ourselves.
>
> [S3L2 Int 28:12]

Rob acknowledged the importance of being familiar with the data. What impact his lack of familiarity with the data had on the learning opportunities for Rob's students is difficult to gauge. Being familiar with the data would, for example: give the teacher more 'resources' with which to evaluate a student's response (and as such would contribute to the teacher's *specialised knowledge of content* with respect to a number of statistical thinking components); enable the teacher to give students guidance with formulating questions that can lead to a worthwhile investigation (i.e., contribute to *knowledge of content and teaching: interrogative cycle*); or help the teacher know what the students might find from the data (i.e., contribute to *knowledge of content and students: investigative cycle*). Consequently, a teacher's lack of familiarity with the data can negatively impact on a number of different categories of teacher knowledge.

6.4 Posing questions for investigation

The unit plan that the teachers used for their lessons either directly followed the provided unit plan (Ministry of Education, 2006) or was an adaptation of this plan. The provided plan gives an outline of the general approach to be used:

> This unit focuses on sorting and organising data sets, i.e., collections of information from a group of individuals. Looking at the data, sorting and organising it first, with things of interest and questions arising from this. This is a different approach than starting with a question then collecting data to see if it is correct.
>
> (Ministry of Education, 2006)

In spite of the unit plan suggesting that students sort and organise first, from which questions for investigation will arise, it is still necessary for students to have some idea, prior to the sorting and organising of the data, of what might be feasible to investigate in the data. Although the teachers adopted slightly different approaches, the posing of questions for investigation proved to be difficult at times for students.

John used the sample data square to make some initial suggestions that might be investigated by the students:

We might look at, if he is left handed, then perhaps left handed people can't whistle. Or perhaps, not all left handed people can whistle. There are things that we could make up statements or predictions about that as well.

[S2L1 V 9:37]

In this situation, John encouraged consideration of two variables, namely handedness and whistling. Further on in the same lesson, having had some discussion about the data collection questions, John asked the students for their suggestions as to what might be investigated.

Student: What children are the youngest, middle and oldest.

Teacher: What could we do with that?

Student: Then boy/girl.

Teacher: Yes, putting the data squares into some groups and seeing if there are some relationships between middle, oldest, youngest with left handed/right handed, boy/girl, left handed people can whistle.

[S2L1 V 13:27]

John initially asked the student to elaborate on the response to do with position in the family. The student suggested a second variable that could be included in the investigation. However, John then pushed the possible investigation to a new level, involving all four variables. This suggestion increased the complexity of the investigation significantly. In the follow up interview with John [S2L1 IntB 24:32], it was revealed that he had not considered how sorting by more variables would reduce the frequencies in the subcategories to low numbers, and that making comparisons between groups would become much more difficult. This indicates a potential problem with *common knowledge of content: transnumeration*. John was aware of the need to encourage the students away from univariate investigations, but had not thought through the implications for the investigations (particularly in relation to *transnumeration* and *reasoning with models*) of trying to consider too many variables at once. So a problem with *common knowledge of content* subsequently impacted on all the other categories of teacher knowledge.

The teachers often were unsure of how much to guide and model for the students as opposed to leaving the students to 'discover' and develop questions for themselves. John commented, following Lesson 2,

Teacher: At that stage, I didn't want to point out, say too much ... if I gave examples, the students would just copy those. I didn't want to tell them too much, let them experiment and see what they could come up with. … I found that bit difficult to explain and get going. I guess it is the abstract thinking that you would need to consider 2 or 3 things at a time beyond the obvious. I therefore made an effort to get around nearly every group, to see whether they were looking beyond just making 2 groups eg. there's more boys than girls.

Researcher: And without giving them too much guidance?

Teacher: Yes. There's modelling and giving examples but if you do that too much, that's all you get back.

[S2L1 Int 21:43]

John's comment indicated that his *knowledge of content and teaching* was rather fragile. Lack of experience with teaching through the investigative approach may be one reason for the state of his *knowledge of content and teaching*.

Linda, having introduced the students to the data squares, and having established the data questions that generated the data on the squares, encouraged the students to make some predictions about what they might find in the data. This approach pushed the students into an 'interrogative' frame of mind; they were forced to think about possible connections within the data. For example, Linda suggested to the students that a possible question to investigate could be, "Can all boys whistle?" Later in the lesson, Linda and the class revisited the students' predictions, once they had sorted the data and made some statements about what the data showed. Some predictions were verified as correct for the data, while other predictions were refuted. Linda's approach ensured that the students were constantly being encouraged to think about the data, before, during, and after sorting and examining the data. This gave a clear purpose to the sorting of the data, and once this had been completed, those initial predictions helped the students focus on what the data actually revealed. By specifically revisiting the predictions, the students were encouraged to think through the investigative cycle; this indicated that the teacher was aware of and thinking about both the *investigative cycle* and the *interrogative cycle* (although not necessarily in these same terms) in relation to both *knowledge of content and students*, and *knowledge of content and teaching*.

Whereas Linda and John encouraged the students to think about the data square and to propose a way of sorting or a question that might be worthwhile to investigate, Rob took a different approach. He encouraged the students to sort the data, but not necessarily with a particular purpose or outcome in mind. After the class was back together, following the group investigations, Rob sought more suggestions as to what could be investigated. The proposals included:

Student 1: How many left handers there are?

Student 2: The number of boys [that] are the youngest and can whistle.

Student 3: The number of left handed boys who can whistle.

Teacher: Or it might be about, all the oldest children in this class can whistle.

[S3L1 V 4:16]

Rob did not respond in any evaluative way about these suggestions from the students. Student 1's suggestion was limited to one variable, and to finding only a frequency rather than making a comparison, although it is not known whether the student was implying that a comparison could be undertaken. Students 2 and 3 each suggested three variables, but in a similar way to Student 1, appeared to be restricting the investigation to finding a frequency rather than a comparison. Using three variables would allow quite a number of different comparisons potentially to be undertaken; however it is not known whether the students intended this.

As well as not responding to the students' suggestions, Rob did not refer the students back to the main questions of comparing 'our class' results with the previous class data. Had he made direct links to the findings from Lesson 0's data, those findings could have been used as the basis for the planned investigation of this class's data.

In a subsequent lesson in which the data to be used included numerical data, Rob gave little guidance to the students when posing questions or giving suggestions of what might be investigated. Some of the students' suggestions were:

Student 1: Put them [the data cards] into year and age and gender.

Teacher: What are you trying to find out?

Student 1: We will figure that out once we have sorted the cards.

Student 2: Like if there are more girls who are Year 6 than boys who are Year 6.

Student 1: We can add them together and do averages.

[S3L2 V 3:24]

Rob challenged the students to think about what they could look at or find out from the suggested sorting. However, he did not encourage thinking about the actual data in relation to the context of that data, in order to consider what was feasible to investigate. Rob could have extended Student 2's suggestion by asking about the other comparisons that could also be made involving gender and year level.

Another situation arose from a student suggesting a feasible investigation of a multivariate data set that included reaction time, year level, age, and gender:

> Teacher: So do you think you could find anything about these different age groups?
>
> Student: The 12 year olds have the quickest reaction times.
>
> Teacher: So on average you are probably right.
>
> Student: And the 13 year olds.

[S3L2 V 6:50]

In this example as with a number of other similar examples, Rob did not encourage the student to refine the question for investigation. Although the student appeared to have a reasonable question in mind from what his response implies, it may have been worthwhile for Rob to seek clarification from the student; requiring the student to refine the question could have given the subsequent investigation more direction and purpose.

In another lesson, Rob did assist a student to refine a question further. The class was considering the data squares involving the variables of gender, foot size, hand span, and age. One student suggested an investigation:

> Student: If there are more B who don't have hand span or foot.

[S3L4 V 0:00]

Although the student's suggestion was not clear, Rob appeared to have some idea of what the student was meaning, so repeated back to the student what he thought the student meant:

> Teacher: Yes ... [pause] so you are saying, "If their hand spans are the same size as ... "
>
> Student: if there are more people who doesn't have the same hand span as ...
>
> Teacher: the same size hand span as their foot size?
>
> Student: Yes.

[S3L4 V 0:00]

Rob managed to make some sense of the intention of the student, so responded with further related ideas:

Teacher: Yes so you can look at the differences between your hand spans and your foot sizes. ... You might see a pattern there, that hand spans are the same, or that on average your hand span is 2cm smaller than your foot. Or most people's hand spans were 1cm smaller than their foot. There's another word, "Most."

[S3L4 0:00]

In this example, the teacher actively sought clarification from the student, assisting him to state the question more clearly. By doing so, the student was given more support for the impending investigation of the data.

The posing of questions for investigation is related to the interrogative cycle, which subsequently and significantly impacts on the investigative cycle. Realising that students are likely to find it difficult to pose appropriate investigative questions is one aspect of the *knowledge of content and students: interrogative cycle*; how to help students with the posing of such questions relates to the *knowledge of content and teaching: interrogative cycle*. No research literature could be identified in relation to students posing questions for statistical investigation. Although Chick, Pfannkuch, and Watson (2005) acknowledge that going from a data set to a representation that will reveal information is challenging (i.e., the process of transnumeration), the posing of questions that occurs immediately before that transnumeration is not considered, but is nonetheless important; students need to think about what the data may reveal and what would be therefore worthwhile to 'formally' investigate. For example, the multivariate data set that includes gender, year level, age, and average reaction time could result in a number of investigations using two variables. Some of these investigations may not be as interesting or productive as others. For instance, investigating the relationship between year level and age would not be as illuminating or interesting as that of the relationship between gender and reaction times, or between age and reaction times. For students to recognise the interest and potential in an investigation compared with another is a key factor in posing investigation questions. Teachers therefore need particular knowledge around students and the interrogative cycle so as to help students develop the skills and understanding necessary for successful investigation.

6.5 Students' handling of category and numeric data

The unit plan (Appendix 2) used by the teachers included a number of multivariate data sets. The first set used by each teacher consisted of four category variables. Subsequent data sets included numeric variables in addition to category variables. Most of the teachers used a data set consisting of one category variable (gender) and three numeric variables (age, arm span, and height). For learning to take place, it is important for teachers to know how to manipulate the different types of data so that they can respond to and guide their students as necessary; knowledge of effective *transnumeration* of data (both category and numeric) would include knowing the difference between the types of data and how these differences might impact on the sorting of data (in this case using the data cards). Whereas category data could be sorted, for example, by two dichotomous variables to form four groups, from which frequencies (or relative frequencies) could be compared, numeric data is not so straightforward.

It became apparent during the research that the students struggled more with the numeric data sets than the exclusively category data sets. One interesting observation was that when students sorted the numeric data into groups (e.g., with heights, all data cards with the same height were grouped together), they often left the groups of cards in a 'random' arrangement rather than ordering the groups numerically. Such ordering of the groups of cards would have enabled the students to notice more about the distribution of the numeric data. John commented in one interview:

> They couldn't explain what they were looking at. The way they sorted wasn't going to help them. They needed more modelling from me. They went into smaller groups. … No real method to what they were doing.
>
> [S2L4 Int 11:23]

Although he had recognised that the students' sorting was not adequate and that modelling was needed, John did not provide such guidance at the time. He lacked the knowledge of how to effectively sort the data cards in order to check for a possible relationship between two numeric variables; this suggests a lack of *common knowledge of content: transnumeration* with regard to bivariate numeric data.

Like John, Rob was surprised that the students found more difficulty with numeric data than category data. His students had found some interesting patterns with category data, but when faced with numeric data, they struggled. Rob also commented (during a follow-up interview) on the need for more modelling by him of sorting. However he was unsure of the most appropriate way to sort the numeric data to help investigate relationships, as he had not attempted this for himself with the data cards.

Dealing with data at the sorting stage is a component of transnumeration. Students' difficulties with handling data, particularly the challenges of moving from category to numeric data, aligns with similar findings from one study (Chick et al., 2005) of Grade 6-9 students investigating a multivariate data set, and another study of Grades 1 - 3 students (Nisbet, Jones, Thornton, Langrall, & Mooney, 2003). *Knowledge of content and students: transnumeration* can be expanded further to include an understanding that students are likely to struggle more with numeric data than category data; more sophisticated techniques are needed to deal with numeric data than deal with, at the simplest level, a dichotomous category variable such as gender. If teachers have such *knowledge of content and students* based on examples like those given above, the next component of knowledge to be developed would be *knowledge of content and teaching: transnumeration*. This aspect of knowledge is based on how to help students deal appropriately with numeric data through developing their understanding and skills of transnumeration, which will subsequently assist their reasoning with models.

The situations described above in relation to both John and Rob illustrate the recognition of a problem, a mismatch between expectations and current experiences (Swann, 1999), with regard to the logic of learning model as discussed in Chapter 3, *Methodology*. Their expectations centred on believing that the students would manage the sorting task, and the current experience revealed that this not was so. However, in spite of the perceived problem, neither teacher was able to develop a new trial solution to solve the problem, that is, eliminate the error. Consequently, the solution was that the status quo remained, with the problem essentially not solved.

6.6 Sorting data: Moving from noticing individual data to group features and relationships

6.6.1 Focus on individual data

It is well recognised that part of a student's development of understanding in statistics involves the move from focusing on individual data to being able to consider group features (e.g., Hancock, Kaput, & Goldsmith, 1992; Konold & Higgins, 2003). In this research, it became apparent that the teacher had an important role to play in helping students move to developing a focus on group features. Teacher questions, explanations, or responses (and including lack of responses) sometimes promoted, unintentionally, a continuing and unwarranted focus on individual data.

Situations arose where students could have interpreted a teacher's question or suggestion to be related to individual data. For instance, John encouraged the class to look at the data beyond a single variable and towards relationships between two or three variables. He suggested sorting the data initially by one variable, and then by another to form four groups of cards. At this stage John proposed a number of possible questions to investigate, including (in relation to reaction times),

> Who's the fastest, who's the slowest?"
>
> [S2L4 V 8:21].

Students could have interpreted this question as meaning that they were to look at individual data values, instead of the intended meaning as an encouragement, or even an instruction, to focus on groups. Another similar example from Rob, after talking about comparing Year 6 and Year 8 girls' reaction times, was his suggestion to sort the cards and find:

> Who has the faster reaction time?
>
> [S3L2 V 10:47]

In both cases, the teachers may well have been meaning 'which group' has the faster reaction time rather than 'which person is fastest or slowest', but the wording of the question led some students to look for individual data values. Some of the subsequent statements from students, which referred to individual data cards for the fastest or slowest reaction times, showed that they had interpreted the question as a focus on individuals.

In an exchange with a group of students while looking at reaction times as related to age, Rob was confronted with the issue of using either individual data values or group features:

Teacher: Are you finding fastest reaction times?

Student: Yes.

Teacher: So these are all 13 years old with reaction times of 13. ... Is that the fastest reaction time there or the slowest?

Student: Slow ... in the middle ... there's also 14 and 11 and only one that's 9.

Teacher: So 9 is the slowest.

Student: Isn't 9 the fastest?

Teacher: Yes, fastest, you are right ... get it into an order so that ... You can keep sorting like that, then you might see to be able to compare something. Like: Maybe there are more girls who are slower, or the older you are the faster your reaction time is.

[S3L2 V 5:19]

There was some recognition by the students of the groups' reaction times, but also of the reaction times for some individuals. Rob tried to encourage a focus on the complete group, but complicated the comparison by the suggestion of bringing in another variable, gender, rather than continuing with relating only age with reaction time. Rob did not assist the students with any suggestion as to the best way to sort the cards to investigate such a relationship.

In the subsequent whole class discussion, Rob encouraged the students to recognise the inadequacy of using an individual data value to support the conjecture about whether boys or girls had the faster reaction time:

Teacher: Did anyone find whether the boys or the girls had the best reaction times?

Student: A girl did, fastest time was 9.

Teacher: Did you use that to say that the girls had the fastest reaction times?

Student: Yes.

Teacher: What was the slowest reaction time then?

Student: 17, and it was set by the girls.

Teacher: So we would have to have a look of what the average of the girls reaction times and the boys' average reaction time, so that we could get a comparison, … so we could see what group had the fastest reaction times.

Student: The average of the girls was 11 and the boys had two 11s. ... oh, the boys had one 10 but no 11s or 9s.

Teacher: So what was the average of the boys' reaction times?

Student: 12 I think.

Teacher: So we could say that the girls had the faster reaction times?

Student: No the girls' average was 13.

Teacher: Maybe we need to have another look at that.

[S3L2 V 11:37]

Rob did not explicitly deal with the use of an individual data value as the justification for the claim that girls have faster reaction times than boys. However his response implied that in spite of a girl having the fastest reaction time overall, as a whole the group of boys might have faster reaction times than the group of girls. The discussion from that point went off in a different direction and did not resolve the issue of the inappropriateness of using individual data values to support a conjecture about groups.

In the interview following this lesson, Rob commented on this episode:

> I think I talked about the girl with the slowest time of 17. ... Rather than using an individual data value to justify a statement, must use all the data to see where it is grouped or spread.

[S3L2 Int 9:12]

The next two short examples also show situations in which Rob's students noticed individual values within a group, but unlike in the previous scenario the students were not encouraged to think about the group data:

Student: One male is only 1cm taller.

Teacher: So the tallest person is a male by 1cm. What else - the females?

[S3L3 V 1:04]

and

Student: Longest arm span goes to a boy with 185cm, and tallest person, boy, that was 181cm.

Teacher: Was there anything you found by comparing arm span and height?

[S3L3 V 10:12]

Rob, although at times accepting students' statements about individual data values, had some *specialised knowledge of content: reasoning with models* in connection with this; he recognised the inappropriateness of the students' arguments based on individual data. He sometimes utilised *knowledge of content and teaching:*

reasoning with models to steer the discussion away from only individual values to consideration of group features in the data. In the examples from three of Rob's lessons, these aspects of knowledge were not in evidence consistently; explicit guidance did not always eventuate when working with groups of students, although there was some indication of Rob's understanding in this area. One possible explanation for such a phenomenon is that Rob, in some situations, may not have recognised that the students were referring to individual data. Consequently, by under-hearing the students, he was unaware of a problem, and therefore was not aware of a need to draw on *knowledge of content and teaching*.

6.6.2 Sorting by one variable

Teacher knowledge about the students' ability to look for and describe relationships between variables, and about how to facilitate the students' learning varied between the four teachers. In general, the students struggled with bivariate relationships and tended instead to focus on univariate data. This was manifested through students finding frequencies of such groups or making simple univariate comparisons.

Louise's students, in Lesson 1, had investigated a given data set in relation to three questions, each of which were based on two variables. Following a similar investigation in Lesson 2 using their own class data and for which they had predicted possible findings prior to undertaking the investigation, they shared their actual findings. Although the first group's finding involved two variables (gender and place in the family), the next groups' findings were all univariate – one dealt with gender ("There's an even amount of girls and boys"); another with whistling ("9 people can whistle and 6 people can't"); and a third with handedness ("there's more right handed people than left handed"). In the interview, Louise acknowledged that the students had difficulty seeing past a single variable, in spite of the previous lesson's focus questions. Although suggesting the reason could be that this type of data investigation was new to her students, Louise was unsure of how to deal with this in her teaching. Her *knowledge of content and teaching: transnumeration* was insufficient with regard to moving the students from dealing with univariate data to bivariate data.

The same type of situation arose with John's class. When the class reported their findings in Lesson 4, the first finding involved three variables (the student had noticed something about the reaction times of the Year 6 girls and the Year 8 girls), but the subsequent findings reverted to one variable, such as: "2/3 of the class are girls"; "there are no Year 7 students" in the data; and "the most common age was 12". John commented later in the interview [S2L4 Int 15:20] that he knew the students encountered "difficulties with considering more than two things at once. They start, then forget what they were doing with it." Back in John's first lesson, he had observed the students' tendency to notice things in relation to only one variable. In that lesson, John had realised the need to encourage and push the students to look beyond one variable. However the situation had not really changed by Lesson 4. Although John had developed some *knowledge of content and students: transnumeration* in relation to recognising the students' difficulties with bivariate data, insufficient *knowledge of content and teaching: transnumeration* meant that he could not really assist students to overcome that difficulty.

6.6.3 Sorting by more than one variable to look at relationships

It is recognised that students find it difficult to move from using univariate data to investigating relationships with bivariate data, that is, 'association' between bivariate data (Chick et al., 2005). For teachers to assist students with such a move, various aspects of teacher knowledge are involved.

In contrast with both Louise and John (as discussed in the previous section), Linda had considered the question of how the students might or might not handle an investigation of relationships between two variables, and anticipated that the students might struggle. As a result of this *knowledge of content and students: transnumeration*, Linda's strategy was to spend some time in the lesson talking with the class about relationships (including what their background knowledge of what the word meant and how it might be used), and to discuss with the students some examples of relationships that they might consider were worth investigating. As Linda reflected on this after the lesson [S1L2 Int 4:04], she considered that it had been time well spent in the lesson, as she believed that the students reasonably understood the idea of relationships and what might be possible to examine in the data. If she had not spent that time on the discussion, she thought that she would have had to spend a significant amount of time answering questions throughout the investigating phase as the students attempted to make sense of what to look for in the data. As a consequence of her teaching strategy, Linda's students generally handled transnumeration of the bivariate data well, and were able to find some interesting relationships. This indicates that Linda had and used appropriate *knowledge of content and teaching: transnumeration* for investigating relationships in bivariate data. An example arose with one of Linda's students not appropriately sorting by two variables. Linda noticed that the student had sorted her cards into two piles, one corresponding to the data of 'barefeet' (from the preference of what you like to run in), and the other for 'blue' (from the preference of colour). Linda talked with the student about not being able to make comparisons between these two groups. She guided the student to first sort the data by footwear preference for running, into the two groups of 'barefeet' and 'shoes'; then the student went on, still with Linda's guidance, of splitting each 'footwear' group into two subgroups for colour preference. At this point the student was able to see that the four groups of cards would enable her to make statements to compare pairs of groups. Linda had used *specialised knowledge of content: transnumeration* and *reasoning with models* when she recognised that the student's initial sorting would be inappropriate for making comparisons; she then used *knowledge of content and teaching: transnumeration* to guide the student through the steps of sorting the data appropriately for the two variables.

As discussed in the previous section, John was aware that students tended to look at a single variable. In an effort to move his students away from these univariate investigations and findings, John tried to push the students towards examining relationships between at least two variables.

> Teacher: What I want you to do is consider 2 or even 3 parts of the data square at a time. Put them into groups and make 3 statements up. Now what might be some possible things we could look at?
>
> Student: What children are the youngest, middle and oldest.
>
> Teacher: What could we do with that? … What could we do next?
>
> Student: Then boy/girl.

Teacher: Yes, looking at the different answers we have on the data square and putting them into groups and seeing if there are some relationships between middle, oldest, youngest with left handed/right handed, boy/girl, left handed people can whistle. So looking at more than one thing, seeing if we can make up groups and make statements.

[S2L1 V 13:27]

John's suggestion was not sufficiently explicit for students to know what John intended. A number of possible relationships that could be examined were combined together within John's statement, and consequently were essentially lost. In the subsequent investigations, some students still focused on single variables, so John again addressed this with the class:

Try to think a bit deeper than: there are 8 boys, there are 10 girls; or there are no left handed boys. See if you can see some sort of relationship between 2 or 3 parts of the data square.

[S2L1 V 14:49]

At this point, John pointed to the data square on the board, and drew some arrows between adjacent parts of the data square. (Figure 6-2 shows a reproduction of John's diagram.)

