Timothy Jacobbe - Elementary school teachers' understanding of essential topics in statistics and the influence of assessment instruments and a reform curriculum upon their understanding

ELEMENTARY SCHOOL TEACHERS' UNDERSTANDING OF ESSENTIAL TOPICS IN STATISTICS AND THE INFLUENCE OF ASSESSMENT INSTRUMENTS AND A REFORM CURRICULUM UPON THEIR UNDERSTANDING

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Curriculum and Instruction

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by
Tim Jacobbe
August 2007

Accepted by:
Dr. Bob Horton, Committee Chair
Dr. Donna Diaz
Dr. Elizabeth Edmondson
Dr. Suzanne Rosenblith

ABSTRACT

With the increased emphasis on statistics at the elementary school level as recommended by the GAISE document, it is important to consider the current status of teachers' understanding of statistics. This study explored teachers' understanding as they implemented new curricula materials which have ten lessons focused on statistical topics at each grade level. These curricula materials provided a medium for discussion that helped reveal whether or not elementary school teachers possess an adequate understanding of essential topics in statistics to implement the standards they are now called upon to teach.

The results of this study indicate that elementary school teachers may not be prepared to teach statistics at the level of depth described in the GAISE document. There are several implications that are drawn from the results including the importance of a statistics course during teacher preparation programs, sustained professional development, and careful consideration during the creation of curricular materials. Teachers who are unprepared to teach material at the level the GAISE document now prescribes face an unfair situation where they have not mastered the material they are now called upon to teach. The standards should not be increased without addressing teachers' preparation to meet those standards.

DEDICATION

This dissertation is dedicated to my loving wife Elizabeth and our two wonderful children, Hannah and Nathan. Without Elizabeth's patience, support, understanding, and love, I would not have had the time and energy to complete this project. Hannah and Nathan have patiently awaited the completion of this dissertation and are looking forward to more play time with Daddy. The completion of this program would not be as fulfilling if it were not for the happiness they bring to my life.

ACKNOWLEDGMENTS

This dissertation could not have been written without Dr. Bob Horton who not only served as my advisor but also encouraged and challenged me throughout my academic program. He and the other members of my committee, Dr. Donna Diaz, Dr. Elizabeth Edmondson, and Dr. Suzanne Rosenblith guided me through the dissertation process. In addition to constructive feedback, Dr. Donna Diaz provided me support and encouragement throughout the process.

I would like to thank Dr. Bill Moss and Dot Moss who provided support for my entire family while going through the final stages of the dissertation process. I would also like to thank my brother, Mike, and his wife, Julie, who provided tremendous support for my family. Their support provided comfort during a difficult time of transition.

In addition, my colleagues at Educational Testing Service provided support during the research process. In particular, Dr. David Banach and Jeff Haberstroh provided insight and guidance during the data collection process. Through my interactions with various programs with Educational Testing Service, I had the honor of working with Dr. Chris Franklin, Dr. Brad Hartlaub, and Dr. Calvin Williams. They provided support and guidance whenever I had questions.

Finally, I cannot thank Dr. Barbara Moses enough for sparking my interest in mathematics education. I would not have reached this point in my career without the influence of Dr. Moses. She not only served as my advisor during undergraduate and graduate studies at Bowling Green State University, but she has become a lifelong friend.

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CHAPTER ONE

INTRODUCTION

Based on the groundbreaking efforts of the Quantitative Literacy Project (Schaeffer, 1986), the National Council of Teachers of Mathematics (NCTM) has gradually increased the depth of statistical topics covered in elementary, middle, and secondary schools (1989, 2000). The Data Analysis and Probability Standard in the *Principles and Standards for School Mathematics* (NCTM, 2000) encourages instructional programs from prekindergarten through grade 12 [that] enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data; AND
- understand and apply basic concepts of probability. (NCTM, 2000, p. 48)

In order to support the objectives set forth in the *Principles and Standards for School Mathematics,* the American Statistical Association released a curriculum framework for PreK-12 Statistics Education known as the Guidelines for Assessment and Instruction of Statistics Education (GAISE) (Franklin et. al., 2005). “This *Framework* provides a conceptual structure for statistics education which gives a coherent picture of the overall curriculum. This structure adds to but does not replace the NCTM recommendations” (Franklin et al., 2005, p. 5).

The GAISE suggests an increase in the level of sophistication for the teaching and learning of statistics in K-12 classrooms. The authors indicate "a major objective of statistics education is to help students develop statistical thinking. Statistical thinking, in large part, must deal with the omnipresence of variability; statistical problem solving and decision making depend on understanding, explaining and quantifying the variability in the data" (Franklin et. al., 2005, p.5). The goals set forth by NCTM are dependent upon students developing a sophisticated sense of variability in data. However, an examination of the specific goals set forth by NCTM reveals no direct identification of variability. The GAISE document helps clarify the path students should take in order to develop a deeper understanding of variability in data. Without such a framework in the past, elementary school teachers may not have developed the necessary level of understanding to teach statistics with an eye toward variability.