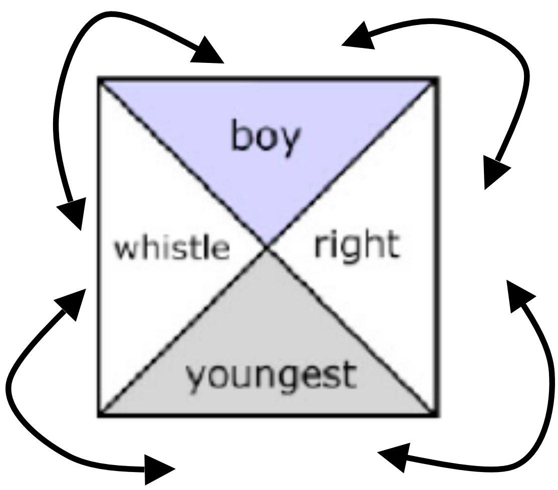

Figure 6-2: A reproduction of John's diagram of arrows on the data card, for assisting students with relationships

In the post-lesson interview, John explained how he intended that the arrows would help the students see how the variables interacted [S2L1 IntB 23:24]. However, he acknowledged that the arrows might have limited the students' views of possible relationships to the 'side-by-side' ones and to only looking at two variables at a time. Later in the same lesson that he had drawn the diagram with arrows, John talked with a group about the initial part of their investigation, at the point where they had completed their sorting of the data cards. In order to push the focus on relationships, he asked:

> Are there any relationships between girls being the oldest, right handed and whistling?
>
> [S2L1 V 15:35]

This was, however, an overly complicated and impractical suggestion for the students, as it involved all four variables. As such, the required transnumeration and subsequent reasoning with models would have been beyond the students. John had not considered these aspects when he made the suggestion. His intention had been to encourage them to not stop once the cards were sorted, but to keep looking at what the groups of cards were 'saying' about the data [S2L1 IntB 24:32]. John's *knowledge of content and teaching: transnumeration* showed some development through his attempt to encourage and assist the students beyond univariate findings. However he did not have sufficient *specialised knowledge of content: transnumeration* to realise that subdividing a relatively small data set (24 cases, each with four variables) would result in potentially 16 groups (although some of these would be 'empty' for this particular data set) into which the 24 cards could be sorted. So although John, like Linda, showed *knowledge of content and students: transnumeration* in understanding that the students tended to univariate investigations, he did not have such good *knowledge of content and teaching: transnumeration* as Linda, to be able to successfully enable the students to handle the bivariate relationships adequately.

A situation with too many variables under consideration arose in Louise's class. Her students had been sorting and grouping the cards to check what they could find in the data. One group reported that they had sorted by all four variables. Louise checked how many different groups they had obtained (there were 10) and she responded,

> Teacher: That's cool. You've used every question to group the data. Anyone done it a different way?
>
> [S4L1 V 8:44]

Louise's response was positive and complimentary that the students had used all four variables to sort the data. In spite of this, in the follow up interview, Louise discussed how she recognised that they would have difficulty because of having used too many variables to sort [S4L1 Int 6:01]. She hadn't expected the students to "get so advanced so quickly." Louise therefore realised that she would need to encourage the students to use only two variables for sorting. It is not known why she did not respond in such a way to the students when they talked about having used all the variables for their sorting. It is a possibility that the demand to respond appropriately, 'on the spot' and in the real time classroom interactions, was outweighed by the need to continue seeking responses from groups. This exemplifies O'Connor's (2001) claim about teacher decision making being affected by a need to reduce student discomfort (by not prolonging a decision for too long) and to keep the lesson moving. This indicated that Louise's *knowledge of content and teaching: transnumeration* was developing but still somewhat tentative. By not following up on the student's comment about sorting with all variables, Louise showed that her *specialised knowledge of content: transnumeration* was insufficient for that particular situation, as the students really needed re-directing at that point.

In Louise's subsequent teaching in that lesson as well as the following ones, she addressed the need to focus on sorting by two variables. She did this by suggesting a way of sorting the cards into four groups through the use of a diagram (Figure 6-3). She explained:

> Teacher: This might be a way of doing it. [Draws the two intersecting lines to create four regions.] You might have in this section [top left] all the girls, this section [bottom left] all the boys, and then you might have all the left handed ones over this side [and puts the word 'left' into the two right hand regions]. So if everyone does that, then we will have a look at what we can find out about that data.
>
> [S4L1 V 10:14]

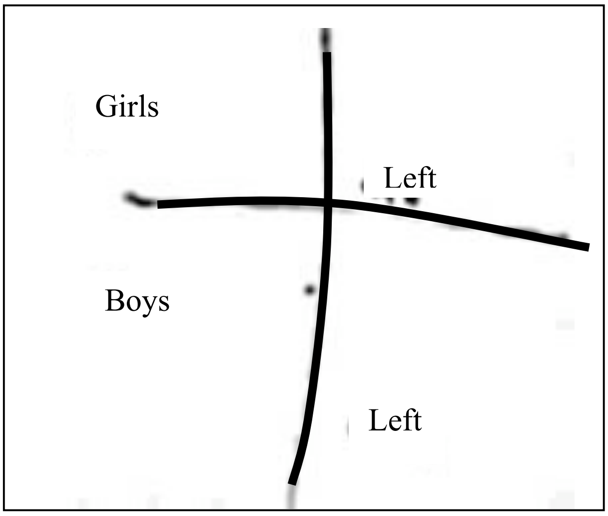

Figure 6-3: Louise's diagram of how to sort using two variables

The attempt to guide the students' sorting was only partially successful, as the diagram and explanation of how to sort were neither complete nor particularly clear. Only some of the students understood, as shown by them setting their cards out in the way that Louise had suggested. The other students tended to create four piles of card in a line along the desk. Such a linear arrangement was not particularly conducive to the students making valid statements about the four groups of cards. This illustrates Louise's use of *knowledge of content and teaching: transnumeration,* rather than being strong and robust, was developing, but insufficient to guide her students adequately.

6.7 Students' difficulty with data-based statements

6.7.1 Introduction

A large number of classroom episodes were identified in which the students had noticeable difficulty with making clear and valid statements, once they had sorted the data. Some difficulties could be attributed to students' language ability and their lack of awareness of the need to be precise with their statements. Others could be linked to the way in which the data had been sorted and arranged, or whether the students had created new representations of the data. Irrespective of the nature or source of the difficulty, teacher knowledge, particularly in relation to *reasoning with models*, was critical. Whether the teacher understood the students' difficulties, and had knowledge of strategies for helping overcome these difficulties, were the aspects of teacher knowledge pertinent to the students' dealing with data-based statements.

This section discusses a number of factors and issues relevant to the students' data based statements. They include: whether the students merely described groups of data or actively compared groups; their use of, and precision with, comparative statements; and teachers' strategies for assisting students with their statements, for instance through revoicing of statements, suggesting alternative representations of the data to refer to, and appropriate modelling of language.

6.7.2 Describing groups or comparing groups?

In a number of instances, having sorted the data squares into groups, the students described the features of each group, independently of the others, rather than notice and make comparisons between the groups. Being able to make such comparisons is essential in the development of statistical reasoning, particularly as a precursor for inference (Konold & Higgins, 2003; Watson & Moritz, 1999).

One example of students describing rather than comparing groups came from John talking with a group of students about the groups of data cards that they had sorted:

> Teacher: Now what are these two here? ... They're exactly the same, so they are off on their own. ... It looks like you've got 3 groups. ... Tell me about the 3 groups.
>
> Student: This is all the youngest in the family.
>
> Teacher: And these 3 cards?
>
> Student: The oldest and cannot whistle.
>
> Teacher: And these?
>
> Student: They are youngest and can whistle and are right handed.
>
> Teacher: So you've got ... I see that they can all whistle, and they are all boys. Is there anything else that would put them in that group?
>
> Student: They are right handed.
>
> Student: All the boys are right handed.
>
> Teacher: So what is not the same about the boys in that group?
>
> Student: Youngest and oldest ...
>
> Teacher: So there's a mixture there. ... So what have you written? There's only one girl who is in the middle [of the family] who can whistle and write with her right hand. Where's that girl?
>
> Student: There are only 3 boys who cannot whistle and all are the oldest.
>
> Teacher: So are these all separate groups?
>
> Student: Yes.
>
> Teacher: Is that one on her own?

Student: She's left handed and can't whistle.

Teacher: Oh so she's the same as those but she is left handed?

Student: Yes.

[S2L1 V 18:40]

In this episode, the students had a number of groups of cards, which had been sorted by all four variables. The descriptions that they gave John were of the features of each group, rather than comparisons between the groups. Because so many variables had been used for the sorting, it was very difficult for the students to draw comparisons and make worthwhile findings about the data set.

Another group of John's students sorted the cards by three variables: gender, age, and reaction time. They described some of the groups:

Student: All year 6 girls who are 11 have reaction time 13.

Teacher: Interesting.

Student: All year 8 girls except for one who is 12 have a reaction time of 11.

[S2L4 V 6:00]

The students had made two statements about different groups, but did not consider linking those two statements into a comparison between the groups; they could have compared the reaction times of the Year 6 and the Year 8 girls, or of the 11-year-old and the 13-year-old girls. Additionally, John did not encourage any such comparison or any further description of the groups that had been identified. He could have challenged the students to consider whether both age and year level were relevant to reaction time, or whether one of those variables would have been sufficient and valid to relate to the reaction times. This type of consideration would have pushed the students to integrate their statistical and contextual knowledge, thereby enriching their statistical thinking.

Another teacher, Rob, recognised the difficulty that he had had in trying to encourage the students to make comparisons between groups:

> [It] has been difficult. Maybe I don't have a certain idea. Having lined up all the cards, some students are still looking at numbers and how they're different, instead of looking at all, to compare the different ones ... quite a challenge to focus on groups. ... Maybe I needed to bring in average and look at that sort of thing or even looking at how the ... I'm not giving them enough scaffolding to get there. I should [have] stopped them earlier and have modelled those with data on the board.

[S3L3 V 0:00]

Rob's admission about the challenge of how to move the focus towards group comparisons links to his *knowledge of content and students: reasoning with models* as well as his *knowledge of content and teaching: reasoning with models*. The first component related to Rob's lack of knowledge about what students might find a challenge with group comparisons, and about the students' development of statistical understanding of group features and comparisons. The second component, related to teaching, is linked to Rob's uncertainty about how to deal with this in a teaching situation.

As discussed in an earlier section, Linda had spent time carefully modelling how the cards could be sorted by two dichotomous variables into four groups. She also encouraged comparisons between pairs of groups of data cards. Consequently her students, in spite of being younger, did not have such a problem as other students with drawing comparisons between groups. *Knowledge of content and teaching: reasoning with models* has a prerequisite in the form of *knowledge of content and teaching: transnumeration*; without the latter, the former cannot and did not feature strongly in teachers, in spite of their understanding the need to push for more comparisons between groups of data.

6.7.3 Comparative statements

The need for precision and accuracy with data based statements became apparent to some of the teachers. Incomplete comparative statements were recognised by some teachers, who then followed these up with students so that they would complete the statements. The statistical component of *reasoning with models* was in evidence in such situations. For example, Linda recognised that a number of comparisons were possible when the statements were based on two variables. Linda discussed in the follow-up interview why she had pushed the student to complete the comparison by asking, "Than what?":

> [It was] important because whatever he was saying, eg. more right handed girls, I didn't know if he was comparing with left handed girls or right handed boys. There could have been 2 things that he was comparing them to. It was important to distinguish which one it was for his statement to be a correct statement.
>
> [S1L2 Int 22:48]

Similarly, Louise encouraged a student to complete a comparative statement:

> Student: Girls have more people that whistle.
>
> Teacher: Compared with what?
>
> Student: ... Boys.
>
> Teacher: Who can check that against their own statement?

[S4L2 V 6:17] [4]

Both Louise and Linda commented that there is a need for precision and accuracy with statements. They understood the need to encourage the students to develop that same understanding, of being able to justify the statement by the data, and of the need for the statement to be understood by others. The simple strategy that both Linda and Louise employed, with it therefore being part of their *knowledge of content and teaching: reasoning with models*, involved responding to the students with "compared with what?" This pushed the students to reconsider the statements and then complete the comparisons, and also provided the students with the chance to develop alternative comparisons using the same groups of data.

6.7.4 Teachers' strategies for assisting

When students struggled with making statements from the data, teachers assisted the students through the use of a number of different strategies.

Revoicing

Sometimes, the teacher 'revoiced' a student's statement (Forman, 2003; O'Connor & Michaels, 1996), a strategy by which a teacher repeats the statement (possibly in modified form) to "make it more accessible (less ambiguous, better formulated, more canonical) to the other students" (Forman, 2003). At other times the teacher completed the statement for the student if it was reasonably apparent what the student was struggling to say, thereby making it more accessible to the student him or herself, as well as to others. For example, a student proposed a possible investigation:

> Student: If there are more boys who don't have hand span or foot.

---

[4] The transcripts and notes for this lesson, as well as its associated interview, are included in Appendix 4, as a sample.

Teacher: Yes, ... [pause] so you are saying, "If their hand spans are the same size as ... "

Student: If there are more people who doesn't have the same hand span as ...

Teacher: the same size hand span as their foot size?

Student: Yes.

Teacher: Yes so you can look at the differences between your hand spans and your foot sizes. ...

[S3L4 Int 0:00]

Here the teacher, Rob, helped the student develop the statement in such a way that it became clear what the student was intending to investigate. In a similar way, Linda tried to make sense of a student's statement by asking him to repeat, and by her revoicing, the statement in smaller sections [S1L2 V 12:16]. This revoicing was initially directed back at the student who was making the statement, but later she revoiced the complete statement so that the class could also make sense of what he was saying about the data.

Linda helped one student develop some valid statements for the data by carefully questioning and guiding the student to focus on a number of smaller statements. The student had sorted the data cards by two variables (gender and movie preference) and began to make some statements based on the data as represented in Figure 6-4:

Student: There are more boys and girls who like IceAge2 ... and two boys and girls who like ...

Teacher: So there are the same ...

Student: … number of boys and girls who like IceAge2, and the same number of boys and girls who like Madagascar.

Teacher: Which one is the most popular of those two?

Student: IceAge2.

[S1L4 V 6:03]

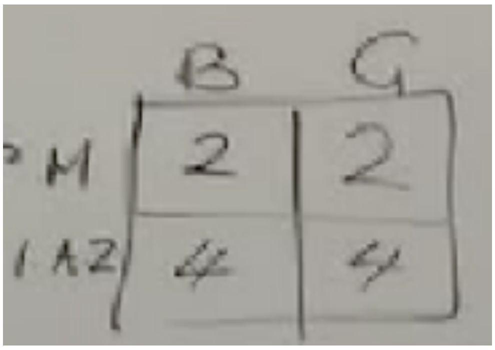

Figure 6-4: Linda’s two-way table of the data for gender (B/G) vs.movie preference (M=Madagascar/IA2=Ice Age 2)

The student’s initial comparative statement was left ‘hanging’ while frequencies of the groups were considered, and was not addressed until the end of the episode. By that stage, the comparison was no longer stated, but was dealt with by the answer to Linda’s question as to which one was the most popular. Her short ‘interruption’ after the initial statement was sufficient to enable the student to clarify the statements he was making.

In each case, the revoicing was dependent on *specialised knowledge of content:* in relation to *interrogative cycle* for Rob, and to *reasoning with models* for Linda. Without such knowledge, they would not have been able to make sufficient sense of the students’ statements to help the students subsequently clarify the statements they were trying to make. In each case, seeing the way forward to help the students required *knowledge of content and students*; any such help cannot occur without a number of types of teacher knowledge in relation to aspects of statistical thinking.

In the examples above involving Rob and Linda, it is possible to identify a link to the logic of learning model (Swann, 1999), which was discussed in Chapter 3, *Methodology*. Both teachers recognised a problem, namely that the students were having difficulty making statements that were sufficiently clear and well-linked to the data. This indicated a mismatch between what they expected the students to be able to do, and the current experience of the recognising that students were challenged with the making of clear, valid statements. Unlike a previous example that was discussed of a mismatch that resulted in no new trial solutions, in this situation each teacher developed a trial solution. In both Rob’s and Linda’s cases, the trial solution consisted of scaffolding the student’s statement by revoicing in parts. Additionally, Linda's trial solution involved the use of a model with which to reason. The trial solutions thus resulted in the 'elimination of error'.

Transnumerating the data into a different representation

Another way in which the teachers were able to assist students with their statements was through encouraging them to use a different representation of the data from which to make statements. Louise expressed surprise [S4L1 Int 15:09] that even when the students had data squares to manipulate and sort, they were still challenged with making statements about the data. If the students transnumerated the data from the cards into another appropriate representation, they acquired another support for developing their statements. Linda used a two-way table representation of data (as shown in Figure 6-4) to support a class discussion of findings. It enabled the class to 'see' the data that the student was making a statement about. Another similar example, also from Linda, arose because she knew that one of her students had difficulty with verbal and written statements in general. Consequently once the data squares had been sorted by two variables into four groups, Linda recommended that he should create a two-way table in his book showing the frequencies of the four groups. This permanent record of the data (as opposed to the physical groups of data squares which were not permanent) gave the student a point of reference for formulating his statements about the data. Linda then showed the class the two-way table (see Figure 6-5) of the number of boys (B) and girls (G) who preferred one of two sports stars, Jerry Collins or Daniel Carter. Linda considered this to be a useful tool for displaying and making various statements about the data, through examining and comparing rows, columns, or pairs of cells [S1L4 V 10:59]. At that point, she asked the class for suggestions as to what the data showed; the students did not have the original data to refer to but were able to make sense of the data as shown in the two-way table, and consequently made a number of worthwhile comparative statements based on the data.

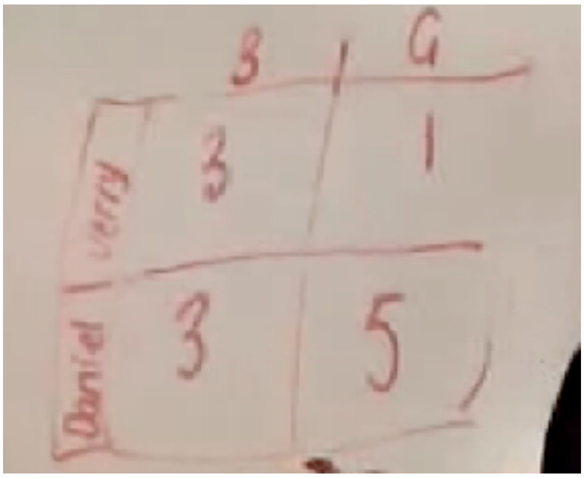

Figure 6-5: Linda's two-way table of gender/favourite sports-star

The students noticed and used column or row totals in the two-way tables as part of their statements, which had not happened when the students reasoned using only the sorted cards. This type of data representation would also be helpful for dealing with proportions of row or column totals, at such a stage that they were ready for using proportional reasoning for conditional type questions.

Some students from both Louise's and Rob's classes found that appropriately sorting and arranging the data cards for a numeric variable created a block graph of the data. This provided an effective visual tool for further noticing and reasoning from the data.

It is clear that for students to progress with making worthwhile statements about the data under investigation, particularly comparisons, teachers need to be aware of other ways of representing the data. With this knowledge, the teachers would then be able to encourage the students to transnumerate the data, thereby creating various representations of the data. Some representations are more useful for revealing 'stories' within the data than is possible with other representations. Helping students' reasoning with models requires teacher knowledge of effective sorting of the data cards, and knowledge of how to subsequently use a variety of representations, such as two-way tables, other types of tables, graphs, and measures such as medians and means.

Modelling of appropriate language by the teacher

The teachers showed awareness of the need to be careful with their own data based statements so as to provide good models for the students. For instance, Linda commented:

> My mind was constantly active to be correct in what I was saying so as to not mislead the kids with what I was saying.
>
> [S1L4 Int 28:03]

Similarly, John recognised the need to be precise and to use correct terminology [S2L4 Int 19:09] as did Rob [S3L2 Int 26:54]. In spite of Rob's comment about precision, in one episode when talking about the median, he instead used the word 'medium':

> Teacher: Explain what the medium is. How did you find it out?
>
> Student 1: Counted from both sides.
>
> Teacher: So it is the number in the middle?
>
> Student 2: Yes.
>
> Student 3: Half, this side here is bigger, this side is smaller.
>
> Teacher: Explain the word medium.
>
> Student 4: It's a type of ... like high, low, medium.
>
> [S3L4 V 12:20]

The first three students appeared to either not notice, or overlook, Rob's use of an incorrect word. Student 4's interpretation, however, seemed to be deviating from the statistical term of median, towards a usage of medium as pertinent to heaters and switches (e.g., "Do you want the heater put on high, medium, or low?"). Whatever the case for Student 4, Rob did not pursue it, as at the time, he was not aware of his use of an incorrect word.

The concept of distribution has been the focus of recent research. For example, distribution was the theme for the Statistics Research, Thinking and Literacy Research Forum (SRTL 4) 2005, and subsequently a complete issue of *Statistics Education Research Journal 5*(2), November 2006, was devoted to papers on distribution. Reasoning about distribution is considered to include dealing with the features of centre, spread, density, skewness and outliers (Bakker & Gravemeijer, 2004; Pfannkuch & Reading, 2006). The term 'distribution' arose in a lesson with one teacher; Rob used it in relation to three subgroups within one data set:

Teacher [to one group of students]: So there are 9 in each row. That's very interesting. So they are evenly spread.

Student [speaking to the class]: When we put them in order from youngest to oldest, there were 9 in each row.

Teacher: So there was an even distribution of the youngest people in the family, there's an even number of the middle people in the family, and eldest. So that's a very interesting thing.

[S3L1 V 7:59]

However in the follow up interview [S3L1 Int 21:36], Rob conceded that he was not sure that he had used the term distribution in the correct way; it had not been a considered and deliberate use of the term. It was used in a naïve yet appropriate way.

Using statistical terms such as distribution, even if the students are not ready for them in a formal sense, contributes to the language development of the students. Over time with continual usage, concepts associated with such terms become more refined (Forman, 2003) and will contribute to the development of students' statistical thinking.

The need for precision with statements so that they were correct for the data was evident in many situations when the teachers were dealing with students' statements. Cases arose when teachers pushed the students to qualify, or slightly change, their statements in order to make them valid for the data. For example, the use of a qualifying word such as 'most', 'more', 'fewer' etc., was at times problematic for the students. When it was recognised by the teacher as necessary, the student was encouraged to reconsider the statement and amend it to make it more valid. For example, Rob commented in an interview that he had noticed the student's statement that 'most of the girls were under 22 [for reaction times]' was incorrect [S3L4 Int 17:25]. The correct statement would have been that, rather than most being under 22, most of the girls were 22 and under or under 23. Rob commented that this type of statement in which the word 'most' is used must be carefully modelled by the teacher so that the students understand the difference between using the word 'most' and another qualifier such as 'more'. In some situations, however, Rob did not recognise incorrect statements, such as, in a later part of the same lesson that was discussed above, when a student referred to the median of 22, she explained that the median showed “most girls are under 22 and most girls are over 22” [S3L4 V 6:59].

On some occasions, to avoid a potential problem with the words ‘more’ or ‘less’ etc., some of the teachers encouraged the students to instead use frequencies in their statements. The use of frequencies gave more precision to the statements and avoided the language difficulties associated with some of the qualifying words. For example, when a student claimed that the majority of the girls were right handed, Louise responded:

> Teacher: Majority of girls right handed. Isn’t that clear! Now who can make a statement that is a little bit more clear than that? Perhaps use exact numbers of how many to how many.
>
> Student: There are 3 left handed girls and 9 right handed girls.
>
> Teacher: So if there are 12 girls in total, how many out of 12 …
>
> Student: Left - one quarter.
>
> Teacher: How many out of 12 were?
>
> Student: One quarter.
>
> Teacher: So 3. Very good.
>
> [S4L1 V 10:14]

Louise felt that the use of a frequency (even though the student ended up giving her a relative frequency of ¼, which she did not comment on) was preferable to using the word ‘majority’. By pushing for a frequency, she indicated a preference for numerical precision compared with the less precise verbal qualifier of ‘majority’.

Teacher knowledge of appropriate statistical language and clear, unambiguous use of language was not always in evidence. When teachers themselves did use such language, students had more opportunity to develop appropriate use of language and subsequently their understanding of statistical concepts. As Anthony and Walshaw (2007) outlined, language is central to enabling students to link their intuitive understandings with accepted mathematical understandings. Consequently, it is clear that if teachers do not have adequate knowledge of appropriate statistical language, their students’ development will be impeded.

6.8 Understanding variation and the development of inference

Aspects of inferential thinking provided significant challenges for all four teachers with regard to teacher knowledge. Two types of inferential thinking were involved in the teaching and learning – drawing inferences about the data set under examination, and drawing inferences from the data set about a population, each of which involves different concepts and thinking (Pfannkuch, 2005). Whichever type of inference was involved, understanding the inherent variation in data was critical to the opportunities for students to develop their inferential thinking. The ability to compare groups of data, in order to make general statements about that data set, is considered an important prerequisite for the later development of formal inference. In this research, such comparisons were, first, between groups of category data, and second, between numeric data sets; how these comparisons were dealt with in the classrooms has been discussed earlier. Aspects covered include: describing and comparing groups, the challenges of comparing numerical data sets (such as, which group, boys or girls, have a faster reaction time), and using medians or means to compare groups (especially when the group sizes are different). In this section, teacher knowledge is examined in relation to drawing inferences about a population based on a given data set (i.e., a sample).

Although not explicitly mentioned in the unit (Appendix 2), inference was 'present' through some questions and notes that were included in the unit. These were:

> All the boys in this group who are the youngest can whistle, does this mean every boy who is the youngest in their family can whistle?
>
> What do you expect to find out about the class? Will the things we found out from Data Set One, be different or similar to our class?
>
> At this point, teachers may wish to discuss the likely difference in results between randomly selecting 24 students from the class and hand picking 24 friends. A quick example is a good way to illustrate this point at this level of the curriculum. The point to get across is that hand picking students to answer a question can give a misleading impression of the class, if it is assumed that it is representative of the whole class.
> For example, the teacher selects five rugby-loving boys in the class and asks them to name their favourite sport. All the boys are likely to say rugby, with the resulting statement make, "Everyone answered rugby, so the favourite sport in the class is rugby" or "Everyone in this class loves rugby."

> This session has the students compiling a new set of data squares based on questions they develop themselves. The questions could be comparing information from the data sets presented during this unit, looking at new information or asking a different group of students the same questions.
>
> (Ministry of Education, 2006)

Most of the teachers asked students whether a finding from the data set could be generalised to a population. Examples of such questions included:

> So do you think in every class there will one left handed girl?
>
> Do you think that in every class there is going to be more right handed girls than right handed boys?
>
> Because all boys in the class can whistle, does that mean that all boys can whistle?
>
> What fraction of boys in the school might like a particular television programme?
>
> All girls in the group like mini-golf, so would all girls in the school like mini-golf?
>
> In this data, all boys who are youngest in the family can whistle, so could we say that every boy in New Zealand who is youngest in the family can whistle?
>
> Will the things we found out from the data squares be different or similar to our class?
>
> Do you think Room 3 next door would have the same results?
>
> Can all right handed people whistle?
>
> Four boys in our class fit this data square; how many people in the school could this data square be exact for?

There were a number of different types of responses given by students to such questions. A typical response was that it is not possible to generalise to another class, because "our class is different", or another class might have different results. This type of response acknowledged variation, although in a way that did not necessarily also recognise the possibility of similarities, trends or patterns. When Linda encountered such a response [S1L2 V 9:00], she did not push the students to consider similarities simultaneously with thinking about how the results might vary with the other class. Likewise, Rob [S3L1 V 5:40] did not probe this type of response further. In contrast, when Louise asked how many people in the school might 'fit' a particular data square (based on four students in this class who had identical responses on their data squares), the student responded:

> Student: I don't know the right answer but probably like 4 people in every class.
>
> Teacher: Yes; other guesses?
>
> Student: There could be 4 in one class and then 3 in another.

Teacher: Right. So we could say that out of 12 classes, 6 of them could be 4s and 6 of them could be 3s. Make our estimation that way.

[S4L1 V 7:43]

The student's initial response focused on similarity or trend. Louise's response to this was to encourage the student to also consider variation. She realised that to make a prediction about other classes, both trend and variation needed to be considered simultaneously. One of Linda's students predicted that, because his data showed exactly half the group were in each of two categories in relation to one variable, he couldn't necessarily say that the same would be true for the whole class. His response showed he was considering only variation. At this point, Linda continued,

Teacher: Do you think that maybe it would be close to half?

Student: Yes could be.

Teacher: Yes. So by going by these six here [in the data set] … How many girls in whole class? … Yes, there's about 12 in the whole class. So you have asked about half the girls in the class already [as his sample of the class]. So maybe the other half of the girls in the class would be the same.

[S1L4 V 13:16]

Linda was obviously not satisfied with the student's answer that had considered only variability; she encouraged him, through her response, to think about trend as well. So like Louise, she had sufficient knowledge to push the student to think of the two aspects simultaneously. She continued further by introducing the idea of sample size, although not explicitly. Her suggestion was that because he had sampled half the girls in the class, there was a 'strength' to the prediction that the other half of the girls could be similar.

Sample size was a factor that arose in some responses. In some cases, it was suggested by students that it was not possible to generalise to a population because the data was based on a small sample. For instance, when Linda asked why a student thought that it was not possible to claim that all boys could whistle when it was true for this class, he responded,

Student: Because it's only this class.

Linda continued:

Teacher: Because our data is just a sample. We can't say that all boys in M. School can whistle. We, as room 12, are just a sample of M. School.

[S1L2 V 9:50].

Although Linda did not appear from this to have knowledge about generalising from a sample, she definitely had such understanding. In the follow-up interviews she talked about the risk of misleading statements from a small sample size [S1L2 Int 13:57], and whether data from 15 boys in the class is sufficient to make a reliable prediction about the whole school or are there other factors such as age that would confound the prediction [S1L2 Int 13:57 and S1L4 Int 21:59].

Sample size was considered during an episode in John's class, along with other factors that influence the validity of generalising:

Teacher: Could we say that every boy in NZ who is youngest in family could whistle?

Student 1: No, it's just the results from this group. They haven't tested everyone. All the youngest boys don't know exactly how to whistle.

Teacher: Okay. What would we have to do to make that statement about NZ?

Student 2: See if everyone can whistle ... in NZ.

Teacher: Would we have to do everyone in NZ?

Student 3: No ... we are only one classroom out of NZ.

Teacher: From only one classroom so we couldn't really say that all boys who are youngest can whistle. We don't have enough data to say that, we would have to do a bigger study, bigger survey, bigger investigation to find that out.

[S2L2 V 9:53]

The responses from Students 1 and 2 as to why generalising was not possible were that the population had not been tested, with the implication that this is necessary. John was not satisfied with this type of response. He questioned the need to ask/test everyone in NZ, at which point Student 3 agreed that it was not necessary as "we are only one classroom out of NZ." It was not clear what the student meant by that response, but one interpretation is that testing everyone in NZ was not a feasible task for this one class to undertake; if that was what the student meant, it was not a relevant response to the question of generalisability. However, John's interpretation of the response was that the student was referring to sample size of this class in comparison to the population size. John's comment about sample size showed a limited understanding of generalising; he was suggesting that it is only possible to generalise if there is a large sample, which does not take into account a number of other issues pertinent to sample size and generalisations. This response provides evidence of John not having strong knowledge of inference, and this relates to *common knowledge of content: variation* and *reasoning with models*.

The inability to generalise without having data from the population was mentioned in other classrooms as well. For instance, one of Linda's students suggested that it was not possible to generalise about television programme preferences because not all students in the school had been asked [S1L4 V 23:02]. Also Rob agreed with one of his students who said that it was not possible to generalise to the whole school because the data has come from only a small portion of the school [S3L1 V 8:51]. Rob did not follow this up any further. On the other hand, Linda encouraged the student to look again at the data from this class, and think whether it might apply to the whole school. The student then agreed that not a lot of boys in the school would watch the programme in question, as not many boys in the sample watched that programme. It appeared that the student understood that a census (data collection from the population, in this case the school) was not necessary; it was possible to make a qualified prediction based on the sample (group within the class) data. The qualified prediction involved the phrase, "not a lot of boys …" based on the sample data showing 1 out of 6. The use of such qualified statements was an indication of informal inference. As well as Linda, the other three teachers in the study understood the need to make such qualified statements, although not all encouraged their students to do so. In one interview, John talked about making statements that included phrases such as, "it is likely but not definite that …", "it is probable that …" [S2L2&3 Int 1:04], but there were no instances in his teaching where he encouraged the students accordingly. Rob talked in an interview about a student who needed to qualify his statement with the word "most"; Rob acknowledged the challenge of encouraging students to think beyond the data [S3L1 Int 25:54]. When asked to suggest how she would answer the question of whether all youngest boys would be able to whistle, Louise said that, "The probability is that a high percentage of …" [S4L1 Int 20:53]. As with John, neither Rob nor Louise, although understanding the need to qualify generalisations (as evidenced in the interviews), dealt with this in the teaching episodes.

Informal inference was present in these various classroom incidents, and called on teachers to use appropriate types of knowledge in relation to inference. Their knowledge in relation to inference can be compared against the four components of informal inference as described by Rubin, Hammerman, and Konold (2006). The four components are: properties of ‘aggregates’ (including: trends/averages and variation; and types of variability); the effect of sample size; controlling for bias in sampling helps ensure a more representative and therefore reliable sample; and the property of ‘tendency’ to help one to distinguish between claims that are always true and those that are often or sometimes true. In this section, the teachers’ knowledge or lack of knowledge in relation to informal inference has been discussed. The first part of the section examined whether the teachers took only variation into account, or whether they simultaneously considered trends and patterns. This corresponds to properties of data aggregates. Next, episodes related to sample size were examined. The final part of the discussion looked at how statements could be adapted through qualifying phrases to make any generalisation more valid in relation to the data from which it has come; this is connected to the fourth property of ‘tendency’. Sampling, and controlling for bias, were generally not in evidence, due to the type of investigation undertaken. Linda’s students collected their own data during Lesson 3, based on data collection questions that each student had formulated. To simplify the logistics of collecting the data, Linda only required each student to survey 12 other students. When the students made statements about the class based on their sample of 12, the opportunity was present for Linda to explore the issues of sampling and bias, but this did not happen.

From the discussion, it is apparent that most of the teachers exhibited parts of informal inference, and were able to assist their students to a limited extent. Of the four teachers, Linda showed the best understanding and use of informal inferential reasoning. It was not complete and sound, but there was evidence of all components being used by Linda. As she commented in the final interview, in response to a question about whether she had to think about statistics while she was teaching:

> Yes, with sample sizes, and generalisations, and the wording of statements/findings. ... I didn't know whether this age group would pick up on that type of thing, and we were already grappling with a number of different concepts as well. My mind was constantly active to be correct in what I was saying, so as to not mislead the kids with what I was saying.
>
> [S1L4 Int 28:03]

It is possible to identify the situations in which a teacher, having a good level of teacher knowledge in relation to inference, could help students move forward in the development of their informal inference. Conversely, the classroom situations for which the students were not pushed and challenged with regard to inference provided evidence of inadequate teacher knowledge of inference.

6.9 Summary

This chapter has centred on a number of significant themes that arose from the analysis in Chapter 5, *Results*. These themes focused on components of and issues related to students' statistical thinking. In each case, it was identified how each of these linked to the teachers' knowledge.

Chapter 7
Conclusions and Implications

7.1 Introduction

This thesis has set out to establish the nature of teacher knowledge that is needed for teaching statistics through investigations. The literature review examined the direction taken by the more recent research on teacher knowledge. Some of the possible research approaches were shown to have certain limitations for examining teacher knowledge as needed and used in the classroom. Research approaches were considered, including those used in the broad domain of mathematics education, through to the narrower and newer field of statistics education, with its more specific, and as argued, somewhat unique aspects. As a result of the review of the literature, a framework was proposed for researching teacher knowledge in statistics. The framework combined elements from two contemporary areas of research, one in relation to statistical thinking (Wild & Pfannkuch, 1999), and the other in relation to teacher knowledge relevant to mathematics education (Ball, Thames, & Phelps, 2005; Hill, Schilling, & Ball, 2004). By combining these two strands of research into one framework, it was proposed that the statistical nature of the work that teachers engage in during the course of their teaching could be examined. In relation to the proposed framework, a research hypothesis was proposed, and research questions were asked.

In Chapter 3, *Methodology in Theory*, the nature and development of knowledge was considered, in relation to the philosophy of Popper and the logic of learning model (Burgess, 1977; Swann, 1999). The appropriateness of these ideas for researching teacher knowledge, and in particular being able to account for the dynamic nature of that knowledge, was argued. Of three different research paradigms considered for this study, one was selected as fitting most appropriately with the conceptions of knowledge discussed. This was a post-positivist realist paradigm. A number of issues in relation to the paradigm and to this study were discussed, including the types of questions on which such research can be based, the generalisation of findings from the research, some of the potential difficulties of situating the research in the classroom, appropriate data collection tools (in particular, video and stimulated recall interviews), and some ethical issues regarding the effect of participation in the research on the teachers' practices and knowledge.

How this theoretical methodological position was enacted for the research was explained in Chapter 4, *Methodology in Practice*. The data were obtained from two sources, one being video recordings of lessons, and the other being audio recordings of post-lesson interviews with the teachers. The interviews were focused on discussion of incidents from the lessons, using stimulated recall based on edited video recordings from the lessons.

The subsequent analysis of and results from the data were presented in Chapter 5, *Results*, and in Chapter 6, *Discussion: Significant Themes*. In these two chapters, as in this chapter, the interpretations of the researcher may not be the only ones possible. However, the researcher's experience as a teacher and familiarity with the statistical content of the teaching unit give some weight to the interpretations as being valid, although not necessarily 'true and correct'. It is therefore acknowledged that the conclusions drawn are tentative and conditional on the interpretations.

This current chapter, *Conclusions and Implications*, examines and discusses the links between the broad aims of the research as derived from the *Literature Review,* the results and discussion from Chapters 5 and 6, the specific research questions, and the contribution that this thesis has made to the research field. Implications are drawn from the research and are proposed, with regard to the state of teacher knowledge for teaching statistics, as being applicable and of interest to practising teachers and teacher professional development providers, to initial teacher educators, and for further research in the area.

7.2 Research hypothesis and questions

Each of the research questions will be addressed, followed by the hypothesis, in relation to the conclusions that can be drawn from this study, and the contribution to research knowledge.

7.2.1 Question 1

What types of teacher knowledge in relation to the components of statistical thinking are needed and/or used in the work of teaching statistics through investigations?

It was clear that, for the type of investigation undertaken by the teachers, specifically one in which the students were given data and made statements of their findings, most aspects of teacher knowledge were needed and/or used. The only exceptions, as discussed in Chapter 5, *Results*, were in relation to two dimensions of statistical thinking, namely *dispositions* and *the need for data*. If other teaching approaches for investigations were adopted, such as those in which students start by posing questions or problems to be solved, *the need for data* would be addressed and capable of being identified in relation to the four components of teacher knowledge.

All remaining twenty-four cells of the framework were identified in at least one teacher's practices, with 21 of those cells represented in either three or four teachers' profiles. It is acknowledged that where one cell was identified for two or more teachers, the particular aspects of statistical thinking and teacher knowledge were not necessarily the same for all those teachers. Generally, within one cell there was a diversity of teacher knowledge pertinent to statistical thinking. Consequently, evidence of teacher knowledge as related to statistical thinking for one cell does not imply thorough and complete knowledge for those aspects in relation to the desirable knowledge associated with the lesson.

Some instances of *common knowledge of content* were inferred from the presence of other components. Where direct evidence was not available, other aspects were examined to see whether they could provide indirect evidence of what appeared to be missing. *Common knowledge of content* was the only type of teacher knowledge for which this was possible, and there were many instances where such indirect evidence was available, generally from *specialised knowledge of content*. Therefore the absence of direct evidence of *common knowledge of content* was generally not of concern, as it was often indirectly found elsewhere. Evidence was found elsewhere for at least one dimension of statistical thinking for all the teachers, and in relation to four dimensions for two teachers. In contrast to *common knowledge of content*, the other three categories of *specialised knowledge of content, knowledge of content and students,* and *knowledge of content and teaching* are all more focused types of knowledge, with specific 'roles', and it is unlikely that evidence of these can be inferred from other types. Consequently, absence of the other types of teacher knowledge was of more concern.

As well as the presence of teacher knowledge in relation to statistical thinking, there were numerous instances of missed opportunities that were described in Chapter 5, *Results*. In fact, 22 of the 24 framework cells were linked to these missed opportunities. The missed opportunities occurred in classroom episodes when components of teacher knowledge were needed but were not in evidence. Some of these corresponded to incorrect knowledge being used, while others related to non-use of teacher knowledge. The non-use of knowledge was due to one of a lack of the necessary knowledge, not recognising that that knowledge could have been used in that situation, and a decision by the teacher to not use the available knowledge. Whatever the reason, appropriate teacher knowledge was not used.

Taken together, the presence of evidence for all framework cells (excluding the two statistical thinking dimensions, as discussed) being present in teaching episodes, and the missed opportunities in a high proportion of the cells indicating that knowledge was needed, clearly show the types of knowledge needed and/or used in teaching statistics. This presents a strong case for the suitability of the framework as a means of identifying the types of teacher knowledge needed and/or used in relation to the components of statistical thinking. No other research literature was able to be sourced in which knowledge for teaching statistics has been examined in the primary school classroom, and in the day-to-day reality of teaching. This study has therefore made a significant contribution through identifying the knowledge in all its forms that is needed for teaching statistics through investigations. It has also identified clearly, through the missed opportunities, that without all the various components of teacher knowledge across the dimensions of statistical thinking, teachers will miss opportunities to enhance their students' learning.

7.2.2 Question 2

What are the features of such knowledge in relation to aspects of statistical thinking?

Broad descriptions of *common knowledge of content, specialised knowledge of content, knowledge of content and students,* and *knowledge of content and teaching* were obtained from the mathematics education literature (Ball et al., 2005; Hill et al., 2004), based on work in the number and algebra strands. These descriptions were generally appropriate for transferring to the statistics education field, although some required adapting and refining. The original descriptions were:

- *Common knowledge of content*: ability to identify incorrect answers or inaccurate definitions, and the ability to successfully complete the students' problems;
- *Specialised knowledge of content*: ability to analyse mathematically whether a student's unconventional answer or explanation is reasonable or mathematically correct, or to give a mathematical explanation for why a process (such as a particular algorithm) works;
- *Knowledge of content and students*: ability to anticipate student errors and misconceptions, to interpret incomplete student thinking, to predict how students will handle specific tasks, and what students will find interesting and challenging;
- *Knowledge of content and teaching*: ability to appropriately sequence the content for teaching, to recognise the instructional advantages and disadvantages of particular representations, and weigh up the mathematical issues in responding to students' unexpected approaches.

Aspects of teacher knowledge in relation to statistical thinking were identified in relation to the framework, from evidence obtained from classroom incidents or interviews with the teachers that re-examined those incidents. Consequently, descriptions of the 24 'pieces' of teacher knowledge were given in Chapter 5, *Results*.

Some examples of each category of teacher knowledge are listed below (along with the data code and source of the evidence). These examples have been derived from the study's data and discussion, and are by no means intended as a complete list of the knowledge that was observed in use or shown as needed in the teaching of investigations. Because so many statistical concepts were covered in the investigative process (from the posing of questions for investigation, consideration of data collection questions, analysis through sorting and other transnumerative processes, and concluding statements), the examples given are a small sample covering a wide variety of statistical concepts.

Examples of common knowledge of content (ckc)

Able to find the three measures of average (mode, median, mean)	ckc: transnumeration	[S3L2 Int 5:55]
Can explain why mode is not useful in certain instances	ckc: transnumeration	[S3L2 Int 27:09]
Considers the effect of sample size on generalising	ckc: reasoning with models	[S1L4 Int 28:03]
Knows that larger sample size leads to statement of greater confidence	ckc: reasoning with models	[S2L2&3 Int 31:56]
Changing the order of wording in conditional statement changes the group total and therefore the fraction (e.g., right handed whistlers or whistling right handers)	ckc: reasoning with models	[S2L2&3 Int 20:46]
Able to make a generalisation to a population	ckc: variation ckc: reasoning with models	[S4L1 Int 20:53]
Suggests reasons why the youngest child in a family is likely to be able to whistle	ckc: integration of statistical and contextual	[S2L2&3 Int 14:55]

Examples of specialised knowledge of content (skc)

Ability to make sense of students' data based statements, with reference to sorted data cards	skc: reasoning with models	[S2L1 V 18:40]
Determines whether suggested data collection question is suitable	skc: investigative cycle	[S1L1 V 6:49]
Recognition of inappropriate comparison of unequal sized groups	skc: reasoning with models	[S3L3 V 5:30]
Ability of evaluate appropriateness of inferential statement	skc:reasoning with models and skc:variation	[S4L1 Int 3:12]
Explains why measures such as mean or median are used as appropriate summary of data	skc transnumeration:	[S4L4 Int 0:00]
Ability to link student's question about 'unusual cases' in relation to data collection question with contextual knowledge	skc: integration of statistical and contextual	[S2L1 V 2:14]

Examples of knowledge of content and students (kcs)

Recognise the need for data collection questions to be closed, with only 2-3 possible responses otherwise students will struggle to sort and group data	kcs: transnumeration	[S1L1 Int 4:20}
Ability to anticipate students will struggle with making accurate inferential statements	kcs:reasoning with models and kcs:variation	[S3L1 Int 18:03]
Recognise that students will have difficulty with sorting to explore relationships between two variables	kcs: transnumeration	[S1L2 Int 1:06]
Recognise that students may find some data collection questions ambiguous	kcs: investigative cycle	[S2L1 V 10:16]
Need to encourage students to examine the data, continually looking for patterns, interesting aspects	kcs: interrogative cycle	[S2L1 IntB 19:51]
Recognise need for students to make links between what is found in the data with what they know about the real world	kcs: integration of statistical and contextual	[S3L1 V 6:13]
Recognise that students find difficulty with making valid statements from data	kcs: reasoning with models	[S4L1 Int 18:45]

Examples of knowledge of content and teaching (kct)

Uses discussion with students to evaluate suitability of data collection question, and how to refine the questions to make them unambiguous	kct: investigative cycle	[S1L1 V 4:51] [S2L1 V 2:14}
Can pose suitable questions to encourage inferential thinking	kct: reasoning with models and variation	[S1L2 V 9:50] [S3L1 V 2:16]
Encourages students to predict what might be found in data, and revisits those predictions after sorting data and making data based statements	kct: investigative and interrogative cycles, and reasoning with models	[S1L2 V 18:07]
Uses 2x2 table as suitable representation for helping make statements from data	kct: transnumeration and reasoning with models	[S1L4 V 7:14]
Considers the statistical implications for data collection from student's questions about 'unusual' family situations (e.g., how would you answer the data collection question about your position in family if you have ½ brothers/sisters, if a brother/sister has died, …)	kct: investigative cycle and integrating statistical and contextual	[S1L1 V 10:45]
Gives examples of statements involving two variables that would be suitable for investigating to help encourage students with posing conjectures to investigate	kct: reasoning with models, interrogative cycle, and investigative cycle	[S2L4 V 8:21}
Shows students a way to sort data by two variables, and suggests possible statements that can made from such a representation	kct: transnumeration and reasoning with models	[S4L1 V 10:14}

7.2.3 Question 3

Are there types of teacher knowledge in relation to components of statistical thinking that are not in evidence in the classroom and, although absent, do not impact on the potential learning opportunities for students?