One of the primary concerns that motivated the creation of the GAISE document was that "statistics...is a relatively new subject for many teachers who have not had an opportunity to develop sound understanding of the principles and concepts underlying the practices of data analysis that they are now called upon to teach" (Franklin et al., 2005, p.5). With these expansions to the K-12 curriculum, it is important to examine what teachers know about the subject matter and how their awareness of the adequacy of that understanding is influenced by the exposure to more advanced content.

This chapter provides an overview of the inclusion of statistical topics in the K-12 setting as well as the issues associated with the increased expectations of teachers. Furthermore, the issue of problematizing teachers' awareness of their understanding is discussed in relation to previous research on teacher change. Finally, the research questions guiding this study are identified.

Statistics in the K-12 Setting

The inclusion of statistical topics, at least at a surface level, has been gaining momentum since the first half of the 20th century. In 1947, a National Research Council report indicated that an introduction to statistics should be included in the school curriculum. However, the report indicated such an inclusion should occur "as soon as there is a sufficient supply of trained teachers." As the calls for inclusion of statistics in the curriculum increased, a joint committee between the American Statistical Association (ASA) and the NCTM was formed to define objectives for the K-12 curriculum in the area of statistics (Garfield & Ahlgren, 1994). The joint committee led to the Quantitative Literacy Project (QLP) which resulted in the development of supplemental materials for grades 6-10. This project helped lay the foundation to change which culminated in the inclusion of statistical topics in NCTM's *Curriculum and Evaluation Standards* (1989) and *Principles and Standards for School Mathematics* (2000) (Scheaffer, 2001).

Since statistics has not received much attention in the school curriculum until recently, there is limited research on the variables related to teaching statistics in the K-12 setting. Garfield (1988) identified four issues related to teaching statistics: (1) the role of probability and statistics in the curriculum, (2) links between research and instruction, (3) the preparation of mathematics teachers, and (4) the way learning is currently being assessed.

The first issue identified by Garfield influenced the second issue for both teachers and researchers. Many of the studies (e.g. Fischbein, Nello, & Marino, 1991; Fischbein & Schnarch, 1997; Jones et al., 1999; Shaughnessy, 1985, 1992, 1993) have focused on probability, but not on statistics. Studies that have involved statistics (e.g., Doerr & English., 2003; Mevarech, 1983; Watson et al., 2003) centered on students' understanding of statistical content at the secondary level or beyond. There is limited research in the area of statistical understanding at the elementary school level.

As stated previously, the third issue identified by Garfield was one of the motivations which led to the GAISE. It is not known whether teachers have an adequate understanding of statistical topics in order to teach the content at the recommended level of depth. Furthermore, it is likely teachers have not seen the statistical topics they must teach since they were students in school themselves, if they saw them at all (Franklin, personal communication, February 7, 2007). Despite the claim by national organizations and authors of standards documents that teachers do not have an adequate understanding of statistics, a search for relevant research on elementary school teachers' understanding of statistical content revealed no previous studies.

The issue of teachers' preparation is also raised by Shaughnessy when he comments that "teachers' backgrounds are weak or nonexistent in [statistics] and in problem solving. This is not their fault, as historically our teacher preparation programs have not systematically included either [statistics] or problem solving for prospective mathematics teachers" (1992, p. 467).

"To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (NCTM, 2000, p. 17). Elementary school teachers often associate adequate content knowledge with an understanding of the material at the level they teach. "Their conceptions of the nature of mathematics shape the kinds of questions they ask, the ideas they emphasize, and the sorts of tasks they choose for their students" (NCTM, 1991, p. 71). With a superficial understanding of the content, teachers are not able to expand on the ideas introduced by their students, nor are they able to correct subtle misconceptions that arise during classroom discussions (Campbell & White, 1997).

The point raised by Campbell and White relates to the fourth issue identified by Garfield. If teachers do not have an adequate understanding of the content, then they are only able to assess students at superficial levels of learning. Questions in textbooks and on standardized tests tend to ask students to find certain values or to calculate simple measures of center (Konold & Khalil, 2003). These types of questions are not consistent with the increased expectations presented in the GAISE. Teachers must now have a deeper command of statistical content in order to help ensure their students develop the understanding expected of them.

According to Dewey, "[The teacher's knowledge] must be wider than the ground laid in textbooks or in any fixed plan for teaching a lesson. It must cover collateral points, so that the teacher can take advantage of unexpected questions or unanticipated incidents" (1910, p. 275). During content courses for elementary mathematics teachers, preservice teachers revisit material they should already have learned in school, but they revisit the material at a deeper level. In order to understand the mathematics they teach, teachers need to possess an understanding that runs deeper than the level of understanding they developed as elementary school students.

Particularly in elementary school, “the prevailing assumption is that the content of the K-12 curriculum is already understood by teachers and. . .is relatively simple. However, in reality, what teachers have learned about mathematics in their pre-college mathematics classes is not adequate for teaching mathematics [or statistics] for understanding” (NCTM, 1991, p. 74). Hatano (1982) indicated that it is a mistake to assume that individuals who know mathematical procedures understand the underlying concepts of those procedures.