The need for data

Recognising *the need for data* to answer questions was the only component of statistical thinking that was not in evidence in the classroom. A reason has been put forward, and discussed in Chapter 5, *Results*, as to why this aspect of statistical thinking was not needed or seen in the observed lessons; this reason is centred on the teaching approach adopted for these lessons. The adopted approach involved students in investigating multivariate data sets that they had been given, or had collected themselves, in order to find interesting things in the data. It is proposed that, given a different approach to the teaching of statistical investigations, the *need for data* would be observed. Such an approach would be based on a question or problem being posed, after which students would recognise that data is needed so must be collected, and analysed, in order for the question or problem to be solved.

Within this study, it is reasonable to conclude that, although *the need for data* was not observed, this did not impact on the learning opportunities for students. Also, it is conjectured that, under a different approach to the teaching of statistical investigations (as outlined above), absence of teacher knowledge in relation to *the need for data* would significantly impact on students' learning.

Dispositions

Another component of statistical thinking that was not identified specifically in relation to the four categories of teacher knowledge was *dispositions*. These dispositions include scepticism, imagination, curiosity and awareness, openness, a propensity to seek deeper meaning, being logical, engagement, and perseverance (Wild & Pfannkuch, 1999). It is suggested that such teacher dispositions are important for motivating and challenging students, for engaging them in the tasks of investigating data, and for helping engender these same dispositions in students. Although not unique to statistics, these dispositions link with the interrogative cycle, and are important for successful data investigations. However, in Chapter 5, *Results*, it was explained how the statistical thinking component of *dispositions* was observed in the classroom in a general way, rather than in relation to the four teacher knowledge categories. Because this study supports the claim that these dispositions are important in the classroom, it would be preferable to modify the framework rather than delete dispositions from the framework. Such a modification would indicate that it is a component of statistical thinking that teachers require, but would show that it runs across all four categories rather being identifiable and describable for each of the four categories of knowledge.

Unobserved knowledge

For each teacher, there were cells of the framework, corresponding to particular categories of teacher knowledge and components of statistical thinking, that were identified as absent. These were shown on the teachers' profiles as blank cells. However, these missing aspects of knowledge were not needed in relation to the teaching that was observed, otherwise they would have been classified as missed opportunities. Consequently, although not observed, they did not impact on the learning opportunities for those students.

It is important to consider therefore whether there are aspects of knowledge on the framework that are not needed in any situation for teaching statistics through investigations. Although unobserved knowledge was 'identified' for each teacher, the four profiles together reveal that all aspects of knowledge were needed by at least one teacher in the observed lessons. The sequence of lessons that the four teachers planned and delivered, although very similar because they were based on the same unit plan, showed differences in content and approach, as would be expected of different teachers, planning for different students who were of different ages. Therefore it is reasonable to conclude that all aspects of teacher knowledge are needed in the teaching of statistics through investigations, although not necessarily for one short sequence of lessons.

Missed opportunities

For an individual teacher, any aspect of knowledge that was identified as needed in a particular situation, but was not in evidence, was classified as contributing to a 'missed opportunity'. For each teacher, the missed opportunities were described in

Chapter 5, *Results*. In most cases, those missed opportunities were interpreted as having impact on the potential learning opportunities for the students.

It appears that there are two possibilities for the occurrence of a missed opportunity. First, the teacher lacked the knowledge needed, and consequently was unable to take the learning in the direction that was required. Alternatively, the teacher had the required knowledge, but for some reason, did not use it. For some of the missed opportunities, follow up discussions in the interviews were able to confirm (as examples of data triangulation in action), one way or the other, whether the missed opportunities were due to lack of knowledge or the non-use of available knowledge. For those that were confirmed as non-use of available knowledge, the reasons behind such non-use were not always revealed. It is recognised that a teacher's decision making, particularly in regard to responding to a student, is affected by a complex set of factors (O'Connor, 2001) that often interact simultaneously. Consequently, it is possible that, for some of these missed opportunities, the teacher made a conscious decision, for other pedagogical reasons, to not use particular, yet available, knowledge.

From the patterns of missed opportunities for each teacher (see Figure 7-1), and the descriptions of the missed opportunities, the state of his or her knowledge can be explored and compared with the other teachers. In some cases, the missed opportunities extend along a row in the profile, which corresponds to a component of statistical thinking across the four knowledge categories. In other cases, the missed opportunities extend down a column in the profile, indicating that a particular category of teacher knowledge was deficient across a variety of statistical thinking components.

Linda had fewest missed opportunities, and some of these were interpreted as having no impact on the learning opportunities for her students. John's missed opportunities were observed in all four categories of knowledge in relation to both *transnumeration* and *reasoning with models* (as represented by rows in the profile), in most knowledge categories related to the *investigative cycle*, and in half of those related to the *interrogative cycle*. Rob had the greatest number of missed opportunities, and these covered all knowledge categories related to *transnumeration*, most categories related to *reasoning with models* and *interrogative* *cycle,* and some related to the *investigative cycle*. Also, his missed opportunities covered all but one statistical thinking component for *knowledge of content and students* and *knowledge of content and teaching* (as represented by columns in the profiles). Louise's missed opportunities covered all statistical thinking components of *specialised knowledge of content*, and most knowledge categories related to *transnumeration* and *reasoning with models*.

Table 5-2: Summary of Linda's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				M
	Reasoning with models			M	
	Integration of statistical and contextual				M
Investigative cycle					M
Interrogative cycle					
Dispositions					

Table 5-3: Summary of John's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				
	Reasoning with models	M	M	M	M
	Integration of statistical and contextual				
Investigative cycle		M	M		M
Interrogative cycle			M		M
Dispositions					

Table 5-5: Summary of Rob's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				M
	Reasoning with models		M	M	M
	Integration of statistical and contextual			M	M
Investigative cycle				M	M
Interrogative cycle		M	M	M	
Dispositions					

Table 5-6: Summary of Louise's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M		M
	Variation	M	M		
	Reasoning with models		M	M	M
	Integration of statistical and contextual		M		
Investigative cycle			M		
Interrogative cycle			M		
Dispositions					

Figure 7-1: Profiles of the four teachers

Significant Themes

As discussed in Chapter 5, *Results*, most of the missed opportunities impacted on the potential learning opportunities for students. The missed opportunities revealed a number of common themes, which were discussed in Chapter 6, *Discussion: Significant Themes*. In some cases, the themes concurred with what is already known from the research literature, although the research literature more often focuses on students' rather than teachers' knowledge.

Teachers listening to and interpreting students' statements

Three types of listening problems were identified in relation to missed opportunities. First, the teacher did not hear, or misheard, a student's question or comment, and consequently responded in a way that was different from and inappropriate for the student's comment. Second, the teacher did not evaluate a student's answer, thereby sometimes allowing incorrect ideas to go unchallenged and unchecked. Third, in situations where it was not clear what the student was saying or meaning, the teacher did not respond to seek further clarification from the student.

In some of the examples that were discussed in Chapter 6, *Discussion: Significant Themes*, it was found through the interviews that non-use of knowledge, rather than absence of knowledge, contributed to the hearing problem. For example, following Louise's classroom discussion on the range of a set of data, the interview revealed that she had *common content knowledge: transnumeration (range)*, although from the classroom episode it appeared that she did not. It would seem, therefore, that in this type of situation, other unknown factors are affecting the teacher's listening, although the end result is that another type of teacher knowledge, namely *specialised knowledge of content: transnumeration*, was not used.

Other episodes discussed in relation to lack of listening or interpreting of students' statements provided clear evidence of the absence of various categories of teacher knowledge. Although Wallach and Even (2005) list various possible factors that contribute to hearing problems (and these factors are teacher knowledge, dispositions, feelings about students, expectations, beliefs about mathematics learning and teaching, and the context in which the hearing takes place), this study strongly suggests that for these four teachers and these sequences of lessons, teacher knowledge is one of the main contributors to the listening problems. The evidence to support this claim is that for each and every 'listening problem' that occurred, an aspect of teacher knowledge was identified in relation to that listening problem. If, however, at least one listening problem could not be linked to an aspect of teacher knowledge on the framework, then other factors from Wallach and Even's list could be held responsible. As no such listening problem was found, the claim that teacher knowledge could be responsible could not be refuted. So if, for example, a teacher does not evaluate a student's response (as one type of listening problem), it is reasonable to conclude that the teacher may not have the knowledge of how to go about evaluating the correctness, or otherwise, of that response. The teacher requires *specialised knowledge of content* in various aspects of statistical thinking to be able to evaluate responses, such as being able to use various representations of the data to check the answer (i.e., using *transnumeration* and *reasoning with models*).

In contrast with the listening problems discussed, Linda's classroom episodes revealed no listening problems. In instances where a student's explanation was not clear, or was incomplete (there were four such instances), Linda sought extra clarification from the student. The evidence shows that Linda had reasonably sound teacher knowledge in most areas of statistical thinking; and there is no evidence available (although sought) to negate this claim.

It is acknowledged that factors, other than teacher knowledge, contribute to listening problems of the types described, such as when it is found that the teacher has the required knowledge but did not use it in a particular situation. However, this study has provided evidence that sound teacher knowledge in various components of statistical thinking can help prevent teacher listening problems of the types described.

Teacher familiarity with the data

It was clear that adequate lesson preparation, in terms of the teacher having reasonable familiarity with the data that the students were to investigate, was important for the 'flow' of the lessons, in terms of the teacher's ability to provide appropriate responses and learning opportunities for the students. The more subjective nature of statistics, and the unpredictability of sampling results (as one example), when compared with mathematics, impacts on this need for a teacher to be ready and prepared for what might arise in statistics lessons. Being familiar with the data contributes to: *specialised knowledge of content* in relation to various statistical thinking components (thereby increasing the teacher's ability, for instance, to analyse from a statistical point of view whether a student's answer is feasible); *knowledge of content and teaching* (such as, in relation to the *interrogative cycle*, being able to guide students to formulate worthwhile questions for investigation); and *knowledge of content and students* (e.g., relating to the *investigative cycle,* knowing some of the possible findings from a data set that the students should be able to determine).

The analysis of the lessons in this study established that a teacher's lack of familiarity with the data could impact negatively on a number of different categories of teacher knowledge, thereby affecting the possible learning opportunities that are presented throughout a lesson.

Posing questions for investigation

The 'first' step of the investigative cycle, namely developing a question for investigation, presented more difficulties for students than the teachers expected. This therefore linked to a number of different teacher knowledge categories and statistical thinking components. As the first step of the investigative cycle, the posing of questions requires thinking in relation to the *interrogative cycle*. Knowing the difficulty that students may face with posing suitable questions, particularly with multivariate data sets, involves *knowledge of content and students*, and ways of guiding students through this requires *knowledge of content and teaching*. Each of these can be seen to be dependent on *common knowledge of content*. For example, John had not considered either the complexity of considering too many variables simultaneously, or its effect, not just on the question being posed, but also on the subsequent analysis of the data.

Examples were given in Chapter 6, *Discussion: Significant Themes* of situations in which a lack of appropriate teacher knowledge impeded students' investigations; and similarly, examples showed where good teacher knowledge encouraged and enabled students to make links between questions, predictions, and actual findings – a constant 'dialogue' occurred between the students and the data, involving the *investigative* and *interrogative cycles*. No research literature on the posing of investigative questions has been found. This study highlights the nature and role of teacher knowledge that is needed in relation to the *investigative cycle* and the *interrogative cycle*, specifically when considering students' posing of questions for investigation.

Differences between students' handling of category and numeric data

Previous research has shown that students find difficulty with the transition from dealing with category data to numeric data (e.g., Chick, Pfannkuch, & Watson, 2005; Nisbet, Jones, Thornton, Langrall, & Mooney, 2003). Although this was also clear in this study, this study specifically revealed the impact and importance of teacher knowledge in assisting students to develop the confidence and skills with handling numeric data. In some cases, particularly those involving association between two numeric variables, it was found that teachers lacked *common knowledge of content: transnumeration* – they were unsure of how to handle and sort the data cards for themselves to investigate possible association. Although teachers recognised the difficulty that students were experiencing (i.e., as teachers developed *knowledge of content and students: transnumeration*), generally they had no strategies available for assisting students appropriately (i.e., their *knowledge of content and teaching: transnumeration* was inadequate).

Sorting data: Moving from noticing individual data to group features and relationships

Teacher knowledge was shown to impact on whether the students were encouraged to think beyond individual data, as a way of justifying statements about a data set. It is known from the literature that students struggle with moving from a focus on individual data to being able to deal with group attributes where individual data are often no longer identifiable (e.g., Hancock, Kaput, & Goldsmith, 1992; Konold & Higgins, 2003). However, more importantly, this study found that teachers were generally unaware of this, that is, they did not have *knowledge of content and students* with regard to the challenge that students would face with regard to moving from noticing individual data to dealing with group features and summaries. Examples were given to show how various components of statistical thinking across the various teacher knowledge categories, if lacking, resulted in situations in which students' learning was disadvantaged; students were not discouraged from making statements about individual data.

In relation to developing a thorough understanding of distribution, students should be encouraged to simultaneously notice group features and individual data values (especially unusual values, or outliers for numeric data). Such an approach was often missing because of insufficient teacher knowledge. Conversely, it was shown that in situations where a teacher had sufficient knowledge of the inappropriateness of using individual data to support an argument, students were challenged by the teacher and tended to progress towards taking on a 'wider' view of group data.

There was evidence that some teachers (for example Rob), during the course of the teaching, developed knowledge of the challenges that students experienced. However, having such knowledge did not necessarily mean that teachers had the corresponding *knowledge of content and teaching* to be able to encourage the students' development with this aspect of handling data and making appropriate statements based on that data.

This study found that, in spite of the intention of the unit for students to investigate association between two variables, students often reverted to statements involving a single variable. Some teachers lacked appropriate knowledge of how to overcome such a tendency. Too often, the teachers' inadequate knowledge restricted their ability to recognise problems with students' sorting of data, or to guide the students sufficiently to deal initially with two variables, before extending to three variables. In contrast, Linda's sound knowledge that was evidenced across a number of categories enabled her to support her students (in spite of being younger than the students of each of the other teachers) to achieve good results from sorting by two variables. It is suggested that sound teacher knowledge enhanced the learning opportunities for these younger students in relation to bivariate association of category data. The variety of categories of the teacher's knowledge in relation to the statistical thinking components, and the depth of that knowledge, resulted in those younger students' learning opportunities being capitalised on and advantaged.

Students' difficulty with data-based statements

The major focus in the teaching unit was on students investigating data in order to make statements about what they found in the data. Generally the teachers did not expect, and were surprised, that the students struggled as much as they did with formulating clear statements, even when the students had ready access to the data in the form of data cards. This study found that students had a tendency to make statements that described a group of sorted data cards, rather than make statements comparing two groups of data. When the students did venture into comparative statements, these were often left incomplete, with the teachers needing to encourage the students to complete them. Such precision and accuracy with statements did not come naturally to students, and teachers had an important role to play in ensuring that students recognised the need for accuracy.

With the types of statement difficulties identified, teacher knowledge was needed in a variety of ways for assisting students with their statements. If the teacher had sufficient knowledge (*specialised knowledge of content* and/or *knowledge of content and students*) to recognise a problem with a student's statement, *knowledge of content and teaching* was required to help the student overcome the problem. A variety of strategies were identified that teachers used, or needed but did not use, in relation to this category of knowledge. One such strategy was revoicing (Forman, 2003; O'Connor & Michaels, 1996), whereby the teacher assisted the students to make the statement more accessible to others as well as him or herself. Another strategy was to use an alternate representation of the data, as an aid for students to 'see' the data in another form, and for the students to refer to in order to check the validity of the statement. Because of sorting of data by two variables, one representation that was successfully used was a two-way table. This gave the students a more permanent representation of the data than the piles of sorted data cards, and the frequencies in the table were useful and easy for referring to by the students. Often the students noticed more from the two-way table than they had noticed from the sorted data cards. A third strategy used by teachers was good modelling of appropriate language. As widely recognised in the research literature (e.g., Anthony & Walshaw, 2007), careful attention to language is critical to help students develop their intuitive understandings towards more sophisticated ideas.

When teachers used their available strategies (i.e., the teachers had appropriate *knowledge of content and teaching*) in relation to the types of difficulties that students experienced with making data based statements, students' understanding and skills related to making accurate data-based statements developed. However, it was also seen that when teachers either did not have or did not use the required knowledge, students' learning was not given the chance to develop.

Understanding variation and the development of inference

In the teaching units developed by the teachers, the development of informal inference was a goal (although the teachers may not have recognised or understood this). Two types of inference were involved: making general statements about the data set that was being investigated; and making statements about a hypothetical population, based on the data set as a sample. The difficulties for students related to: comparing groups, more so with numeric data than category data; considering a number of statistical aspects simultaneously, such as the centre of data and unusual or extreme values; and using data as a sample to make valid statements about a population. Some of these difficulties for students, and their links to teacher knowledge, have been covered in earlier sections. The aspect to be discussed in this section relates particularly to the need to consider variation when comparing groups or generalising to a population from the sample data.

There were challenges for students, and teachers, with comparing two groups to draw an inference about the data set (e.g., "Do the boys or the girls have a faster reaction time?"). Such comparisons required students to consider a number of aspects simultaneously, particularly those linked to the concepts of variation and distribution, and which include the centre and spread of data (Bakker & Gravemeijer, 2004). Through these comparisons, teacher knowledge was needed to help students push the boundaries of their understanding, through developing the ability to consider a number of relevant aspects simultaneously. Similarly, with considering the data as a sample from a population, teacher knowledge was shown to be essential if students were to develop an understanding of possible trends while, at the same time, acknowledging variation in data; and the ways in which statements about a population needed to be expressed tentatively, as possibilities rather than certainties, required such understanding on the part of the teachers.

The study showed that most of the teachers had some knowledge in relation to informal inference (when compared against the definitions of its components from Rubin, Hammerman, & Konold, 2006). It was seen that when such knowledge, although not complete and robust, was used by Linda, the learning opportunities for her students increased and were reasonably effective. It is known that young students are capable of informal inferential thinking (Watson & Moritz, 1999), and this study has shown that this is more likely to occur if the teacher has the requisite knowledge to develop the students' conceptual understanding. Conversely, it was seen that in the situations where the teacher did not have the needed knowledge, the learning opportunities were lost and students' understanding was not enhanced.

Summary

It has been clearly shown that if aspects of knowledge in relation to statistical thinking are not in evidence in the classroom, there is some impact on the potential learning opportunities for students. This was achieved through evidence being presented and discussed not only of examples of good practice (as it recognised that from a logic of learning perspective, confirming evidence is not sufficient proof), but also from examples of when knowledge was not enacted.

7.2.4 Question 4

Does teacher knowledge grow in the course of teaching? If so, what are the conditions or events that caused the growth of teacher knowledge?

Numerous situations arose in which a teacher's knowledge appeared to develop during teaching. Evidence for this was obtained from and verified by sequences of classroom incidents, or classroom incidents and follow up interviews, through which the teacher knowledge development was 'triangulated'.

The most common category of knowledge to develop was that related to students' difficulties, that is, *knowledge of content and students*. Often, the teachers did not know of or expect the areas in which the students would be challenged. Therefore, when these situations arose, the teachers realised the difficulties that the students were having, and as a result the teachers' *knowledge of content and students* developed, in relation to the relevant component of statistical thinking. In relation to Popper's theories and the logic of learning approach (Swann, 1999) adopted for this research, the teachers became aware of a 'problem'; that problem was a mismatch between their current knowledge, or expectation, of students, and what they observed students struggling with. When that mismatch was recognised, there was desire on the part of the teacher to resolve the mismatch. Therefore the conditions were 'right' for the teacher's knowledge to develop. The 'tentative solution' for the teachers was a change in their knowledge about students' handling of the current statistical task.

Some aspects of *knowledge of content and students* that developed, and which have already been discussed, were related to students' challenges when they were involved in: posing questions for investigation; sorting data cards, especially when involving numeric data, and when two or more variables were under consideration for possible association; and making statements that were valid and accurate for the data. However, when such an aspect of teacher knowledge did develop, in many cases the teacher was faced with a problem of inadequate *knowledge of content and teaching* in relation to that newly developed *knowledge of content and students.* The teaching knowledge was inadequate because the teacher had not needed it previously, due to a lack of awareness that students typically had difficulties with the particular concept or skill. Again, in relation to Popper's theories, the teacher became aware of a mismatch between the state of their current knowledge of content and teaching (or in this case, the inadequate level of it) and the current experience (of realising that some knowledge of content and teaching was required). The tentative solution to this problem was that the teacher had nothing to replace the inadequate knowledge at that time, and so the current state of affairs was maintained.

One example of the development of teacher knowledge was briefly discussed earlier in relation to Rob and the students' difficulty with moving from a focus on individual data to dealing with group features. Another example was with John, when he became aware of students' difficulties with sorting and considering bivariate data (*knowledge of content and students*: *transnumeration*). He had found that many of the statements made by students focused on a single variable. Through this recognition, he realised that he needed a strategy to help students consider two variables simultaneously. His 'tentative solution' to this was to draw a data card on the board. He explained to students that, rather than finding something like, "There are 8 boys, there are 10 girls …", they should see whether there was some sort of relationship between two or three parts of the data, and at this point he drew some arrows linking adjacent variables (see Section 6.6.3 and Figure 6-2). Then he suggested that they should see if they could organise the data squares, maybe by moving them around on the desk and putting them into different groups. The diagram and the explanation constituted John's tentative solution, as new *knowledge of content and teaching*. It was an attempt, albeit a relatively unsuccessful one, to help the students with their sorting of the data cards by two variables. Subsequently, after further sorting of cards and statements from students, John again was aware of the inadequacy of his tentative solution of using the arrows to help suggest relationships to be examined. He knew that he would have to try another, clearer way of explaining and encouraging the students to consider and sort two variables (i.e., he would have to develop another tentative solution for his problem).

The various situations that provided evidence of the growth of teacher knowledge showed that it can and does develop through teaching. However, the categories of knowledge in which growth occurred were limited to *knowledge of content and students*, and to a lesser extent, *knowledge of content and teaching*. This suggests that the category of *common knowledge of content* is not likely to develop through teaching. The situation with *specialised knowledge of content* is less clear than for *common knowledge of content*. It could be possible, that as a teacher's *knowledge of content and students* develops (with regard to difficulties that students encounter), a teacher may develop *specialised knowledge of content* that better enables a teacher to, for instance, evaluate a student's explanation with regard to its statistical merit. In this study however, there was no clear evidence of the development of *specialised knowledge of content*.

In this section, some examples have been discussed that illustrate the dynamic nature of knowledge (Cochran, DeRuiter, & King, 1993; Fennema & Franke, 1992; Hiebert & Carpenter, 1992; Manouchehri, 1997), and its evolution through teaching. Further examples of knowledge development were discussed in previous chapters (not always in such explicit terms, but certainly with evidence that it was occurring), in relation to the particular context in which that knowledge was needed and/or used. It can be claimed, therefore, that this study has shown that teacher knowledge is dynamic, and its growth is responsive to classroom conditions, such as student questions or comments.

7.2.5 Hypothesis

All aspects of teacher knowledge in relation to the components of statistical thinking are necessary for the work of teaching statistics through investigations, and the *absence of any aspect will impact negatively on the learning opportunities for students.*

The discussion in this chapter pertaining to the four research questions has indicated that this study has provided good evidence that all components of statistical thinking, with the exception of dispositions, are necessary for the work of teaching statistics. Also, from the discussion of missed opportunities, it is strongly argued that the absence of appropriate teacher knowledge can impact negatively on the learning opportunities for students. Similarly, evidence has been sought that might refute this hypothesis, but that evidence was not found.