Researchers have focused on the importance of knowing mathematics in order to teach effectively. However, much of the work on elementary school teachers’ content knowledge has found them lacking (e.g. Ball, 1988, 1990, 1991; Ball, Hill, & Bass, 2005; Collopy, 2003; Fennema & Franke, 1992; Remillard, 2000; Shulman, 1987). Particular research has focused on multiplication and division (Ball, 1990; Graeber, Tirosh, & Glover, 1989; Simon, 1993; Simon & Blume, 1994; Wheeler & Feghali, 1983; Zazkis & Campbell, 1996), fractions (Ball, 1990; Borko et al., 1992; Lehrer & Franke, 1992), decimals (Thipkong & Davis, 1991; Tirosh & Graber, 1990); measurement (Enochs & Gabel, 1984), and area and perimeter (Menon, 1998; Reinke, 1997) (in Bush, et. al. under review). There is a gap in research focused on teachers’ understanding of statistics.

As Shaughnessy points out, “the real barriers for improvement of [statistics] teaching. . .are fundamentally (a) getting [statistics] into the mainstream of the mathematical science school curricula at all, (b) enhancing teachers' background and conceptions of probability and statistics, and (c) confronting students' and teachers' beliefs about probability and statistics" (1992, p. 467). The efforts of the QLP and NCTM have addressed the first barrier for the improvement of statistical teaching. However, there is a gap in the research that speaks to the barrier involving teachers' understanding of statistical topics and how to confront teachers' awareness of their understanding.

Problematizing Teachers' Awareness of their Understanding

Teachers must acknowledge a deficiency before they are willing to dedicate time and energy to learning more advanced content (Richardson & Placier, 2001). Clarke pointed out that "when it came to [teachers] integrating mathematics to solve problems within the context of a 'realistic' unit, they had difficulty. This led to situations where their mathematical understanding was challenged" (1995, p. 157). The same can be said about teachers as they become exposed to statistics. When they are asked to look beyond simple calculations they have performed in the past, teachers may begin to question the adequacy of their understanding.

Teachers must be willing to re-examine their own understanding of mathematics (e.g. Ball, 1989; Thompson, 1991). Many teachers were educated in a system which promoted the memorization of rules and procedures. Their notions of what it means to be a mathematics teacher are influenced by their previous experiences. As teachers are required to increase the level of sophistication involved with covering statistical topics in accordance with the GAISE and NCTM's *Principles and Standards for School Mathematics,* they may experience a conflict. There could be a dramatic difference between the level of understanding they were expected to develop during their educational process and what is now required of them to teach.

Elementary teachers' "knowledge of mathematics is shallow and…this deficiency represents a real impediment to achieving reform" (Cooney, 1994, p. 11). One or two semesters of mathematics in college, even if taught in a manner consistent with the NCTM vision, does not significantly alter individuals' views and understanding of mathematics (Borko et al., 1992). Therefore, teachers must be exposed to experiences which challenge their existing views and understanding of statistics through professional development and other activities (Horton, 1997).

According to Richardson and Placier, "the long-term collaborative, and inquiry-oriented programs with inservice teachers appear to be quite successful in changing… conceptions, although not all teachers respond well to such approaches" (2001, p. 921). In order for any intervention to be successful, teachers must be willing to change. This willingness to change is associated with an awareness of a need for change. Schon (1983) described a reflective practitioner as someone who reflects upon himself or herself with an eye toward improving as a teacher.

It takes a reflective practitioner to re-examine the adequacy of his or her understanding in light of the recent reform movements in statistics education. Moreover, in light of these recent movements there is need for continued research on elementary school teachers' understanding of essential topics of statistics.

Purpose of this Study

With the increased emphasis on statistics at the elementary school level, it is important to consider the current status of teachers' understanding of statistics. In order to explore teachers' understanding, research was conducted with teachers implementing new curricula materials, *Math Out of the Box* (Moss et al., 2005), which have ten lessons focused on statistical topics at each grade level. These curricula materials provided a medium for discussion that helped reveal whether or not elementary school teachers possess an adequate understanding of statistics to implement the standards they are now called upon to teach. The questions that guided the data collection for this study are:

Research Questions

1. What is the understanding of elementary school teachers in the following areas of statistics: data displays, measures of central tendency, and measures of variation?
2. Does the implementation of the curricula materials and exposure to advanced assessment instruments influence elementary school teachers' awareness of their understanding of statistics?

CHAPTER TWO

A REVIEW OF THE LITERATURE

In order to consider the level of understanding required to teach statistics at the elementary school level, it is helpful to explore what the GAISE are expecting students to learn in elementary and middle school. It is reasonable to expect teachers to possess at least this level of understanding if they are to teach statistics effectively (Franklin, personal communication, February 7, 2007). With the GAISE providing a medium to define the content of statistics, different types of content knowledge are discussed. Finally, specific "types" of content knowledge are chosen from the literature to serve as the foundation for discussion of elementary school teachers' understanding of statistics.