It is acknowledged that any component from the framework will have a number of aspects associated with it, particularly when the multiple phases of a statistics investigation are considered. In the teaching of statistics investigations with a particular class and at a particular level, some aspects of statistical thinking relevant to a category of teacher knowledge may not be needed. When the necessary knowledge was examined across all four teachers in this study, it was clear that not all teachers needed the same knowledge for their teaching. However, it can be claimed that for teaching over a longer period of time than four lessons, and with other changes in the teaching context (such as differences in the students, time of the year, age level, data being used by the teachers or students, or teaching approach), a fuller extent of knowledge would be needed by each teacher than was observed for that teacher teaching those lessons to that particular group of students.

It might be suggested that a lack of content knowledge on the part of a teacher could be compensated by other pedagogical strategies. Such a suggestion, if true, would mean that teachers would not need some categories of knowledge. For instance, when a student responds to a question, and the teacher does not have the required *specialised knowledge of content* to evaluate that response, by putting the student's response back to the class for their consideration and debate, the teacher has mitigated the effects of inadequate specialised knowledge of content. In such a situation, the other students may provide a justification for why the response is valid or otherwise. However, without the teacher having suitable *specialised knowledge of content*, it would be possible for a student, who presents a convincing but invalid argument, to mislead the rest of the class and therefore contribute to the development of misconceptions. So, even in a classroom environment that encourages full participation in discourse practices, the teacher requires *specialised knowledge of content* in order to know whether a student's response is valid, and therefore be able to make a teaching decision as to whether intervention, on the part of the teacher, is needed. This example, and the discussion of missed opportunities and themes, suggests that other general pedagogical strategies cannot compensate for inadequate teacher knowledge.

7.3 Contribution

It is often suggested in the research literature that research on teacher knowledge should be conducted in the context in which that knowledge is used (Ball & Bass, 2000; Barnett & Hodson, 2001; Borko, Peressini, Romagnano, Knuth, Willis-Yorker, Wooley et al., 2000; Cobb, 2000; Cobb & McClain, 2001; Fennema & Franke, 1992; Foss & Kleinsasser, 1996; Friel & Bright, 1998; Marks, 1990; Sorto, 2004; Vacc & Bright, 1999). However such a suggestion is often followed by an acknowledgement that that type of research is difficult to conduct, and beyond the scope of the research being reported. This study, by conducting research on teacher knowledge in the classroom in which that knowledge is used, has provided a significant contribution to the research field. Literature searches have been unable to locate any other research in statistics education that focuses on teacher knowledge at the primary school level, and that is classroom based. This study therefore appears to be unique, and provides important insights to what knowledge a teacher needs for teaching statistics, based on the reality of the classroom context.

A case has been made and argued for a framework that can be used to investigate the nature of teacher knowledge needed for and used in the classroom. The framework has been shown to be a useful model for determining the knowledge needed for teaching statistics through investigations. Using investigations to teach statistics is recommended by recent research, and has been adopted in New Zealand, as well as a number of other countries. The use of the framework, as well as enabling the identification of knowledge that was used in the classroom, also enabled the identification of knowledge that was needed, but not used. The consequences for students' learning, in relation to the non-use of teacher knowledge, were discussed.

The study revealed findings that add to the available research knowledge about teacher knowledge in relation to statistics investigations.

7.4 Limitations

This study is based on one researcher's interpretations of data from classroom video recordings and stimulated recall interview audio recordings. Other interpretations may be possible. However, the researcher's experience as a teacher, and teacher educator, and background of mathematics and statistics, along with the study's approach of searching for disconfirming evidence of conjectures, means that the potential for major flaws in interpretation has been minimised.

With regard to the classification of knowledge through the framework, the category of *specialised knowledge of content* provided challenges in differentiating it from *common knowledge of content*. It is not possible, with what is known or not known about the common statistical knowledge of the 'typical' educated person, to be certain about the boundaries between *common knowledge of content* and a teacher's *specialised knowledge of content*. The research literature documents a considerable amount about statistical misconceptions, and the general need for a greater level of statistical literacy in today's world (e.g., Ben-Zvi & Garfield, 2004). This study's classification of and distinction between these two categories of teacher knowledge may need redefining. The mathematics education research has, with regard to mathematical concepts, described classroom based scenarios that mathematicians were not able to make sense of in the same way that teachers could (e.g., Hill et al., 2004). This provided clear evidence of the existence of *specialised knowledge of content* for the relevant areas of mathematics. Further research in the statistics domain would be needed to clarify the distinction between *common knowledge of content* and *specialised knowledge of content*. The suggested differences between these two categories, as discussed mainly in the *Results* chapter, are therefore tentative until proven inadequate.

The small convenience sample of four teachers indicates that broad generalisations to the teaching community could be fraught. However, the findings are presented in such a way, that disconfirming evidence, if found, would reveal those findings as inadequate and ready to be replaced by a new tentative theory. Also somewhat countering the possible limitation due to the sample size are the benefits obtained from being able to examine data across the four teachers, for similarities and differences. As was discussed in Chapter 3, *Methodology in Theory*, this exemplifies the use of cross-sectional time triangulation (Cohen, Manion, & Morrison, 2000). The comparison of data from the four teachers enabled a greater level of confidence in the conclusions that could be drawn about teacher knowledge.

No research can be conducted in the classroom without being intrusive, and therefore impacting on that classroom environment. It is impossible to determine the research's effect on the learning and teaching in the classrooms in which this research was conducted. Would the teacher and the students have acted differently had the researcher not been there with a video camera? What effect did this presence have on the knowledge used and needed by the teacher? Such questions are not possible to answer. However, rather than having only negative effects, the research process could also have had positive effects on what was being studied. The post-lesson stimulated recall interviews gave the teachers the opportunity, among other things, to view and reflect on classroom incidents, to share ideas and interpretations of what was occurring, to consider alternative strategies, and to seek advice about future lessons. Such opportunities may well have contributed to different practices in subsequent lessons from what might have happened should that viewing, reflecting, and sharing have not occurred. The research process created an intervention in the usual teaching and learning sequences. This short-term intervention may have had some negative effects, but as argued, may have also resulted in positive effects for the teacher (and therefore the students). A hypothetical question arises: Would any of the teachers' future teaching on statistical investigations benefit from this research intervention, given the input that the teacher received from the researcher and the research process?

7.5 Implications and further research

This study focused on the knowledge for teaching statistics of teachers early in their teaching careers. As the teachers were all in their second year of teaching, and therefore relatively inexperienced, their knowledge profiles have clear implications for initial teacher education. Although some of their current knowledge could be attributed to development from the teachers' teaching experience or from knowledge that developed prior to their initial teacher education, the role and responsibility of initial teacher education programmes is critical. This study's findings can provide guidance for what particular aspects of knowledge development should be the focus of initial teacher education programmes. As most initial teacher education students have not had the advantage of learning statistics through investigations, their *common knowledge of content* should be developed through immersing the students in investigations. As their *common knowledge of content* develops, their *specialised knowledge of content*, particularly for listening to and making sense of students' responses, will develop. The use of videos showing students involved in aspects of investigations, especially making data based statements, would be particularly useful for helping the development of initial teacher education students' knowledge for listening to and making sense of school students. *Knowledge of content and teaching* (e.g., teaching sequences, advantages and disadvantages of various alternative data representations, and knowing how to respond from a statistical viewpoint to students' ideas, especially the unconventional ones) is dependent on *knowledge of content and students* (e.g., understanding the aspects of investigating data that present particular challenges for students, knowing the common misconceptions, or errors that students are liable to make). Although this study showed that *knowledge of content and students* develops in the classroom (particularly recognition of challenges for students), initial teacher education students are unlikely to be in the situations in which such knowledge would develop. Consequently, these two categories of knowledge should also be a focus in initial teacher education programmes. Overall, all aspects of teacher knowledge must be targeted, as the connections between the categories of knowledge mean that individual categories of knowledge cannot operate in isolation.

Teaching statistics through investigations is a recent development in school statistics curricula. As most experienced teachers would have had little opportunity to teach statistics in this way, there are implications for teacher professional development, irrespective of the length of teaching experience of the teachers. Targeting teachers' professional development in relation to *knowledge of content and students* and *teaching* simultaneously with building their own *common knowledge of content* through investigations (and consequently also *specialised knowledge of content*) is considered an optimum approach, which other research findings support (Timperley, Wilson, Barrar, & Fung, 2007 forthcoming). In fact, Timperley et al.'s review of the professional development literature showed overwhelmingly that no professional development that focused solely on general pedagogy was successful in raising the achievement levels of students, and conversely that the most successful professional development, in terms of student achievement, involved the development of both the content knowledge and the pedagogical content knowledge of teachers (and that this latter category was particularly critical). Similar to the recommendations above for initial teacher education, this study's findings provide a strong argument regarding the content and approach of professional development programmes.

Further research could: broaden to investigate teacher knowledge in statistics at other school levels; develop assessment items to measure the various categories of teacher knowledge in statistics; and examine and measure the effect of teacher knowledge on student outcomes.

7.6 Final word

This thesis set out to examine the nature of teacher knowledge needed for and used in the teaching of statistics. The evidence showed that such knowledge exists and is needed in the classroom. It also showed, very importantly, that when it is missing, students' learning opportunities are affected. The presence of certain types of knowledge cannot adequately substitute for the missing components.

This study has increased our knowledge of teacher knowledge. In the words of Goethe, the German novelist, theorist, humanist, scientist, painter, and polymath:

Knowing is not enough; we must apply!

http://www.wisdomquotes.com/cat_knowledge.html

This thesis has argued how the increased knowledge gained from this study can be applied to initial teacher education and the professional development of practising teachers, and how that knowledge contributes to the research field.

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Appendix 1
Information sheets and consent forms

The following Information sheets and consent forms are attached:

- Information sheet for Principal/Board of Trustees and Teacher
- Consent form for Teachers
- Information sheet for Parents/Caregivers
- Information sheet for Students
- Consent form for Students and Parents/Caregivers

Information Sheet – Principal/Board of Trustees and Teacher

My name is Tim Burgess. I work at Massey University's College of Education as a senior lecturer in the School of Curriculum and Pedagogy. As part of my doctoral research, I intend to undertake a project that will look at aspects of teachers' knowledge related to the teaching of statistics.

For this research project, I would like to be in a classroom for a week, watching some mathematics lessons. I will use a video camera to record the teacher teaching a Statistics Unit to the class. After videoing the lessons, the teacher and I will look at the video and talk about things related to their teaching. This will be audiotaped. The teacher will also be asked to provide copies of lesson plans of the lessons to be videotaped.

The teacher or the principal will have the right at any stage to terminate their involvement in the project.

It is possible that while videoing the teacher, children may appear on camera whilst talking to the teacher. Therefore I will be seeking the approval of the children as well as their parents/caregivers to be involved in this pilot study. The following points will be followed:

- Any child can withdraw from being involved at any stage, without having to give the teacher or me any reasons.
- Any child can ask at any stage to not be videoed.
- The teacher, my supervisors and I are the only people who will see the video.
- There will be no reports written in which any child, the teacher or the school could be identified.
- My supervisors or I can be contacted at any stage (details are below) with any questions about the study.

The teacher and I will make arrangements (such as through re-arranging the seating) to minimise the possibility of recording any children for whom written permission is not received from themselves or their parents/caregivers. In the event that any of these children are recorded, post-editing of the video will occur, so that their image is not recognisable, and anything they say will not be used in the data analysis.

This project has been evaluated by peer review and judged to be low risk. Consequently it has not been reviewed by one of Massey University's Human Ethics Committees. As the researcher, I am responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you wish to raise with someone other than me, please contact Professor Sylvia Rumball, Assistant to the Vice-Chancellor (Ethics and Equity),

phone 06 350 5249, email humanethicspn@massey.ac.nz.

If you would like further information about the study, you can contact me or my supervisors:

Tim Burgess, School of Curriculum and Pedagogy, ph. 350 5799 ext 8863, email t.a.burgess@massey.ac.nz

Supervisors:

Dr Margaret Walshaw, School of Curriculum and Pedagogy, phone 350 5799 ext 8782, email m.a.walshaw@massey.ac.nz

Dr Glenda Anthony, School of Curriculum and Pedagogy, phone 350 5799 ext 8600, email g.j.anthony@massey.ac.nz

Consent Form - Teacher

I have discussed the research project with Tim Burgess and I fully understand the purpose and extent of the project, and accept the intended level of my involvement:

- Providing a copy of lesson plans or a unit plan prior to the teaching of the lessons;
- Being videotaped while teaching a sequence of three or four statistics lessons;
- Being interviewed (and audio taped) in relation to my viewing of parts of the videotaped lessons as well as the lesson plans.

I give my approval to be involved in the study, on the conditions outlined below:

- The principal/Board of Trustees has given written consent for the project to be carried out in my classroom.
- I can ask Tim or his supervisors any questions about the study.
- The only people who will see the video other than myself will be Tim and his supervisors.
- There will be no reports written in which the school or I could be identified.
- I can withdraw from being involved at any stage, without having to give any reasons.

Signed:………………………………………………. Date: ……………………….

Information Sheet – Parents/Caregivers

My name is Tim Burgess. I work at Massey University's College of Education as a senior lecturer in the School of Curriculum and Pedagogy. As part of my doctoral research, I intend to undertake a project that will look at aspects of teachers' knowledge related to the teaching of statistics.

For this research project, I will be in your child's classroom for a week, watching some mathematics lessons. I will be using a video camera to record the teacher teaching a Statistics Unit to the class. After videoing the lessons, the teacher and I will look at the video and talk about things related to their teaching.

It is possible that while videoing the teacher, your child may appear on camera whilst talking to the teacher. Therefore I would like to ask for your approval for your child to be involved in this pilot study. If you agree to this involvement, the following points will be followed:

- Your child can withdraw from being involved at any stage, without having to give the teacher or me any reasons.
- Your child can ask at any stage to not be videoed.
- You or your child can ask my supervisors or me (details are below) any questions about the study.
- The teacher, my supervisors and I are the only people who will see the video.
- There will be no reports written in which your child could be identified.

To give your consent for your child to be involved, the attached consent form must be signed and returned to your child's teacher. If you do not return the sheet by ……………., the teacher and I will make arrangements (such as through re-arranging the seating in the classroom) so that your child will not be included in the video recordings of the Mathematics lessons. If your child happens to be recorded, I will edit the video so that your child cannot be recognised, and anything your child says will not be used.

This project has been evaluated by peer review and judged to be low risk. Consequently it has not been reviewed by one of Massey University's Human Ethics Committees. As the researcher, I am responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you wish to raise with someone other than me, please contact Professor Sylvia Rumball, Assistant to the Vice-Chancellor (Ethics and Equity),

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Information Sheet - Students

My name is Tim Burgess. I work at Massey University's College of Education as a senior lecturer in the School of Curriculum and Pedagogy. As part of my research for my doctorate, I intend to undertake a project that will look at aspects of teachers' knowledge related to the teaching of statistics.

As part of my research, I am going to be in your classroom for a week, watching some mathematics lessons. I will be using a video camera to record your teacher teaching a Statistics Unit to your class. After videoing the lessons, your teacher and I will look at the video and talk about things related to their teaching.

It is possible that while videoing the teacher, you may appear on camera while you are talking to the teacher. Therefore I would like to ask for your approval to be involved in this pilot study. If you agree to be involved, the following points will be followed:

- You can withdraw from being involved at any stage, without having to give your teacher or me any reasons.
- You can ask at any stage to not be videoed.
- The only people who will see the video are your teacher, my supervisors, and me.
- There will be no reports written in which you would be able to be identified.
- You can ask my supervisors or me (details are below) any questions about the study.

To give your consent to be involved, the attached consent form must be signed and returned to your teacher. If you do not return the sheet by ……………., your teacher and I will make arrangements (such as through rearranging the seating in the classroom) so that you will not be included in the video recordings of the Mathematics lessons. If you happen to be recorded, I will edit the video so that you cannot be recognised, and anything you say will not be used.

This project has been evaluated by peer review and judged to be low risk. Consequently it has not been reviewed by one of Massey University's Human Ethics Committees. As the researcher, I am responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you wish to raise with someone other than me, please contact Professor Sylvia Rumball, Assistant to the Vice-Chancellor (Ethics and Equity),

phone 06 350 5249, email humanethicspn@massey.ac.nz.

If you would like further information about the study, you can contact me or my supervisors:

Tim Burgess, School of Curriculum and Pedagogy, ph. 350 5799 ext 8863, email t.a.burgess@massey.ac.nz

Supervisors:

Dr Margaret Walshaw, School of Curriculum and Pedagogy, phone 350 5799 ext 8782, email m.a.walshaw@massey.ac.nz

Dr Glenda Anthony, School of Curriculum and Pedagogy, phone 350 5799 ext 8600, email g.j.anthony@massey.ac.nz

Consent Form for Student and Parent(s)/Caregiver(s)

Student

I, ………………………………………………………………... have read and understand the Information Sheet about the research project to be carried out by Tim Burgess. I give my approval to be involved in the pilot study, on the conditions outlined below:

- I can withdraw from being involved at any stage, without having to give any reasons.
- I can ask at any stage to not be videoed.
- I can ask Tim Burgess or his supervisor any questions about the study.
- The only people who will see the video are the teacher, Tim Burgess and his supervisors.
- In any reports written about the pilot study, it will not be possible to identify me.

Signed

Student: ……………………………………………… Date: ……………………..

Parent(s)/Caregiver(s)

I/We, ………………………………………………………………… have read and understand the information sheet about the pilot study to be carried out by Tim Burgess. I/We give approval for ……………………………………………….. to be involved, based on the conditions outlined above.

Parent/caregiver ……………………………………… Date: ……………………..

Parent/caregiver ……………………………………… Date: ……………………..

Appendix 2
Unit Plan

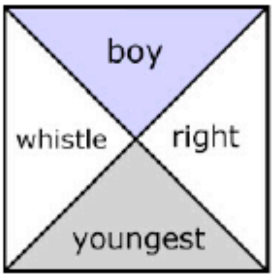
2.1 Introduction

The following unit plan was obtained from the nzmaths website (Ministry of Education, 2006) and used as the basis of the units taught by the four teachers.

2.2 Unit Plan

Data Squares

Statistics, Level 3



Overview of Unit

This unit provides a way of looking at information from a group of individuals. Data squares hold several pieces of information about individuals. Sorting and organising a set of data squares allows things to be found out about the group and questions about the group answered. The data squares allow students to consider issues needing more than one category to be considered at the same time.

Relevant Achievement Objectives

- collect and display discrete numeric data in stem-and-leaf graphs, dot plots, and strip graphs, as appropriate
- use their own language to talk about the distinctive features, such as outliers and clusters, in their own and othersâ€™ data displays
- make sensible statements about an assertion on the basis of the evidence of a statistical investigation

Specific Learning Objectives

- collect information
- sort information into categories
- answer questions by sorting, organizing and arranging information
- make sensible statements about the information with supporting evidence

A description of the mathematics explored in the unit

This unit focuses on sorting and organising data sets, i.e. collections of information from a group of individuals. Looking at the data, sorting and organising it first, with things of interest and questions arising from this. This is a different approach than starting with a question then collecting data to see if it is correct. The data squares allow students to consider issues needing more than one category to be considered at the same time.

Understanding the difference between individual data and group data is central to the unit. The goal is to have students make statements about the group in general.

Increasing studentâ€™s ability to accurately describe aspects of a data set, including developing statistical vocabulary, is part of the unit. As students become comfortable with making statements and describing data, more precise vocabulary is to be encouraged.

Resources

- paper and pencils
- scissors
- Data Set One Master
- Data Set Two Master
- Data Set Three Master
- Blank Data Square

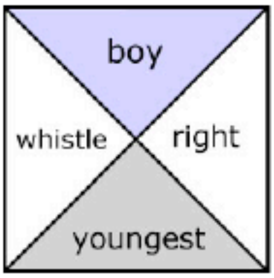
Teaching Sequence

Session One

Part One â€" Making Class Data Squares
Show the following data square to the class and explain what a data square is, i.e. a square piece of paper contains four pieces of information about one person.

The information on this data square comes from four questions:

- Are you a boy or girl?
- Can you whistle?
- Are you the oldest, youngest or a middle child in your family? (Only children are classified as oldest)
- Which hand do you write with to produce your neatest work?



Ask the class to tell you something about this student.

- Does anyone in the class fit this data square?
- Do you know someone that fits this data square not in this class?
- How many people could this data square be correct for?

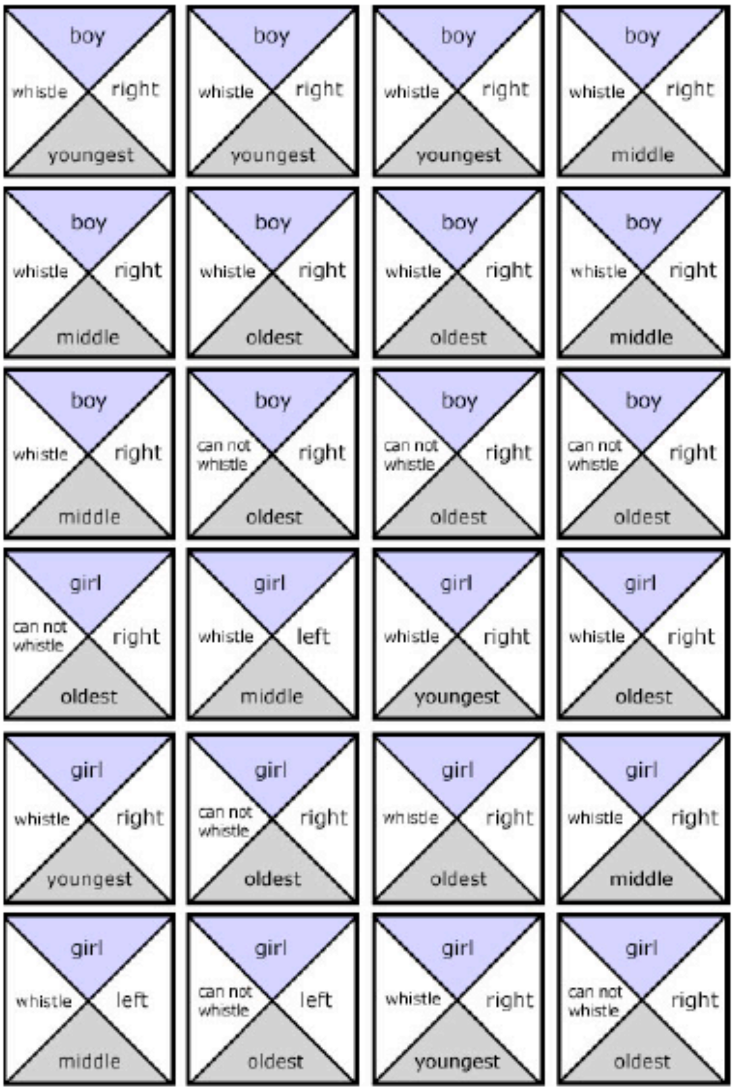
Discuss the importance of knowing exactly what each piece of data is about, i.e. the importance of specific questions. Discuss how some students could answer the same question differently, e.g. "Are you right handed or left handed?" could give two different answers for the person who throws a ball with one hand and writes with the other. Questions need to be specific, with no ambiguous answers.

What would a data square about you look like?

Hand out a blank data square for each student to fill out. Once completed collect all data squares.

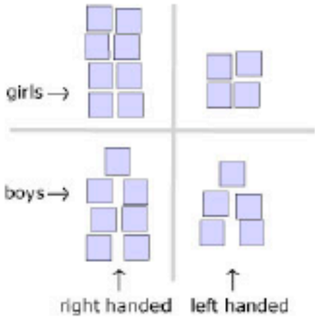
After this session the teacher needs to photocopy all the data squares onto a piece of paper, one set for each pair of students in the class. Photocopying onto coloured paper is suggested to make it easy to recognise the classâ€™s data set. This data set will be used during the next session.