What do the Standards Say?

At least in some arenas of mathematics education, there seems to be some confusion over what constitutes the study of statistics. If an elementary school teacher were asked to define statistics, he or she would probably say it involves the calculations of mean, median and mode (McGatha, Cobb, & McClain, 1998). Although the calculations of such measures of center are important in the study of statistics, they are not the foundation of the study. Such confusion is rather surprising and disappointing based on the efforts of the Quantitative Literacy Project, QLP, (Schaeffer, 1986) and the NCTM (1989, 2000).

The QLP identified a set of guidelines for teaching statistics. The guidelines were:

1. Experiences (activities) for students should be focused on asking questions about something in the students' environment and then finding quantitative ways to answer the question.

2. Problems should be approached in more than one way with an emphasis on discussion and evaluation of these different methods.

3. Real data should be used whenever possible in any statistics lesson, and classroom presentations should give students hands-on experience in working with data.

4. Traditional topics in statistics should not be taught until students have experienced and worked with simple counting and graphing techniques, and have established a foundation for those traditional ideas.

5. The emphasis in teaching statistics should be on good examples and building intuition, not on probability paradoxes or using statistics to deceive.

6. Student projects should be an integral part of any work in statistics.

7. The emphasis in all work with statistics should be on the analysis and the communication of this analysis, not on a single answer. (Scheaffer, 1991) (Note: Although this reference is from 1991, these guidelines were used in the creation of the QLP materials in 1986 and thus were available in 1989 when NCTM published the *Curriculum and Evaluation Standards for School Mathematics*.)

These guidelines set the foundation for NCTM's (1989) statistics and probability strand. As early as the K-4 grade band, students should "collect, organize, describe, display, and interpret data as well as make decisions and predictions on the basis of that information" (1989, p. 54). The clarity of the message continued with the release of the *Principles and Standards for School Mathematics* (NCTM, 2000). There, as early as grades 3-5 "students should move toward seeing a set of data as a whole, describing its shape, and using statistical characteristics of the data such as range and measures of center to compare data sets" (NCTM, 2000, p. 177). Even though substantial efforts were made by the QLP and NCTM, confusion still existed. For that reason, the GAISE document was created to further clarify the content necessary to develop statistical understanding in the K-12 setting.

The GAISE Document

The message from the statistical community is stronger with the release of *A Curriculum Framework for PreK-12 Statistics Education* (Franklin, et. al., 2005). This document identifies three levels of statistical development (Levels A, B, and C) that students must progress through in order to develop statistical understanding. Grade ranges for attainment of each level are intentionally unspecified. Students must begin and master the concepts at Level A before moving on to Levels B and C. A discussion of these levels sheds light on the level of statistical understanding expected during the elementary years, based on recommendations from the GAISE document. It is paramount for students to have worthwhile experiences at Levels A and B during their elementary school years in order to prepare for future development at Level C at the secondary level (Franklin, personal communication, February 7, 2007). "Without such experiences, a middle [or high] school student who has had no prior experience with statistics will need to begin with Level A concepts and activities before moving to Level B" (Franklin, et.al., 2005, p.13).

The following discussion outlines what the GAISE are calling for at each level. Level A serves as the focus for this discussion, since it is reasonable for the objectives of Level A to be realized in the primary grades (Franklin, personal communication, February 7, 2007). The objectives of Level B are also discussed as these build upon the understanding students develop at Level A. For Levels A and B, topics are discussed in the following categories: data displays, measures of central tendency, and measures of variation.

Level A

The objectives of Level A are:

1. [Students] need to develop data sense – an understanding that data are more than numbers. Statistics changes numbers into information.

2. Students should learn that data are generated with respect to particular contexts or situations and can be used to answer questions about the context or situation.

3. Students should have opportunities to generate questions about a particular context (such as their classroom) and determine what data might be collected to answer these questions.

4. Students should learn how to use basic statistical tools to analyze the data and make informal or casual inferences in answering the posed questions.

5. Students should develop basic ideas of probability in order to support their later use of probability in drawing inferences at Levels B and C. (Franklin, et.al., 2005, p. 23)

Data Displays

Throughout their experiences at Level A, students should be exposed to a variety of displays for exploring distributions and association (Franklin, et. al, 2005, p. 24). These displays should include frequency tables (p. 24), bargraphs (pp. 25-26), stem-and-leaf plots (p. 27), dotplots (p. 28), scatterplots (p. 31), and time plots (p. 32). The GAISE specifically indicate that students at Level A should not be exposed to pictographs or circle graphs as these "type[s] of graph[s] require a basic understanding of proportional or multiplicative reasoning (p. 25)." In addition, students should be exposed to the proper use of a bargraph versus a histogram. "A bargraph is used to summarize categorical data. If a variable is numerical, the appropriate graphical display with bars is called a histogram, which is introduced at Level B. At Level A, appropriate graphical displays for numerical data are the dotplot and stem-and-leaf plot" (Franklin, et. al., 2005, p. 35).