Part Two â€" Data Set One
Organise the students into pairs, hand out to each pair a set of data squares, Data Set One Master, and get them to cut out all the data squares. Once cut out, have the student sort and organise the data squares to find out things about this group of students.



Encourage the students to look for multi dimensional interesting things. This means looking for interesting things within different categories rather than simply counting the number in categories. For example, rather than seeing if there are more girls than boys or more whistlers than non whistlers, look to see if more boys than girls are left handed or if there is a link between place in family and the ability to whistle.

Arranging the data squares like below, is one way to help see things in the data.



The teacher is to move around getting the students to explain and show what they have found out. The teacher is to encourage students to add detail to their observations. This could include thinking proportionally. For example, rather than "One more girl is right handed than boys", "A larger proportion of girls are right handed, 8 out of 12 girls in comparison to 7 out of 12 boys are right handed." More able students are to be encouraged to think proportionally when the number in comparing groups is not the same, e.g. 8 out of 20 is a small proportion than 7 out of 9.

The following question could be asked to encourage thinking:

- Are there more whistling right-handers or whistling left-handers proportionally?
- Is there anything interesting when comparing place in the family and whistling?
- All the boys in this group who are the youngest can whistle, does this mean every boy who is the youngest in their family can whistle?

On a large piece of paper write up what the students discover or get each pair of students to write down what they found out about this group. Keep this information, as it can be used in the next session to compare with the class data set.

Session Two

During this session, students will be sorting and arranging data squares about themselves, i.e. the students own data squares. Before the class data set is handed out, remind the students about what they found out about Data Set One in Session One and how they organised the data square to see things. Briefly discuss what they expect to find out about their class.

- What do you expect to find out about the class?
- Will the things we found out from Data Set One, be different or similar to our class?

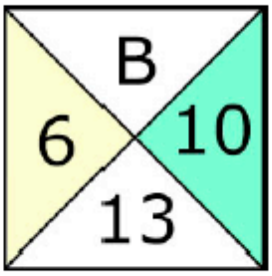
Hand out a set of class data squares to each pair of students. The pairs are to cut out the data squares, sorting and arranging them to look for things of interest. The teacher is to move around getting students to explain and show what they have found out.

Conclude the session by considering the statements the students made at the beginning of the session and sharing other things of interest.

Session Three

Show the following data square to the class, telling them it is information from a student in a class like ours, then ask them the following questions:

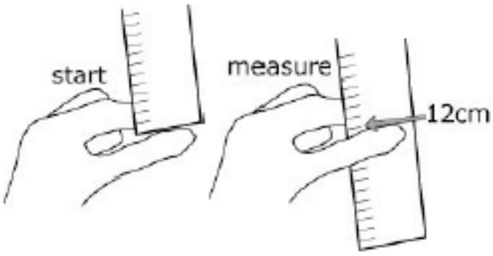
- What do you think the letter and numbers mean?
- Why are letters and numbers used instead of words?
- What specific questions could give the answers: B, 6, 10 or 13?



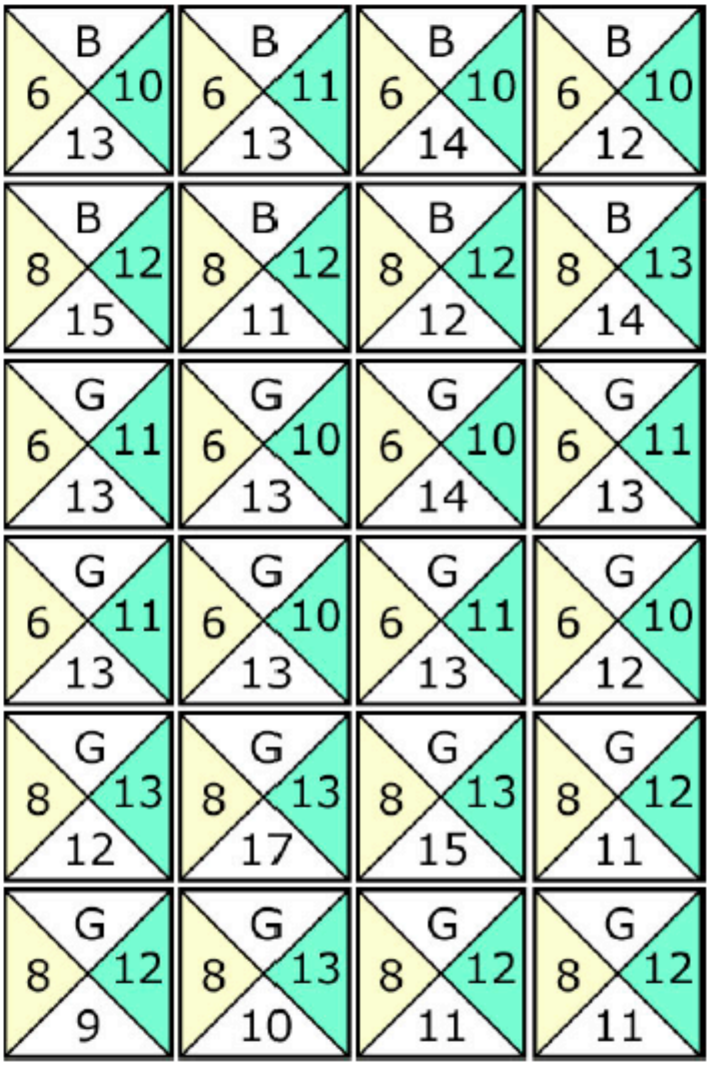
Explain that the four questions from this data square are:

- Are you a boy or girl? - B
- What year level are you at school? â€" 6
- How many years old are you? â€" 10
- What is your reaction score for catching a ruler? â€" 13

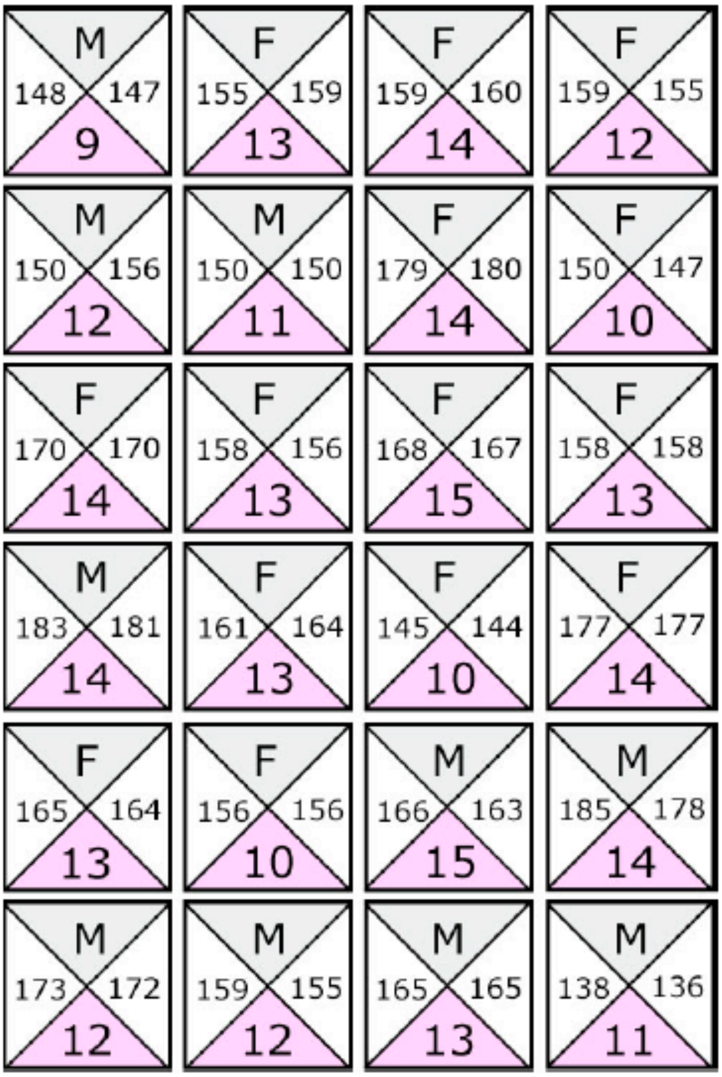
The reaction score is the average length a ruler falls, before being caught, when it is dropped four times. To work out the reaction score, one student holds a ruler vertically above the test studentâ€™s first finger and thumb; the bottom of the ruler is in line with the top of the thumb. The ruler is released and the test student closes their finger and thumb as quickly as they can to catch the ruler. The number of centimetres the ruler falls through the finger and thumb is the score. This is repeated four times, with the scores averaged to give the reaction score. For example, if the ruler fell 12 cm first time, 15 cm second time, 11 cm third time and 14 cm fourth time, the average is 12 + 15 + 11 + 14 = 52, 52 ï¸ 4 = 13, therefore, the reaction score is 13.



Hand out a set of data squares, Data Set Two, to each pair of students. The pairs are to cut out the data squares, sorting and arranging them to look for things of interest. The teacher is to move around getting each pair getting students to explain and show what they have found out.



Data Set Three â€“ Optional
A third data set has been included for teacher wishing to repeat the activity in this session. The data for this set was obtained from:
www.censusatschool.org.nz.
Data Set Three is a data set of 24 students. The data is: top â€“ male/female, left â€“ arm span in cm, right â€“ height in cm, bottom â€“ age in years.



Session Four

Today the students, in pairs, will design and compile their own data square set. Each pair of students needs to design three questions to ask 24 other students in the class. The first question will be "Are you a boy or a girl?" with three new questions added.

Discuss and brain storm suitable questions

Sample questions:

- How many centimetres tall are you?
- How many centimetre is your right hand?

Specific instructions will be needed with questions like this, so it is clear where to start and finish measuring.

- How fast can you run 100m?
- What is your favourite . . . ?

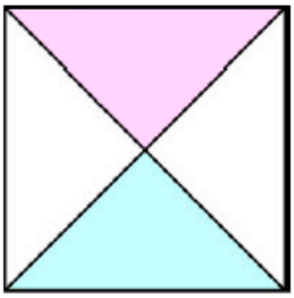
  A list of possible favourites to select from is best with questions like this.

- What time did you go to bed last night?

  When organising the data from questions like this, categories may be needed, e.g. before 8 pm, 8 to 9 pm, 9 to 10 pm, 10 to 11 pm and later than 11 pm.

Before starting to collect data each pair of students needs to write three statements about what they expect to find out about the class.

Each pair of students needs to collect information and make 24 data squares from students in the class.

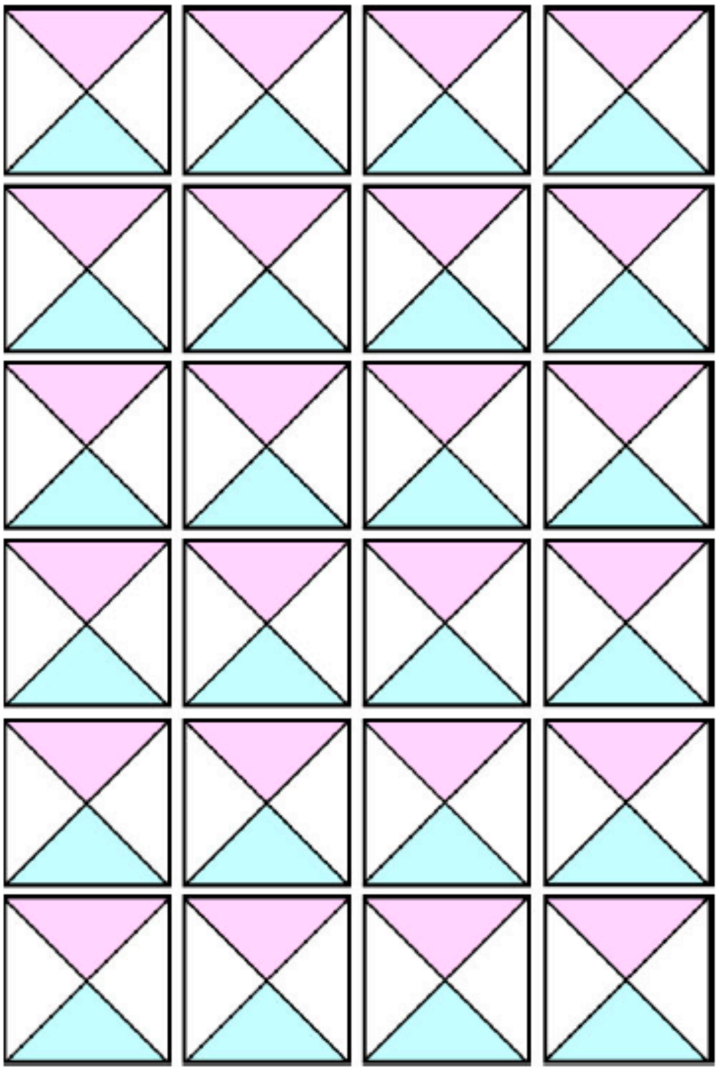


At this point, teachers may wish to discuss the likely difference in results between randomly selecting 24 students from the class and hand picking 24 friends. A quick example is a good way to illustrate this point at this level of the curriculum. The point to get across is that hand picking students to answer a question can give a misleading impression of the class, if it is assumed that it is representative of the whole class. For example, the teacher selects five rugby-loving boys in the class and asks them to name their favourite sport. All the boys are likely to say rugby, with the resulting statement make, "Everyone answered rugby, so the favourite sport in the class is rugby" or "Everyone in this class loves rugby".

Once the data squares are completed, students are to sort and arrange them to look for things of interest. The pairs of students are to prepare a brief report of the things they have found out.

Session Five

This session has the students compiling a new set of data squares based on questions they develop themselves. The questions could be comparing information from the data sets presented during this unit, looking at new information or asking a different group of student the same questions.



Appendix 3

Data Detective Poster

This resource was given to the teachers to use in their classes.

Source: http://www.censusatschool.org.nz

Appendix 4

Sample transcripts and notes from Annotape

Explanation

The following are samples of the transcripts and notes from the video or interview data for particular lessons, which were coded using Annotape software. The text in Annotape notes were printed to PDF files, one file per record (video or audio), from which this Appendix was generated.

The samples include the transcripts and notes from one video and its associated interview for each teacher. The names of the teachers and the lesson numbers for the samples are as follows:

Linda (School 1)	Lesson 4
John (School 2)	Lesson 2
Rob (School 3)	Lesson 2
Louise (School 4)	Lesson 2

Title: Sch1Lesson4E.mov

Type: Video
Time: 29-Aug-06 9:06

KCS: ReasonModels

S talks to T about the sorting of the cards. He had two piles - all girls who like Fords and Play Station 2 (PS2) and all the boys who like Holdens
and X-box.
T asks question: Does it go that way? ... If you sort the boys who like Holdens, is it a given that they like X-box as well better than Play station? ...
T was trying to make sense of why the student had sorted into those two quite different sets to compare. T pointed to a card for a boy who like
Holdens but also like PS2.
T suggests that S look at boys and girls who like Holdens and boys and girls who like Fords. Then go into whether they like X Box or PS2.
S sorting was incomplete so T was having to make sense of what he was thinking in relation to the sorting he had done.

SKC: InvestCycle

T notices that she is sorting cards into two piles, one with children who like to run in barefeet and the other with children who like blue.
T realises that this sorting is inadequate for making comparions later on. T asks S questions about what the data collection was and what were the
alternative responses that children could give. It was then suggested by the T that these two alternatives should be used as the basis for the first part
of the sorting. Then later, the second variable could be used to further sort the data and make comparisons between the groups.

KCS: ReasonModels

T interprets S incomplete comparison statement about the data.
S had cards arranged as:
Boys Girls
IceAge2 4 4
Madagascar 2 2
S says: There are more boys and girls who like IceAge2 ... and two boys and girls who like ...
T: So there are the same ...
S: number of boys and girls who like IceAge2, and the same number of boys and girls who like Madagascar.
T: Which one is the most popular of those two?
S: IceAge2.
T helps the student to work through valid statements of comparison for the data.
statement difficulty
no extra clarification - positive case

KCT: Transnum

T puts a 2x2 factor table on the board for the class, as a representation of S sorting of cards.

KCT: ReasonModels

T encouraging stduents to make statements based on transnumerated data (into 2x2 factor table showing frequencies).
T also encouraged and modelled proportional thinking. T asked questions to get Ss to focus on the proportions.

KCS:Transnum

T tries to interpret incomplete S statements based on sorted cards. T knows that this students has difficulty with verbal/written statements, so
recommends he create a 2x2 table of the data so that he can use that to help formulate statements.
T helps S to identify what column and row labels that could be used with the 2x2 table.
T then shows the class the 2x2 table that would be used to help show the data.
statement difficulty
no extra clarification - positive case

KCT: ReasonModels

T shows class the 2x2 table that S had been encouraged to draw to show the data. Then T asks Ss in class to make statements about the data using
the 2x2 table.
Number of children who like Daniel Carter or Jerry Collins
Boys Girls

Daniel Carter 3 5
Jerry Collins 3 1
S: There are a greater number of girls who like Daniel Carter than boys who like Daniel Carter.
T: There are more girls who like Daniel Carter than ... what else?
S: There are more boys who like Jerry Collins than girls who like Jerry Collins.
S: There are more boys and girls who like ...
T: There are more girls who like what?
S: There are more girls who like Daniel than Jerry.
T: Yes. So there are 3 or 4 things that you could write underneath that diagram.
KCS: ReasonModels
T tries to make sense of incomplete S statement. T appears to misinterpret the S statement.
Missed opportunity
no extra clarification - positive case, asks student to repeat and clarify
statement difficulty
KCT: Variation
T encourages inferential thinking and asks Ss to justify whether a generalisation can be made.
Acknowledges that variation could occur in the data
but that the trend could be found in the larger group.
KCT: ReasonModels
T notices patterns in data that would be suitable for encouraging inferential statements. Encourages Ss to make statements about the data, then
pushes Ss to consider whether the statements could be generalised to the whole class.
KCS: ReasonModels
T tries to interpret S statement based on incomplete/incorrect sorting of 3 variables at a time.
T listens to S statements; T notices that some cards sorted incorrectly. S statement about the people sorted by 3 variables (kittens, soccer and
Holdens) and compared with other responses (puppies. rugby and Fords). T talks with S about this statement of comparison does not tell us much -
same frequency in each group.
T asks S to suggest that to compare two things would make the comparisons much better and easier. So S suggests Fords/Holdens with girls/boys.
After resorting, the S now has:
Fords Holdens
Girls 4 2
Boys 4 2
Now S can see that same number of girls and boys like Fords, ditto for Holdens.
T asks: Which is the most popular car?
S: Fords
T; How do you know?
S: Because that's my favourite car.
T: No, how do you know that that is the more popular car?
S: Because there are 4 and 4 making 8, but for Holdens there only 2 in each making 4.
T encourages S to see double the number ofpeople like Fords compared with Holdens. Or, half the number of people like Holdens compared with
Fords. T also encourages S to notice that double the number of girls like Fords compared with those who like Holdens, and same for boys.
statement difficulty - positive case
KCS:Transnum
T tries to interpret S statement based on incomplete/incorrect sorting of 3 variables at a time.
T suggests that to make comparisons between groups, it would be easier to sort by just two variables to start with.
S resorts data cards, into Holden/Ford, boy/girl. After resorting, the S now has:
Fords Holdens
Girls 4 2
Boys 4 2
This helps the S make good statements of comparison between various groups.
statement difficulty
KCT: ReasonModels
T, after making sense of S statements, encourages S to make inferential statement about boys in the school who might like to watch Sticky TV (based
on her data of 1 out of 6 who liked to watch it).
S suggests that in the school, not a lot of boys would want to watch it.
Another S suggests that he doesn't know because we haven't asked all students in the school. T suggest he

looks at the S's results. Then he agrees
that not a lot of boys would watch Sticky TV.
inferential thinking
SKC: Transnum
T tries to make sense of S statement based on her sorting and transnumerating of the data into a table that tries to cover every variable.
Four columns - G B G B
Four rows for another 2 variables each with 2 possible responses. On right hand side of grid, another variable listed. No numbers in any of the
cells.
T recognises that S is trying to compare Boys and girls who have different lengths of hair with the colour of their skin.
no extra clarification - positive case
statement difficulty

Title: Sch1Lesson4Int.aif

Type: Audio
Time: 29-Aug-06 9:06

KCT: InvestCycle

R and T discuss the size of the Ss' samples (12 data collected) in relation to the sorting of data into categories. T had realised when Ss were
designing their data collection questions that if there were 3 possible responses to a number of the questions, there would be very small frequencies
in the subcategories. T encouraged the Ss wherever possible to change the number of possible responses to 2 instead of 3.
At data question design time, the Ss need to be aware of the need to restrict the number of possible responses to the questions.

KCT: ReasonModels

R and T discuss the size of the Ss' samples (12 data collected) in relation to the sorting of data into categories. T had realised when Ss were
designing their data collection questions that if there were 3 possible responses to a number of the questions, there would be very small frequencies
in the subcategories. T encouraged the Ss wherever possible to change the number of possible responses to 2 instead of 3. With too many
subcategories, it would be much harder for the Ss to make statements/reason adequately with the data.

KCT: InvestCycle

T aware of the management of data collection within the classroom. Ss able to collect data from 12 others in reasonably easy manner. T was aware
that different Ss were at different stages of the data colection phase: some were still writing questions, others were collecting.
T had checked Ss' questions before they embarked on data collection. T aware that questions were suitable. Could only recall one S who had asked
a question for which there was only one possible response. Time spent earlier on data collection questions was well worthwhile.
If Ss had collected data from 20 children instead of 12, data collection phase would have taken longer, but would not make much difference to the
sorting phase (in terms of time) but it may have been preferable in terms of number of data within subcategories.

KCS: ReasonModels

[S1L4V3:57]
T recognised that S had not sorted the data adequately for the statement he was trying to make. T had interpreted the S's statement as being
inappropriate for the way the data was sorted.
T tried to get S to "back track" and sort the data properly to make correct statements.
statement difficulty

SKC: ReasonModels

[S1L4V 5:14 SKC:InvestCycle]
S had tried to sort data but T had realised that she would not be able to make comparisons between groups based on the way she was sorting the
data.
T discusses how to compare groups, you need to sort the data into groups representing the alternative choices for responses (eg., in this case,
barefeet vs. shoes for running, rather than barefeet vs. like the colour blue).