Measures of Central Tendency

Graphical displays provide information students can use to calculate descriptive statistics, including measurements of central tendency. The GAISE document indicates that students should be able to determine means, medians, and modes. Respectively, students at Level A should understand these measurements as a fair share (p. 30), middle point (p. 29), and values that occur most often (p. 26). In their analysis of the Sixth Mathematics Assessment of the NAEP, Zawojewski and Heckman (1997) found that students in 7^{th} and 11^{th} grade do not understand the mean, median, mode, and range. Their findings were based on the examination of students' performance on the NAEP which evaluated the percent correct and response rate for all questions related to data analysis and statistics.

An analysis conducted by McGatha, Cobb and McClain (1998) indicated that when students are asked to find the "center" of a set of data, they most often choose the mean regardless of the context. This analysis was based on a study involving 7th grade students. In their study, McGatha, Cobb and McClain used performance assessments (which presented data within a context) in 3 sessions with the 7^{th} grade students. The number of students involved in the study was not reported. The assessments were administered by a former middle-school teacher who was part of the research group. The students worked in groups of 3 to 6 on the tasks. The fact that the students' understanding of the center of a data set was limited to the mean is telling since the students worked in groups of 3 to 6. If the students were unable to consider values other than the mean while working together as a group, then it is likely they would not consider values other than the mean while working individually. Despite the ability to work with one another, the students did not utilize other measures of central tendency regardless of the context in the task.

In an analysis of the Seventh Mathematics Assessment of the NAEP, Zawojewski and Shaughnessy (1999) found similar results. However, in some contexts it is inappropriate to calculate a mean. For example, the mean is not an appropriate measure of central tendency for categorical data. Students often try to compute the mean or find the median of a categorical set of data. For example, if a set of data is categorized by gender where the numbers 1 and 2 represent a female and male, respectively, they may calculate a mean by adding up all the values and computing a mean that falls somewhere between 1 and 2, which is meaningless in this context.

Furthermore, students do not realize that in some contexts the median may be a more appropriate measure of center (Zawojewski & Heckman, 1997). For example, to determine the "average" salary in the United States it is more appropriate to use the median than the mean. In this context, extremes (outliers), like some professional athletes' salaries, increase the mean and possibly misrepresent the center of the data (Franklin, et. al., 2005, p. 35). The most likely reason why students often calculate the mean without thinking of the specific context is that they have been exposed to only noncontextual situations where the objective is to correctly perform a calculation, rather than use a statistic to analyze a set of data (McGatha, Cobb, & McClain, 2002; Zawojewski & Shaughnessy, 1999).

Further evidence of students not understanding the concept of mean is evident in the work of Gal, Rothschild, and Wagner (1990). In their study they interviewed students of age 8, 11, and 14 to determine the usefulness of the mean. In particular the students were presented with two data sets and asked to indicate whether the means were different. Of the students who were able to successfully calculate a mean, only 50% of the 11 and 14 year olds used the measures of center to compare the values.

Another misconception regarding measures of central tendency is the notion that an average is a typical score. Konold and Higgins (2003) found that younger students often choose the mode to summarize a distribution of data because they associate "typical" with the value that occurs most often. The mode is an appropriate measure of center; however it is not the only appropriate measure of center. Other research has shown that students associate the center of data to be in some range or cluster of values (Cobb, 1999; Konold, Robinson, Khalil, Pollatsek, Well, Wing, & Mayr, 2002; Mokros & Russell, 1995; Noss, Pozzi, & Hoyles, 1999; Watson & Moritz, 1999). It is important that students be exposed to a variety of contexts so they can determine which measure of center best summarizes the data for a particular context.

Research indicates that students do not know which statistic to use in specific situations. One of the reasons the mean may be the most informative measure of center is that it includes all values of a data set. However, it is more likely that students simply perform the calculation out of some set procedure they have memorized (McGatha, Cobb, & McClain, 2002). Zawojewski and Heckman (1997) found that students do not understand when to use the median. Students should understand the advantages and disadvantages of each measure of central tendency for a given context (Zawojewski & Shaughnessy, 2000).

Measures of Variation

In addition to developing an understanding of center, the authors of the GAISE document recommend that students at Level A should become familiar with variation through considering the maximum and minimum values of a data set. At Level A, students can use these values to calculate the range of the data (Franklin, et.al., 2005, p. 30).

Since the study of statistics exists in order to explore variability (Cobb & Moore, 1997; Konold & Pollatsek, 2002), elementary school students should begin to question why variations occur in data. If variations are discovered in data sets, students should be encouraged to examine what factors may have caused these variations. Variations may be due to errors in data recording, natural occurrences, or the results of something interesting to explore. "The notions of error and variability should be used to explain the outliers, clusters, and gaps that students observe in the graphical representations of data. An understanding of error versus natural...variability helps students to interpret whether an outlier is a legitimate data value that is unusual or whether the outlier is due to a recording error" (Franklin, et.al., 2005, p.33).

According to Watson, Kelly, Callingham, and Shaughnessy (2003) there is little research on students' understanding of variability. Shaughnessy claims that the absence is due to the fact that the K-12 curriculum has focused on measures of center rather than on variation. As students move through Levels A, B, and C it is important to return to the concept of variation and what role it plays in statistical analysis.

Level B

The concepts discussed in level B are a continuation of the experiences students are exposed to at Level A. At Level B:

1. Students become more aware of the statistical question distinction (a question with an answer based on data that vary versus a question with a deterministic answer).

2. Students make decisions about what variables to measure and how to measure them in order to address the question posed.

3. Students use and expand the graphical, tabular and numerical summaries introduced at Level A to investigate more sophisticated problems.

4. Students develop a basic understanding of the role that probability plays in random selection when selecting a sample and in random assignment when conducting an experiment.

5. Students investigate problems with more emphasis placed on possible associations among two or more variables and understand how a more sophisticated collection of graphical, tabular and numerical summaries is used to address these questions.

6. Students recognize ways that statistics is used or misused in their world. (Franklin, et.al., 2005, p.37)

Data Displays

The data displays introduced to students at Level A are expanded upon at Level B. Students are introduced to histograms (p. 44), frequency tables (p. 44), grouped frequency and relative frequency tables (p.45), boxplots (p. 46) and time-series plots (p.55). Students at Level B should be exposed to misuses of graphs in the media. In particular students should be exposed to misuses of pictographs which compare distributions inappropriately (Franklin, et. al., 2005, p. 57).

Measures of Central Tendency

The measures of central tendency introduced at Level A are also expanded upon at Level B. The biggest "expansion" to the measures of center introduced at Level A is that students should begin to see the mean as a "balance point" rather than as a "fair share" (Franklin, et.al., 2005, p. 41). The following activity provided by the GAISE gives an example of how students should visualize this concept.

> Nine students were asked: How many pets do you have? The resulting data were 1, 3, 4, 4, 4, 5, 7, 8, 9. [These data are summarized in a dotplot]
> If the pets are combined into one group, there are a total of 45 pets. If the pets are evenly redistributed among the nine students, then each student would get five pets. That is, the mean number of pets is five. [The dotplot is then presented with 9 dots above the 5]
>
> It is hopefully obvious that if a pivot is placed at the value 5, then the horizontal axis will "balance" at this pivot point. That is, the "balance point" for the horizontal axis for this dotplot is 5. What is the balance point for the dotplot displaying the original data? We begin by noting what happens if one of the dots over 5 is removed and placed over the value 7 [They show a dotplot with 8 dots over 5 and one dot over 7]. Clearly, if the pivot remains at 5, the horizontal axis will tilt to the right. What can be done to the remaining dots over 5 to "rebalance" the horizontal axis at the pivot point? Since 7 is two units above 5, one solution is to move a dot two units below 5 to 3, as shown below [A dotplot is shown with 1 dot over 3, 7 dots over 5, and 1 dot over 7]. The horizontal axis is now rebalanced at the pivot point. Is the only way to rebalance the axis at 5? No. Another way to rebalance the axis at the pivot point would be to move two dots from 5 to 4, as shown below [A dotplot is shown with 2 dots above 4, 7 above 5, and 1 above 7].

> The horizontal axis is now rebalanced at the pivot point. That is, the "balance point" for the horizontal axis for this dotplot is 5. Replacing each dot in this plot with the distance between the value and 5 we have [There is a dotplot with dots replaced by the distance away from 5, so there are two 1's above 4, seven 0's above 5, and one 2 above 7]. Notice that the total distance for the two values below the 5 (the two 4's) is the same as the total distance for the one value above the 5 (the 7). For this reason, the balance point of the horizontal axis is 5. Replacing each value in the dotplot of the original data by its distance from 5 yields the following plot [There is a dotplot with one 4 above 1, one 2 above 3, three 1's above 4, one 0 above 5, one 2 above 7, one 3 above 8, and one 4 above 9].
>
> The total distance for the values below 5 is 9, the same as the total distance for the values above 5. For this reason, the mean (5) is the balance point of the horizontal axis. (Franklin et al., pp. 41-43)

Measures of Variation

Students at Level B should be exposed to more sophisticated measures of variation. "At Level B, students should be introduced to the idea of comparing data values to a central value, such as the mean or median, and quantifying how different the data are from this central value" (Franklin, et. al., 2005, p. 44). The GAISE recommends students expand their "tools" for measuring variation from the range to the Mean Absolute Deviation (MAD) (Franklin, et. al., 2005, p. 44). Students at Level B should also be introduced to the interquartile range (IQR) (Franklin, et. al., 2005, p. 47).