SKC: Transnum

[S1L4V 5:14 SKC:InvestCycle]
S had tried to sort data but T had realised that she would not be able to make comparisons between groups based on the way she was sorting the
data.
T discusses how to compare groups, you need to sort the data into groups representing the alternative choices for responses (eg., in this case,
barefeet vs. shoes for running, rather than barefeet vs. like the colour blue).
posing invest. questions

KCT: Transnum

[S1L4V 7:14 KCT: Transnum]
T had the idea that it would be easier for class to make sense of S's statements if there was a

representation of the sorted data using a 2x2 factor
table, rather than a verbal description, particularly with the some cell frequencies being double other cell frequencies.
T acknowledges the difficulty that Ss have with verbal statements based on the sorted cards; 2x2 tables would be helpful with making statements.
T and R discuss how the 2x2 table helped Ss notice row or column totals, which no Ss had done with just the sorted cards.
statement difficulty
KCS:Transnum
[s1L4V 8:59 KCS: Transnum: T has suggested to S that he create a 2x2 table for data and has helped, through guided questioning, with what the
column and row labels should be.]
T realised that S needed help to explain his thinking orally and written. It would be a good tool to help him with writing his findings down.
T decided to show whole class, again as a useful tool for displaying (transnumerating) the data and making statements about the data. Good for
showing relationships in the data.
statement difficulty
KCT: IntegStatContext
[S1L4V 13:09 KCT: ReasonModel and Variation - T pushes the making of inferential statements, and the appropriate language of 'most' girls ...
even though the sample data shows all girls ...]
Statement made: all girls who wanted to live in USA liked to run in shoes, but all girls who wanted to live in UK liked to run bare feet. So many
categories empty; all girls liked minigolf.
So implication, if a girl wants to live in USA then she must like to run in shoes. etc.
T pushed the idea of generalising to whole school.
T wondered if it is a 'girl thing' that girls wanted to play minigolf. T knew that for herself, she would prefer minigolf.
inferential thinking
KCT: ReasonModels
[S1L4V 13:09 KCT: ReasonModel and Variation - T pushes the making of inferential statements, and the appropriate language of 'most' girls ...
even though the sample data shows all girls ...]
Statement made: all girls who wanted to live in USA liked to run in shoes, but all girls who wanted to live in UK liked to run bare feet. So many
categories empty; all girls liked minigolf.
So implication, if a girl wants to live in USA then she must like to run in shoes. etc.
T pushed the idea of generalising to whole school because she realised that all the girls surveyed did like to play minigolf.
S's data was that 100% liked minigolf, so did the Ss agree with the prediction that most girls in the school would like minigolf come from the data
or from the real life context?
T thought that both, the data and the context, would probably have affected the Ss' predictions.
S thought that he couldn't make a prediction about the whole school because we haven't asked them all.
T referred back to the data, and said that this data shows that, so what do you think might be the case for the whole school?
inferential thinking
KCT: Variation
[S1L4V 13:09 KCT: ReasonModel and Variation - T pushes the making of inferential statements, and the appropriate language of 'most' girls ...
even though the sample data shows all girls ...]
Statement made: all girls who wanted to live in USA liked to run in shoes, but all girls who wanted to live in UK liked to run bare feet. So many
categories empty; all girls liked minigolf.
So implication, if a girl wants to live in USA then she must like to run in shoes. etc.
T pushed the idea of generalising to whole school because she realised that all the girls surveyed did like to play minigolf.
S's data was that 100% liked minigolf, so did the Ss agree with the prediction that most girls in the school would like minigolf come from the data
or from the real life context?
T thought that both, the data and the context, would probably have affected the Ss' predictions.
S thought that he couldn't make a prediction about the whole school because we haven't asked them all.

T referred back to the data, and said that this data shows that, so what do you think might be the case for the whole school?
inferential thinking
KCS: ReasonModels
[S1L4V 17:39 KCS: Transum]
T realised that S was sorting data into two many categories to make sense of data. S ended up with 6 subsets.
R: With 3 variables and 2 possible responses for each variable, how many different subgroups would there be?
T: Eight.
R: So there were 2 subgroups not represented with data cards. He had sorted the data totally. You realised that at the time?
S: Yes. And so the difficulty he had with trying to explain what he had found. I re-directed him to an easier way of sorting so that we and he could
understand what he was doing .
Once data resorted, much easier to see patterns and relationships in the data and make statements from it.
T asks which car most popular and S responds with Ford (which data shows) but for the reason that that is what he likes.
R: Was that unexpected?
T: No, not really. That was his surface thinking but I needed him to delve deeper into what we were doing, to think about the task, not himself.
Even though, he may stretched his bias to the people he was asking. But we don't know that.
R: The personal attachment to the data, you weren't satisfied with that. You pushed him to use the data, rather than personal experience.
statement difficulty
KCS:Transnum
[S1L4V 17:39 KCS: Transum]
T realised that S was sorting data into too many categories to make sense of data. S ended up with 6 subsets.
R: With 3 variables and 2 possible responses for each variable, how many different subgroups would there be?
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understand what he was doing .
Once data resorted, much easier to see patterns and relationships in the data and make statements from it.
statement difficulty
KCT: ReasonModels
[S1L4 V 23:02 KCT: ReasonModel]
T listens to S statements; then encourages S to make inferential statement about children in whole school based on her data. S suggests that
because 1 out of 6 boys liked to watch Sticky TV, then not a lot of boys in the school would like to watch it. T was trying to encourage the Ss to make that response.
Another S suggests that he doesn't know because we haven't asked all students in the school. T suggest he looks at the S's results. Then he agrees
that not a lot of boys would watch Sticky TV.
T encouraged the S to leave out data about shoes because that complicated the data statements that she could make.
R: I wonder if some difficulty with generalising to whole school might be to do with the different ages (although the Ss did not suggest that)?
T: Mmm, I think that with another situation, I did ask about generalising to our team (so children of similar ages rather than whole school of quite
different ages). That didn't cross my mind with this question. I don't know whether that programme might be an age-related one for younger or older
children. I don't know the programme.
inferential thinking
statement difficulty
KCS:Transnum
[S1L4 V 28:06 SKC: Transnum - S's table in book with many columns and rows for sorting all data]

Various columns and rows: What she going to sort all data?
T: I thought that she was going to just sort the two, and I wondered how she was going to do it. i will check up on her later.
R: Did you see how she handled it later?
T: No, haven't yet.
R: It looked complicated ...
T: yes, and I don't think it will work. But I will see how she does it and she might decide her herself that those columns aren't needed, or... I think
she will see it for herself.
R: So you think she might have problems with it but you are going to leave her to it to find for herself?
T: Yes.
So T, knowing that the S may have difficulty with such sorting, however leaves her to discover this for herself.
SKC: Transnum
[S1L4 V 28:06 SKC: Transnum - S's table in book with many columns and rows for sorting all data]
Various columns and rows: What she going to sort all data?
T: I thought that she was going to just sort the two, and I wondered how she was going to do it. i will check up on her later.
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T: yes, and I don't think it will work. But I will see how she does it and she might decide her herself that those columns aren't needed, or... I think
she will see it for herself.
R: So you think she might have problems with it but you are going to leave her to it to find for herself?
T: Yes.
CKC: ReasonModels
T needed to think about samples size, making generalistions, and ensuring that her own statements were valid for the data.
R: What have you learned about Ss and their understanding of statistics, what they find easy/hard?
T: Teaching the whole class was easier to manage with Statistics, I cannot imagine doing it in groups. They all helped each other out. We all learnt
together really.
R: Specific things you learnt?
T: Ways in which to word the information that we found, in order to make a true and correct statement about the data. I think that the kids have
picked up on as well.
I think that they learnt a lot about questioning. For example, the opposite of left. I hadn't thought of that so learnt something from that about
Children's understanding of statistics - what they found easy or hard?
T: With the kids who went and tried to organise the data into all the categories, they needed re-directing.
R: You were getting them to make it simpler.
T: It's not until they get back to the practical task, in spite of the talking and explaining about it, that the real learning takes place... as they play
around with the data cards.
R: Did you have to think about statistics yourself?
T: Yes, with sample sizes, and generalisations, and the wording of statements/ findings. I was always taught to say a greater number or greater
amount, rather than 'more than' because more means out of the whole lot... I didn't whether this age group would pick up on that type of thing, and
we were already grappling with a number of different concepts as well. My mind was constantly active to be correct in what I was saying so as to not
mislead the kids with what I was saying.
R: Interesting, the words more and less; refine to numerical. May count; other times push to proportions. There's a lot to think about.
T: Their understanding of number is very important. We are not going on to fractions etc until next term, but this would be really helpful with
statistics.
R: There were times when you used words such as twice as many, double, or half etc
T: I think that doing an investigation changed the way that the kids also thought about statistics. From the initial brainstorm, statistics was graphs
and tallies. I think that they have moved from that idea, to statistics is finding information from data. Graphs in isolation mean nothing.

appropriate language
statement difficulty
inferential thinking

KCT: ReasonModels

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appropriate language
statement difficulty
inferential thinking

Title: Sch2Lesson2.mov

Type: Video
Time: 29-Aug-06 9:07

KCT: Dispositions

T, by asking what the Ss noticed in the data, anything interesting

KCT: InterrogCycle

T asks Ss what they did they notice in the data as they were sorting.
S: All the youngest people are RH.
T: Interesting, How did you sort it to find that?
S: Put them into groups like youngest and looked at them.
S: All the boys are RH.
T: Was there anything else that the RH boys had in common?
S: They could all whistle... And if they were LH they couldn't... no just made that up.
T: Yes because you just said they were all RH.

KCT: ReasonModels

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S: They could all whistle... And if they were LH they couldn't... no just made that up.
T: Yes because you just said they were all RH.

KCT: ReasonModels

T has 3 questions on board to help focus Ss investigation of data.
1. Are there more whistling RH or whistling LH proportionally?
T: Proportionally - what does that mean?
S: Most of.
S: True.
S: Common.
T: Proportion. What's a portion?
S: Bit of something.
T: Part of larger group. Like if you cut a pizza into portions, then it's pieces or fractions, parts of. So proportionally, part of a larger group.
Looking at one group in a larger group. So question, means whistling RH or whistling LH out of a larger group. So there might be 18 out of 24 and
12 out of 24. You can do that comparison.
2. Is there anything interesting when comparing place in the family and whistling?
3. All the boys in this group who are youngest can whistle. Does this mean every boy who is the youngest in their family can whistle?
T: The data squares won't tell you that. They tell you what happens in this particular group. I want you to think about that question. Could we say
that?
inferential thinking

SKC: ReasonModels

Question 1 re comparisons proportionally.
T Out of the whole group which was 24 ...
S: 15 RH whistlers.
T: How many LH whistlers?
S: There are none in this one (boys group).
T: Why do you have a boys' group and a girls' group when we are looking at LH and RH?
S: Just easier.
T: Are there RH whistlers over there?
S: There are 6 and 9 here.
T: Are there LH whistlers over there?
S: 2 LH whistlers.
T: 2 out of what? Our whole group is 24. So 2 out of 24 and 15 out of 24. So proportionally there are more?
S: RH whistlers.
Missed opportunity. The question was whistling RH or whistling LH but the Ss here talked about RH

whistlers and LH whistlers. T did not pick up
the discrepancy in the wording of the statements, which totally altered the meaning and therefore answer to the question. These Ss were correct in
their statements but the intention of the original question was lost.
Based on question in lesson plan (obtained from nzmaths website).
misinterpret question
Based on
LH RH Total
whistlers 2 15 17
non-wh. 1 6 7
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SKC: ReasonModels
Question 1 with class.
T reads the correct question.
S: There are more RH people that can whistle.
T: Can we extend our answer at all?
S: There are 15 RH whistlers out of 24 and 2 LH whistlers out of 24.
T: Anybody disagree with that statement? Our biggest proportion is RH whistlers, because 15 out of 24.
S: We put 8 out of 21 can't whistle for RH. And 2 out of 3 can because there were 3 LH.
T: Where did you get your 21 from?
S: um... out of those ...
S: It is supposed to be 24.
S: Was there 6 or 8 out of RH who could whistle?
T: Let's see, we'll check with another few groups.
S: There's 8 so it's 3x8 = 24, which is 1/3 of them. And 1 out of 3 LH ... no 2 out of 3 LH can whistle.
T: Say that again please.
S: There's 8 RH people who cannot whistle out of 24 which is 1/3. And there's 1 out of LH people that cannot whistle which is 1/3.
T: So you've looked at the ones that can't whistle in the results as well, out of the group of RH people. Is that what you've done?
S: I don't know, I'm confused.
T: I'll have a chat to you, I think I know what you've done which is correct. So checking our numbers out of 24 for people that are RH and can
whistle and LH and whistle out of our whole group.
S: The rest of them from our group which was 15 whistlers ...
T: 15 RH?

S: No there was 9 RH and 6 LH and then we ... that means that the rest of the numbers... so that means that the rest of the numbers ... they can't
whistle.
T: So proportionally are there more RH whistlers or LH whistlers?
S: Most of the oldest cannot whistle.
T: So you've moved on a little bit.
Missed opportunity. T had not looked carefully at data himself. Did not ask for a diagram or summary of any sort to verify numbers etc. Some of
the Ss' arguments were based on incorrect numbers but T did not check/verify.
not knowing data
missed evaluation
statement difficulty
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Based on
LH RH Total
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Total 3 21 24
SKC: ReasonModels
Question 1
T: We've got 15 and 2 as a number of LH. How many LH whistlers were there?
S: 2

T: 2 out of 24 ... and how many RH whistlers were there out of 24?
S: 15.
T: 15 out of 24 was our proportion out of the whole group that were RH and could whistle. So proportionally there were more RH whistlers than
LH whistlers. I think what you have done L is look at the portion of RH that can and can't whistle. That's okay, it will provide an interesting sideline
as well.
S: It's 1/3 that cannot whistle.
T: You've taken that a step further which is excellent.
Missed opportunity. T did not go back or check the wording of the question. May be due to not having investigated the data himself to answer the
question before giving the question to the class.
Not knowing data
Based on
LH RH Total
whistlers 2 15 17
non-wh. 1 6 7
Total 3 21 24
CKC: ReasonModels
Question 3 with class - all boys who are youngest in family can whistle. Pushing inferential thinking.
T: Could we say that every boy in NZ who is youngest in family could whistle?
S: No, it's just the results from this group. Haven't tested everyone. All the youngest boys don't know exactly how to whistle.
T: Okay. What would we have to do to make that statement about NZ?
S: See if everyone can whistle ... in NZ.
T: Would we have to do everyone in NZ?
S: No ... we are only one classroom.
T: Only one classroom so we couldn't really say that all boys who are youngest can whistle. We don't have enough data to say that, we would have
to do a bigger study, survey, investigation.
Missed opportunity. Did not follow up on answer from student who said No we wouldn't have to test all NZ.
Not clear whether the T has knowledge about being able to use sample data to make generalisations about the population. There are indications
that T knows that the population would not have to be tested (surveyed, census) but he did not correct/question the Ss who were suggesting the need
to test the population.
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to test the population.

Title: Sch2Lesson2&3Int.mp3

Type: Audio
Time: 29-Aug-06 9:07

KCT: InterrogCycle

[S2 L2 V 0:06 KCT ReasonModels]
T asks class what they had found. Wanted to check that they had found something. Were able to make links between 2 parts of data. Not confident
that they had. Nothing really surprising out of what they had said. Would have expected most to have made 2 or 3 links/relationships. To get from
them .. had they considered Boys RH/Boys LH, and girls, wh/non-wh. Just seen some relationships there.
sorting 2 variables

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sorting 2 variables

CKC: ReasonModels

[S2L2 V 1:19 KCT ReasonModels - 3 questions on board]
Questions from unit plan. Ways of interpreting - some ambiguity. Some answers from Ss - had started to see the question differently.
Your answers to the 3 questions:
T first sorts into wh/non-wh.
2 wh LH out of sample of 24, 15 wh RH out of 24.
I know that Liam said, out of LH, 2 out of 3 can whistle. 2/3 of LH can whistle.
15 out of 21, 6 can't; that portion of RH, 14 is 2/3 of 21. There's slightly more proportionally RH that can whistle as opposed to those that can't.
Back to question: There's 2 out of 3 whistling LH.
The 2 ways of interpreting the question: which is correct?
T: I thought the correct way was wh RH out of the total in the sample.
R: If wording changed - out of the RH, the proportion who can whistle more or less than the proportion of the LH who can whistle.
T: That would lead me to look at it differently.
R: In class I could see the different interpretations happening.
T: Liam's way of thinking about it was great; it was beyond what I thought the class would come up with. I could see that if we started getting into
too much of exactly what it meant, I could see that we would lose too much of the class with that.
R: Numeracy levels - many couldn't compare 2/3 with 15/21. Tricky if we are going to compare the subgroups.
Based on
LH RH Total
whistlers 2 15 17
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Total 3 21 24
Question 2
Based on
non-wh wh total
oldest 7 4 11
middle 0 7 7
youngest 0 6 6
Total 7 17
T: Comparing those that can whistle, there are higher nos. in the middle and youngest than in the oldest out of the total group. And there's an
unusually large no. of oldest children that can't whistle, and all the non-wh are oldest. Could then look at B and G: about even.
R: All youngest in family can whistle. Middle in the family are whistlers. But oldest in family, more likely to be non-wh. Variety of statements.

Missed opportunity. R had to give those later statements, the T did not see or comment on these aspects.
Question 3
T: I would say that from a small sample like that you couldn't make that statement.
R: Could you make any statement, maybe slightly more qualified?
T: It is likely that if you are a boy youngest in the family that you can whistle. Likely, but not definite. Probable. It looks like that if you are
youngest, boy or girl, you probably will be able to whistle. I would look at ... older kids would have taught you.
R: Definite statement about this group but qualified statement about larger group.
inferential thinking
CKC: Transnum
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youngest, boy or girl, you probably will be able to whistle. I would look at ... older kids would have taught you.
R: Definite statement about this group but qualified statement about larger group.
T; The correct way to phrase your statement, avoiding definites, all; I guess some work on probability would help to refine what you are saying, to
make sure you are saying it is true.
R: Big idea in statistics: generalising to a larger group, recognising trends but variation as well.

Sample size will determine how confident we
would be about the generalised statement. Variation occurs: but trends will be there.
T: Media - things can be twisted to what you want it to say.
R: Also proportions - out of what group size. Absolute numbers may not be fair, need to recognise group size. Eg. class 17 boys, 13 girls. 8/17
whistle, 7/13 can whistle. 8 is more than 7 so more boys can whistle than girls. But over half of smaller group are whistlers but less than 1/2 of
larger group.
Missed opportunity - when considering the reasons why generalisations from data might or might not be feasible. Did not consider contextual
factors, such as age.
inferential thinking
CKC: IntegStatContext
T: suggests reasons why youngest child in family likely to be able to whistle.
CKC: IntegStatContext
T refers to way media may examine data without considering other factors.
CKC: ReasonModels
[S2L2 V 3:25 SKC ReasonModels - group had cards sorted into extra, unecessary groups.]
R: RH whistlers and LH whistlers but question was whistling RH and whistling LH. Does it change the question?
T: You are changing the total in a group by the order of what comes first.
R: A person who is a LH whistler is also a whistling LH but in terms of groups relevant to the question, the order does make a difference. It
changes the focus of the which you are looking at.
Based on
LH RH Total
whistlers 2 15 17
non-wh. 1 6 7
Total 3 21 24
Missed opportunity.
KCT: Transnum
[S2L2 V 3:25 SKC ReasonModels - group had cards sorted into extra, unecessary groups.]
T talks about why.
T: Ss said just easier. I thought they had just taken some each to sort and then just sat and looked at them. I asked them if you are interested in
LH/RH and whistling, then why separate them.
R: RH whistlers and LH whistlers but question was whistling RH and whistling LH. Does it change the question?
T: You are changing the total in a group by the order of what comes first.
R: Would be worthwhile to get Ss to sort the cards by one variable (to one side and other side) and then second variable top and bottom.
A person who is a LH whistler is also a whistling LH but in terms of groups relevant to the question, the order does make a difference. It changes
the focus of the which you are looking at.
Based on
LH RH Total
whistlers 2 15 17
non-wh. 1 6 7
Total 3 21 24
Missed opportunity - R talked about suggestions teaching the Ss about how best to sort the cards into regions on the desk. Links to being to reason
with the cards for making statements.
SKC: ReasonModels
[S2L2 V 3:25 SKC ReasonModels - group had cards sorted into extra, unecessary groups.]
T talks about why.
T: Ss said just easier. I thought they had just taken some each to sort and then just sat and looked at them. I asked them if you are interested in
LH/RH and whistling, then why separate them.
R: RH whistlers and LH whistlers but question was whistling RH and whistling LH. Does it change the question?
T: You are changing the total in a group by the order of what comes first.
R: Would be worthwhile to get Ss to sort the cards by one variable (to one side and other side) and then second variable top and bottom.

A person who is a LH whistler is also a whistling LH but in terms of groups relevant to the question, the order does make a difference. It changes the focus of the which you are looking at.
Based on

	LH	RH	Total
whistlers	*2*	*15*	*17*
non-wh.	*1*	*6*	*7*
Total	*3*	*21*	*24*

Missed opportunity.
CKC: ReasonModels
[S2L2 V 9:53 CKC ReasonModels - generalisations, need to test whole population?, sample size]
In future how would you respond to S answer? In fact what is your answer to such a question?
T: How many people would we need to test? Or ask what sort of numbers would need to be surveyed to make more of a generalisation than from a sample of 24.
R: Would you be more confident about making a statement from a sample of 240 than from 24 to do with youngest in the family?
T: Yes, you could be.
R: The more data, the more confident you could be about the statement you make?
T: Yes. 10x the sample size, more of a statement from that.
R: Sample size important. Other reasons why might we not say all? .... age important, like little babies, your daughter 12 mths old, cannot whistle yet.
inferential thinking

Title: Sch3Lesson2E.mov

Type: Video
Time: 29-Aug-06 8:54

SKC: InterrogCycle

T gets suggestions from Ss for possible data collection questions, for mainly numerical data.
T unsure why S said - you are 10 and born on 13/6. You were born on the 13th?
no extra clarification
S: Could be "What's the date today?"
Missed opportunity as this question is not suitable for data question.
missed evaluation
T tells class the data questions.
Related to data square B 10 13 6

SKC: Transnum

T and class talking about average: How is it obtained?
S: By doing that.
T: Do it once to get the average?
S: No, do it multiple times and see what number gets the most scores. If they didn't get the most scores it would the one in the middle.
missed evaluation
no extra clarification about "most"/"middle"
appropriate language
T: The one in the middle? How do they work out the average?
S: Add them all together and divide by the number of tries.
T: Good
Missed opportunity - the comment about which number gets the most scores (mode) overlooked by T. Also no comment re. the S comment about the
one in the middle.

SKC: ReasonModels

T listens to S statements about what they have found with the data and their justifications for those statements.
S: Girls have faster reaction time than boys.
T: How do you know that?
S: 9 is short as [focusing on one individual value rather than group].
T: How are you going to sort them to find out if the boys have a faster reaction time than girls?
Missed opportunity - did not comment about the focusing on an individual data value although pushed then to think about the group of B and G.
S: My reaction time is 1.
T: We're not actually doing reaction times, we are sorting the data we have. ... How are we going to do that?
S: I don't know, we've already done this before and I don't feel like doing it.
T: What else could we find out?
S: Could find out the oldest person. ... Put them into order from youngest to oldest. ... And see what their different reaction scores are.

SKC: Transnum

T listens to S statements about what they have found with the data and their justifications for those statements.
S: Girls have faster reaction time than boys.
T: How do you know that?
S: 9 is short as [focusing on one individual value rather than group].
T: How are you going to sort them to find out if the boys have a faster reaction time than girls?
Missed opportunity - did not comment about the focusing on an individual data value although pushed then to think about the group of B and G.
individual/group data
S: My reaction time is 1.
T: We're not actually doing reaction times, we are sorting the data we have. ... How are we going to do that?
S: I don't know, we've already done this before and I don't feel like doing it.
T: What else could we find out?
S: Could find out the oldest person. ... Put them into order from youngest to oldest. ... And see what their different reaction scores are.

posing invest. questions
SKC: InterrogCycle
T and group - S proposing things that they can investigate in the data.
S: Put them into year and age and gender.
T: What are you trying to find out?
S: We will figure that out once we have sorted the cards. ... Like if there are more G who are Year 6 than B who are Year 6.
S: We can add them together and do averages.
Missed opportunity - T did not ask what data the averages were going to be from.
S: I don't get it.
T: What don't you get?
S: What we are doing.
T: We're trying to see if there is anything interesting about this class. Like: Does it affect your reaction time to how old you are?
S: The older you are the slower you are [- hypothesis rather than data based].
T: Is that true for this class?
S: No idea.
S: Well let's see if we can find that out.
S: We could which one was the highest and sort them out oldest to youngest and see who has the highest.
S: Do a median for this group.
T: So we get it sorted so we can see it, can't we, and then we can see some sort of thing.
Missed opportunity - T does not explore with the Ss how the comparison of groups can be carried out using medians or using the suggested sorting method.
SKC: Transnum
T and group - S proposing things that they can investigate in the data.
posing invest. questions
S: Put them into year and age and gender.
T: What are you trying to find out?
S: We will figure that out once we have sorted the cards. ... Like if there are more G who are Year 6 than B who are Year 6.
S: We can add them together and do averages.
no extra clarification
Missed opportunity - T did not ask what data the averages were going to be from.
S: I don't get it.
T: What don't you get?
S: What we are doing.
T: We're trying to see if there is anything interesting about this class. Like: Does it affect your reaction time to how old you are?
S: The older you are the slower you are [- hypothesis rather than data based].
T: Is that true for this class?
S: No idea.
S: Well let's see if we can find that out.
S: We could see which one was the highest and sort them out oldest to youngest and see who has the highest.
S: Do a median for this group.
T: So we can sort them out from oldest to youngest and to the reaction times. So the fastest ...
S: ... is the median for this group.
T: So we get it sorted so we can see it, can't we, and then we can see some sort of thing.
Missed opportunity - T does not explore with the Ss how the comparison of groups can be carried out using medians or using the suggested sorting method.
individual/group data
no extra clarification
SKC: Transnum
T with group - findings about reaction times and gender.
T: Are you finding fastest reaction times?
S: Yes.
T: So these are all 13 years old with reaction times of 13.... Is that the fastest reaction time there or the slowest?
S: Slow ... in the middle... there's also 14 and 11 and only one that's 9.
T: So 9 is the slowest.
S: Isn't 9 the fastest?

T: Yes, fastest, you are right ... get it into an order so that ... You can keep sorting like that, then you might see to be able to compare something.

Like: Maybe there are more girls who are slower, or the older you are the faster your reaction time is.

Missed opportunity - How can sorting of cards like that help to compare the reaction times of the groups?

individual/group data

appropriate language - wording of question is not really focusing the students on what the teacher wants.

SKC: ReasonModels

T with group - findings about reaction times and gender.

T: Are you finding fastest reaction times?

S: Yes.

T: So these are all 13 years old with reaction times of 13.... Is that the fastest reaction time there or the slowest?

S: Slow ... in the middle... there's also 14 and 11 and only one that's 9.

T: So 9 is the slowest.

S: Isn't 9 the fastest?

T: Yes, fastest, you are right ... get it into an order so that ... You can keep sorting like that, then you might see to be able to compare something.

Like: Maybe there are more girls who are slower, or the older you are the faster your reaction time is.

Missed opportunity - How can sorting of cards like that help to compare the reaction times of the groups?

SKC: ReasonModels

T and group - have sorted cards and found some things.

T: How have you sorted them?

S: Age groups... and highest to lowest in their reaction times. ..

T: What's interesting about this group of cards?

S: They all 11 year olds and all have same reaction time of 13.

T: What group has got the fastest reaction time do you think? ... this group, they have all the lowest times. ...

S: That one card there has the lowest of all the times.

T: What's this group - it's the 10 year olds?

S: Sorting out the time of the different age groups.

T: So do you think you could anything about these different age groups?

S: The 12 year olds have the quickest reaction times.

T: So on average you are probably right.

S: And the 13 year olds.

T: But except for this guy here, he's pretty slow [12 year old with RT 15 - second slowest of all data] ... So let's try and order in some way; maybe

year groups - oh are they all in the same year group if they are in the same age group? .. what about something about the B and the G - who has the

fastest reaction times out of them?

S: Girls do.

T: How do you know that?

S: [points to one data value G with RT 9].

T: But that's one girl ... what about his G here [with RT 17].

S: She's got the slowest RT.

T: So maybe you could compare them somehow like that.

S: Fastest to slowest ...

T: So maybe you could go B and G, their reaction times, the fastest on their reaction times.

Missed opportunity - how can the groups be compared to establish which group is faster? The way the T phrased the question could lead Ss to

looking at individual data values rather than deciding which group is faster.

SKC: Transnum

T and group - have sorted cards and found some things.

T: How have you sorted them?

S: Age groups... and highest to lowest in their reaction times. ..

T: What's interesting about this group of cards?

S: They all 11 year olds and all have same reaction time of 13.

T: What group has got the fastest reaction time do you think? ... this group, they have all the lowest times. ...

S: That one card there has the lowest of all the times.

T: What's this group - it's the 10 year olds?

S: Sorting out the time of the different age groups.
T: So do you think you could find anything about these different age groups?
S: The 12 year olds have the quickest reaction times.
T: So on average you are probably right.
S: And the 13 year olds.
T: But except for this guy here, he's pretty slow [12 year old with RT 15 - second slowest of all data] ... So let's try and order in some way; maybe
year groups - oh are they all in the same year group if they are in the same age group? .. what about something about the B and the G - who has the
fastest reaction times out of them?
S: Girls do.
T: How do you know that?
S: [points to one data value G with RT 9].
T: But that's one girl ... what about his G here [with RT 17].
S: She's got the slowest RT.
T: So maybe you could compare them somehow like that.
S: Fastest to slowest ...
T: So maybe you could go B and G, their reaction times, the fastest on their reaction times.
Missed opportunity - how can the groups be compared to establish which group is faster? The way the T phrased the question could lead Ss to
looking at individual data values rather than deciding which group is faster.
appropriate language - re. group with fastest reaction time
individual/group data
posing invest. questions
SKC: ReasonModels
T and group.
T: Quite a few of the year 6s had the same RT. ...
S: There's more in that box than anything else.
T: Did all year 6 have 13 [RT]?
S; Some had 10 and 14. .. and there are more G.
T: Doubles?
S: Like that [pointing at some cards]. ...
T: So the B are all separate [no two cases identical]... Why is that do you think?
S: Don't know.
S: Less boys.
T: That's probably why. There's a lot less B than G. ... Good ...
Missed opportunity - How relevant it is to have noticed identical cases in relation to the comparisons that they are being asked to notice and draw
conclusions about.
individual/group data
SKC: ReasonModels
T and group
Comment about the number of year 6 G than B etc. Teacher tries to push for a comparison involving variables other than gender and Year group.
Encourages them to compare Year 6 and year 8 G RT and leaves them to sort and "who has the faster RT".
Missed opportunity - the wording of the question could encourage looking at individual data rather than group comparisons.
appropriate language
individual/group data
SKC: ReasonModels
T and class
Two comments from Ss - both focusing on individual data or on frequencies and not on comparison of groups.
When S suggested that fastest RT was from a G then they must be faster, the T asks who has the slowest RT [also G with 17], so .. we would need to
look at average of girls RT and the average of the boys' RT to get a comparison.
S: The average of the G was 11 and the B had two 11s. ... oh, the B had one 10 but no 11s or 9s.
T: So what was the average of the B?
S: 12 I think.
T: So we could say that the G had the fastest RT.
S: No the G average was 13.
T: MAybe we need to have another look at that.

Missed opportunity - did not explore the use of averages to compare - just told them that that was what we should use.
Also did not follow up about the different averages that were being quoted by Ss.
individual/group data
missed evaluation
CKC: ReasonModels
T knows to use an average to compare two groups rather than using individual data values. But he does not explain why other than indicating that
fastest and also slowest times were from G so therefore use average.

Title: Sch3lesson2Int.mp3

Type: Audio
Time: 29-Aug-06 8:54
CKC: InterrogCycle
Missed opportunity - T had not engaged with the data set, which contained mainly numerical data, prior to the lesson and acknowledges how this
had an impact on the lesson and students, in regard to how he could help the Ss handle the comparisons satisfactorily.
I found it probably ... probably with the confidence of myself with it, I hadn't experimented with it [the data] and looked at all the different patterns,
or comparing all the different information myself. Which probably led to me,... I wasn't able to give them scaffolding to find something interesting.
not knowing data
KCS: InterrogCycle
T comments on how the Ss struggled with statements beyond the superficial, of frequencies rather than comparisons.
T: Some of them struggled to see. They were still doing that counting. A lot of the comments were like, "There are more boys ..." I think that that
came up actually more than the lesson before. Before they were looking at two different things, like the whistling and the boys, comparing the boys
to the girls that could whistle. Whereas today they seemed to be more focused on just doing a straight count of, [eg.] there are a lot more girls than
boys.
T talks about the difficulty the Ss face with handling numerical data, whereas they had reasonably handled comparisons with category data. T
surprised about this.
Missed opportunity - T wondered if he had not given enough model of how to undertake such a comparison.
statement difficulty
sorting 2 variables - frequencies or comparisons
category/numeric data
SKC: InterrogCycle
[S3L2 V 0:00 -T does not respond to S's suggestion for data question, "What is the date today?".]
T response to R question as to whether it is a suitable question for data collection: "I don't think so ... it doesn't relate to anything".
Need to determine whether a question is suitable for data collection.
Missed opportunity
CKC: Transnum
[S3L2 V 1:08 - averages]
T aware of the 3 averages - mode, median, and mean; and how to find them.
T talks about how he was not going to introduce these in the lesson, yet it came from a question he asked the class. It would have been a good
chance to introduce these ...
KCT: Transnum
[S3L2 V 1:08 - averages]
T aware of the 3 averages - mode, median, and mean; and how to find them.
T talks about how he was not going to introduce these in the lesson, yet it came from a question he asked the class. It would have been a good
chance to introduce these ...
Missed opportunity - did not explore these further with the Ss.
KCS:Transnum
[S3L2 V 1:50 - boys discussing RT between B and G]
Ss did not make good comparisons between groups.
Ss then went off the think about other variables rather than following through with the comparison between B and G.
G faster RT than B because one G had a time of 9.
T's reaction to this: I think I talked about the G with the slowest time of 17... Rather than using an individual data value to justify a statement, must
use all the data to see where it is grouped or spread.
Ss did not really know how to sort the cards in order to make the comparisons.

T talked about there being not so many B which would affect the comparison between the groups.
statement difficulty
sorting 2 variables - frequencies or comparisons
individual/group data
SKC: ReasonModels
[S3L2 V 1:50 - boys discussing RT between B and G]
Ss did not make good comparisons between groups.
Ss then went off the think about other variables rather than following through with the comparison between B and G.
G faster RT than B because one G had a time of 9.
T's reaction to this: I think I talked about the G with the slowest time of 17... Rather than using an individual data value to justify a statement, must
use all the data to see where it is grouped or spread.
Ss did not really know how to sort the cards in order to make the comparisons.
T talked about there being not so many B which would affect the comparison between the groups.
KCT: Transnum
T and R talk about the sorting of cards by two numeric variables eg., age and RT.
S had hypothesised that maybe the older boys get slower.
Sort into age groups as per a bar graph. Then sort within each group into RT. Check to see how the RT in the 4 groups vary, by looking at centre
and also spread of the data.
Better than having cards in clumps.
SKC: Variation
[S3L2 V 5:18 - Ss had not used all cards in sorting]
Cards in age groups, sorted reasonably well. But when making a comparison, they had noticed that all children in one age group had the same
reaction time, but with one other group, they did not use all cards - they left out some cards for which the RT was different from the rest in that same
age group.
T: I pulled out some cards because in that group, there was a ...
S had commented about RT of 9 so T asked about another with RT of 17.
T challenged Ss to think about the fact that the data varied in the group.
SKC: Transnum
[S3L2 V 5:18 - Ss had not used all cards in sorting]
Cards in age groups, sorted reasonably well. But when making a comparison, they had noticed that all children in one age group had the same
reaction time, but with one other group, they did not use all cards - they left out some cards for which the RT was different from the rest in that same
age group.
T: I pulled out some cards because in that group, there was a ...
S had commented about RT of 9 so T asked about another with RT of 17.
T challenged Ss to think about the fact that the data varied in the group.
individual/group data
KCT: Dispositions
[S3L2 V 6:49 - encouraged Ss to be inquisitive and open minded about the data]
T used question to encourage those dispositions.
KCS: InterrogCycle
T knows that some Ss need more guidance with investigation by having a particular question to guide their thinking.
KCT: InterrogCycle
[S3L2 V 10:45 SKC: ReasonModels - difficulty Ss have with comparing]
Comparisons difficult. T pushed Ss to compare rather than find frequencies.
T encouraged them to look then at RT.
R suggests putting up a statement which the Ss have to prove/disprove, eg., Year 6 have faster RT than Year 8.
How could we go about proving that? .. ways of sorting ... then make statements about comparison of RTs. ...
Interesting things gender vs RT or age vs RT,; no point looking at year level vs age, they will know the relationship with that and so it is somewhat
trivial, not very interesting.
Even gender vs age is not really of great interest.
T: S made statement that the older you get the slower you get.... Some Ss would definitely benefit from having a set statement to investigate. ... Hard

when it is too open, better to channel their thinking.
sorting 2 variables - frequencies or comparisons
posing questions - finding questions of interest in data
KCT: Transnum
[S3L2 V 12:20 - use of averages for measurement data]
Ss notice one data value of 9 so G faster, T then comments about the slowest time of 17 for a G, so need to use averages.
How many Ss in class would be able to calculate mean?
T: Most ... done it in Science investigations ...
R: So Ss could calculate mean. When cards are lined up, also very easy to find median, by counting from each end. .. It would be worthwhile for Ss
to find both mean and median, and compare. ... Median nice one to find with data cards. Making Ss realise that because there is variation then to
compare two groups, we need to reduce data to single value; cannot compare individual data value from group to compare groups.
R: Suggests some comparisons to make, in order to push statistical thinking - individual values but also group tendencies. Can link with Science.
T: Mode is not so useful, most frequent isn't it?
R: Yes, the most common one; depends; in this case, with G, the mode is 13 which is in the middle.
T: Ss already talking about the median, because this is in the middle ... some data higher, some lower.
individual/group data
KCS: ReasonModels
T acknowledges the importance of modelling correct use of language/terminology (mean, median).
appropriate language
SKC: Transnum
T indicates that the 'mode' is not such a useful/valid measure in some instances.
KCS: ReasonModels
R: Model a contentious statement to get them going.
T: I think that did help them the other day.
T: I hadn't played with the cards myself, so I wasn't sure what they would find. I couldn't lead them. After the lesson I thought that I should go back
to using numeric data, instead of going on to something else that we collect ourselves.
R: Ss should have the same set of data so that the Ss are all talking about the same data.
R: Look for relationships such as gender vs height or armspan, or height vs armspan etc. If we know someone's armspan could we make a
prediction about their height?
Notice unusual values in the data, eg., 14 year old who is 183cm tall; or someone with big difference between height and armspan.
not knowing data
individual/group data

Title: Sch4Lesson2E.mov

Type: Video
Time: 29-Aug-06 8:56

KCT: InterrogCycle

T gets Ss to make predictions about what they might find about our class.
S: Most Bs can and most Gs can't whistle.
S: more Bs in our class.
T: We know there are more Bs but good guess, mind you we're not sure who was here and who was away last time [when we collected the data].
S: How many Bs can and can't whistle.
S: Most people might be the youngest.
T: That's a pretty good guess.
sorting 2 variables - univariate fequencies or comparisons

CKC: Variation

T understands that with different sample, the results could be different.
T and class - comparing our data squares with other class - what do you predict?
S: Probably quite different.
T: Why?
S: These weren't the people ... ones that we don't know.
S: Similar, because the questions were the same.
T: Good point, But there could be variation because we're different people.
S: Could be quite siimilar because there are not many different variations that you can have... In our class at least one of each, in other class,
probably one of each as well.
inferential thinking

KCT: InterrogCycle

T and class - comparing our data squares with other class - what do you predict?
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S: These weren't the people ... ones that we don't know.
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S: Could be quite siimilar because there are not many different variations that you can have... In our class at least one of each, in other class,
probably one of each as well.

KCT: Transnum

T and class - T asks why it is not a good idea to pile cards but better to spread them out.
S: Harder to count up.
T: Why else might I not want you to stack them up?
S: Because then you can see if they are all the same.
T: Right. You can really see them. Can have a really good look at them.

KCT: InterrogCycle

S: Could find how many Gs and how Bs can whistle.
T: Okay, so first, how many Bs and how many Gs are there?
S: 11 of each.
T: First statement, there's an even no. of Bs and Gs. Then you can compare how many Bs can whistle with how many Gs can whistle. Or how many
Gs are youngest compared with how many Bs. Then write a statement, make sense? ... No? ... What could we look for and try and find out?
S: How many Gs the oldest.
T: How?
S: By counting.
S: 3 of each.
T: So equal numbers of oldest with Bs and Gs?
S: yes.
T: What else can you see?
R: With G cards why don't you put the whistlers at the top and non-whistlers at the bottom? ... then do the same with the Bs.
T: So what can you see quite clearly now?
S: Most of the Gs can whistle and most of the Bs can't.

T: Is that a good statement Michael?
S; Don't know.
T: Which is more?
S: Gs.
T: So Gs have more people who can whistle than Bs.
S: Yes.
R: You said before, "Most Bs can't whistle". Is that true?
S: No, most ... I don't know.
T: There are more Bs than Gs who can't whistle, but there are more whistling Bs than non-whistling Bs.
KCT: ReasonModels
T and group - interrogating and making statements from the data
S: Could find how many Gs and how Bs can whistle.
T: Okay, so first, how many Bs and how many Gs are there?
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T: First statement, there's an even no. of Bs and Gs. Then you can compare how many Bs can whistle with how many Gs can whistle. Or how many
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T: Is that a good statement Michael?
S; Don't know.
T: Which is more?
S: Gs.
T: So Gs have more people who can whistle than Bs.
S: Yes.
R: You said before, "Most Bs can't whistle". Is that true?
S: No, most ... I don't know.
T: There are more Bs than Gs who can't whistle, but there are more whistling Bs than non-whistling Bs.
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S: 11 of each.
T: First statement, there's an even no. of Bs and Gs. Then you can compare how many Bs can whistle with how many Gs can whistle. Or how many
Gs are youngest compared with how many Bs. Then write a statement, make sense? ... No? ... What could we look for and try and find out?
S: How many Gs the oldest.
T: How?
S: 3 of each.
T: So equal numbers of oldest with Bs and Gs?
S: yes.
T: What else can you see?
R: With G cards why don't you put the whistlers at the top and non-whistlers at the bottom? ... then do the same with the Bs.
T: So what can you see quite clearly now?
S: Most of the Gs can whistle and most of the Bs can't.
T: Is that a good statement Michael?
S; Don't know.
T: Which is more?
S: Gs.
T: So Gs have more people who can whistle than Bs.
S: Yes.

R: You said before, "Most Bs can't whistle". Is that true?
S: No, most ... I don't know.
T: There are more Bs than Gs who can't whistle, but there are more whistling Bs than non-whistling Bs.
statement difficulty
maths/stats
sorting 2 variables - frequencies or comparisons
appropriate language
SKC: ReasonModels
T and class - summaries of findings so far
S: Gs have more people that whistle.
T: Compared with what?
S: ... Bs.
T: Who can check that against their own statement?
statement difficulty - frequencies or comparisons - use of comparative language
S: Amount of Bs who are youngest in family same as amount of Gs youngest.
T: Can that be re-worded?
S: Bs and Gs have same amount of youngest people.
S: Of the youngest people, same amount of Bs as Gs.
S: There are 3 Bs and 3 Gs who are youngest in family.
S: There are 5 Bs and 5 Gs.
S: Oh yes , 5.
T: We could say that, it's giving specific details.
S: More Bs are exactly ... 8 Bs write with RH and 8 Gs write with RH.
T: Same amount of LH Bs and Gs.
S: RH. [T ignores].
Missed opportunity.
missed evaluation
S: Even amount of Bs and Gs.
T: Even as in what?
S: Even as in 11 Bs and 11 Gs.
T: So, instead of saying even, because even makes me think of even numbers and stuff.
S: But it is even numbers.
T: Is it? How many Bs? ... 11 ... Is 11 an even number?
Could get a little confusing. So how could we word it?
S: There are the same amount of Bs and Gs.
Missed opportunity. In previous lesson, T had used the word 'even' in the sense of 'equal numbers', which she is now recommending against using.
appropriate language
sorting 2 variables - focus on univariate
S: 9 people can whistle and 6 people can't.
T: Instead of exact numbers what could we say?
S: More people can whistle than can't whistle ... More whistlers than non-whistlers.
sorting 2 variables - focus on univariate
S: There are more Bs that are RH than Gs.
S: Same amount of LH Bs and Gs.
S: There are 3 Bs and 3 Gs who are middle children.
T: Same amount of B and G middle children.
S: There's more RH people than LH.
S: Already got that.
T: Have we?
S: There's the same amount of LHers.
T: Oh that one can't be a true statement.
S: Yes it can, it can be, I think.
T: Say it again.
S: There's more RH people than LH.
T: True?
Ss: Yes ... not sure.
sorting 2 variables - focus on univariate
no extra clarification
SKC: IntegStatContext
T listens to S discuss the knowledge of what he had seen on TV about Gs who lied that they could whistle. Half of them lied; 90% of them lied.
T: In this class?

S: On TV. ...
T: Interesting but doesn't apply to this data, which is this class.
S: They still could have lied.
Missed opportunity perhaps in terms of discussing validity of data in data collection; applicability of that data to our class - why or why wouldn't it apply to our class.

Title: Sch4Lesson2Int.mp3

Type: Audio
Time: 29-Aug-06 8:56

KCT: InterrogCycle

[S4L2 V 0:00 KCT InterrogCycle - predictions]
T encouraged Ss to make predictions about what the data might show before examining it carefully. This helps Ss to engage with the data having
considered what it might be like, what it might show.
posing invest. questions

SKC: Variation

[S4L2 V 1:22 CKC Variation - S's answer re. similarities/differences between classes' data.
S: Because these weren't the ones that we don't know.
T: This wasn't a good explanation. She could have said, they might be different ages or ... She just thought they were a different group of people.
You can understand that but there wasn't a logical reason as to why.
R: Could be similarities could be differences. Just because they are people we don't know is not a good reason for suggesting the data would be
different.
T did not explore these ideas with the S when she gave that answer. Missed opportunity no extra clarification
T: I was thinking that they would think it's going to be quite similar because questions are the same, only so much variation that can come from
those particular questions.
Good understanding of variation.

KCS: ReasonModels

[S4L2 V 6:17 SKC ReasonModels - 8 summary statements from Ss, and each refined through questioning from T
1. More Gs can whistle cf. Bs
Accurate?
T: Comparatively, because there may have been a different no. of Bs and Gs so could have looked further into that. But most of the class are not
ready for proportions.
R: There are more Gs who can whistle than Bs; or there are more Gs who can whistle than Bs who can whistle?
T: I see what you are saying.
R: Everyone probably interpreted it that way but little things are needed for accuracy/precision/clarity. Ss found some of these statements hard.
T: Yes it is. I didn't expect them to. ... even with data squares there. Some are alright, but the average S in the class doesn't have the greatest grip
on it. .. Just where. you assume they are at by the time they are at this level.
R: Initial statements were comparing 2 variables at a time; then headed towards frequencies of one variable.
T: They were just having trouble seeing past that. Maybe to them, what was the relevance. Perhaps I needed to start with some guiding questions
in the way that can you find out .... I made them look at those variables. I still think they would have struggled with it. It must be something new to
them. Definitely some gaps in their knowledge.
Missed opportunity - T surprised about the way they could and could not handle verbal statements from data.
appropriate language
statement difficulty
sorting 2 variables - univariate, frequencies or comparisons.

SKC: ReasonModels

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them. Definitely some gaps in their knowledge.
KCS: IntegStatContext
[S4L2 V 13:08 SKC IntegStatContext - S talking about things on TV to do with people lying with data. T made conscious decision not to follow up on this, even though she wasn't listening deeply to what he was saying. If we had talked about the
honesty with which the people in this class had answered the question, some could have taken offence to that which would have distracted the
discussion.
Other Ss tune out when he starts going off on tangents, which he does quite often.
listening - missed evaluation - choice