Summary

Although there are not many more graphical displays, measures of central tendency, or measures of variation introduced at Level B, the major difference between Level A and Level B is the sophistication with which students examine data. At Level B students use the foundational understanding developed at Level A to compare groups and make associations between sets of data (Franklin, et. al, 2005, pp. 27, 31, 32, 46-47, 48-49, 49-50, 51-52). They also look at shapes of distributions to determine if outliers are present (without formal calculations) (Franklin, et. al., 2005, p. 48). Finally, they begin to compare the variability between groups (Franklin, et. al, 2005, p. 39) and explore the concept of how repeated sampling may reduce the variability in a set of data (Franklin, et. al, 2005, p. 54).

The objectives of Levels A and B lay the groundwork for students to begin to realize many of the objectives set forth by the NCTM (1989, 2000). This progression should allow students to begin formulating their own questions of interest, collect data, and analyze their results based on the concepts of center and variation. Since it is reasonable for students to realize the objectives of Level A prior to completing elementary school, then, at the very least, it is reasonable to expect elementary school teachers to have an understanding of statistical topics through Level B (Franklin, personal communication, February 7, 2007). In order to begin discussing what understanding should be expected of teachers, one must consider research related to statistics at the elementary school level. What follows is a summary of those findings.

Research Related to Statistics in the Elementary School

Although a review of the literature does not reveal any prior studies involving the statistical understanding of elementary school teachers, there are two studies related to the research presented here. An extensive search of the literature revealed that Teachers Ideas About Teaching Statistics (Begg & Edwards, 1999) was the only study that focused on the teaching of statistics at the elementary school level. Unfortunately, the full reporting of the original study could not be completed since the graduate student, Roger Edwards, conducting this work passed away during the project. His advisor, Andy Begg, continued with his direction and published a paper based on their joint work. The data that was presented was based on "unstructured, semi-structured, and clinical interviews; survey (Likert) scales that provided a guide with respect to the efficacy of the research" (Begg & Edwards, 1999, p. 2). The sample included 22 inservice elementary school teachers and 12 preservice elementary school teachers in New Zealand. The majority of teachers were females (specific number not reported) and many of the inservice teachers had substantial teaching experience (mean number of years not reported).

In general teachers' attitudes toward statistics were negative. Some of the words they associated with the subject were "fear, horrors, uninteresting, boring, and horrible graphs" (Begg & Edwards, 1999, p. 2). In considering the teachers' ideas of average, or measures of center, most teachers were not familiar with the mathematical definitions of the terms mean, median, and mode. When asked about the word average, the most common response given was that it "was in the middle." However, when pressed about their understanding regarding specific measures, the teachers possessed better understanding of the mean than the median or mode (Begg & Edwards, 1999, p. 5).

Begg and Edwards (1999) found that teachers did not rate the importance of teaching statistics at the elementary level very high. Nor did teachers consider the development of a deeper understanding of statistics important. When teachers were asked whether they would prefer professional development which focused on statistical understanding of the topics they taught or on activities for students, the teachers most often preferred obtaining activities. This preference was despite a self-admitted lack of understanding of statistics by the teachers. The lack of understanding was further evident in that most of the teachers were "unfamiliar with one or more of the [statistical] terms taken from the curriculum" (Begg & Edwards, 1999, p. 8).

Greer and Ritson (1994) conducted a different study related to elementary school teachers and statistics. The motivation behind their study was that Northern Ireland, under the United Kingdom education system, had been increasing the amount of statistical topics covered in the K-12 setting without increasing the content covered during teacher preparation. In order to explore the readiness of teachers in Northern Ireland to teach statistics, Greer and Ritson conducted a survey of 16 elementary and 24 high school teachers. The surveys were conducted through interviews rather than through mail. The interviews contained open-ended questions and prompts which raised issues of importance to the teachers. Further details of the methods used by Greer and Ritson were not available. They reported that although there is reason for concern at both levels, "judging by this sample, [elementary school] teachers are ill-prepared to teach [statistics]" (Greer & Ritson, 1994, p. 52). This was based on the teachers' responses which indicated that of the 16 elementary school teachers: 94% felt they were not taught the content during their teacher training courses; 63% felt they had never learned about the topics since then; and 88% felt they did not understand the mathematics necessary to teach the topics.

Summary

Although Begg and Edwards (1999) and Greer and Ritson (1994) did not directly assess the statistical understanding of elementary school teachers, the studies reveal specific areas where teachers are unprepared to teach. Perhaps, the lack of understanding is a result of being educated in a system that does not develop statistical understanding at the level discussed in the GAISE. "Such a limited view of statistics...means that teachers may find it difficult to enable their students to take possession of the content if they have not previously taken possession of the content themselves" (Begg & Edwards, 1999, p. 10).

Content Knowledge

In order to discuss teachers' understanding of statistics, it is necessary to examine the literature with respect to what constitutes knowledge in general. The following discussion of knowledge involves the work of Ryle (1949), Scheffler (1965), Skemp (1978), Hiebert and Lefevre (1986), and Star (2000, 2005). The literature discussed focuses on the dichotomy of knowledge into two categories and how these categorizations have progressed. Finally, the terms procedural and conceptual knowledge are selected to help describe elementary school teachers' statistical understanding in this study.

Gilbert Ryle (1949) first distinguished between "knowing how" and "knowing that." Ryle defines these phrases primarily through examples.

> We speak of learning how to play an instrument as well as learning something is the case; of finding out how to prune trees as well as finding out that the Romans

> had a camp in a certain place; of forgetting how to tie a reef-knot as well as forgetting that the German for 'knife' is 'Messer.' (1949, p. 28)

These distinctions of knowledge type were focused on action or lack thereof. Knowing how involves an ability to perform some type of action and knowing that involves knowledge of a particular fact. "Understanding is a part of knowing how. The knowledge that is required for understanding intelligent performances of a specific kind is some degree of competence in performances of that kind" (Ryle, 1949, p. 54). In regard to this point, is it possible for someone to judge a singing performance if one cannot sing? What constitutes 'competence in performances of that kind' as Ryle suggests? A judge may "know that" of singing, knowing what tune or pitch a song should be sung; however a judge may not "know how" to perform a song in pitch and in tune.

This example is used as it relates to a proposition by Ryle in regard to what it means for someone to know a tune.

> It certainly does not entail his being able to tell its name, for it may have no name; and even if he gave it the wrong name, he might still be said to know the tune. Nor does it entail his being able to describe the tune in words, or write it out in musical notation, for few of us could do that, though most of us can recognise tunes…To describe him as knowing the tune is at the least to say that he is capable of recognising it, when he hears it. (1949, p. 226)

According to Ryle there are varying levels of knowing how and knowing that. A person may know how to do something well or may know how to do something poorly. Similarly, a person may know something (e.g. how to recognize a tune as described above) at varying degrees of complexity (Ryle, 1949).

The work of Ryle was just the beginning of theories related to knowledge. Scheffler (1965) expanded upon the work of Ryle. He was the first to introduce the common distinction of knowledge in terms of concepts and procedures. Scheffler related knowing how to the knowledge of concepts (1965, pp. 19-21) and knowing that to the knowledge of procedures (1965, pp. 14-18).

Richard Skemp (1978) was the first mathematics educator to relate the categorization of knowledge specifically to mathematics. A relationship can be drawn from the distinctions made by Scheffler and Skemp. Because Skemp viewed mathematical knowledge types as "relational" or "instrumental," a relationship can be drawn from the distinctions made by Ryle and Scheffler. Skemp's relational understanding involves "knowing what to do and why" which is analogous to Ryle's "knowing how" and Scheffler's "conceptual knowledge." The instrumental understanding described by Skemp as "rules without reasons" (Skemp, 1978, p. 9) is somewhat analogous to Ryle's "knowing that" and Scheffler's "procedural knowledge."

Skemp identified two types of mathematical mismatches that can occur between teachers and students regarding relational and instrumental understanding. These were:

1. Pupils whose goal is to understand instrumentally, taught by a teacher who want[s] them to understand relationally.
2. The other way about. (Skemp, 1978, p. 10)

According to Skemp, students who want to learn instrumentally simply want some type of rule they can apply in order to obtain an answer. This type of understanding usually involves a multiplicity of rules rather than fewer principles of more general application.

Skemp warns that the other type of mismatch which may occur could be more damaging.

> A less obvious mismatch is that which may occur between teacher and text. Suppose that we have a teacher whose conception of understanding is instrumental, who for one reason or other is using a text which aim is relational understanding by the pupil. It will take more than this to change his teaching style. I was in a school which was using my own text, and noticed that some of the pupils were writing answers like
>
> 'the set of {flowers}'.
>
> When I mentioned this to the teacher (he was the head of mathematics) he asked the class to pay attention to him and said: 'Some of you are not writing your answers properly. Look at the example in the book, at the beginning of the exercise, and be sure you write your answers exactly like that.' (Skemp, 1978, p. 11)

Skemp's work in regard to relational and instrumental understanding led to Hiebert and Lefevre revisiting the terms introduced by Scheffler in order to further define knowledge types in reference to mathematics (1986). Hiebert and Lefevre clearly distinguished between procedural and conceptual knowledge within the context of mathematics.

As one can see from this discussion, there are various terms to distinguish between knowledge types. Hiebert and Lefevre felt that the terms procedural and conceptual knowledge would be useful within the context of mathematics (1986, p. 3). Since it is difficult to define knowledge in terms of this type or that type, they "do not believe…that the distinction provides a classification scheme into which all knowledge can or should be sorted" (Hiebert & Lefevre, 1986, p.3). However, these terms provide a means of discussing varying knowledge types within the context of mathematics.

According to Hiebert and Lefevre, "conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (1986, pp. 3-4). They distinguished between two levels of conceptual knowledge: primary and reflective. Primary conceptual knowledge involves "constructing knowledge at the same level of abstractness (or at a less abstract level) than that at which the information itself is represented" (Hiebert & Lefevre, 1986, p. 4). Reflective conceptual knowledge involves relationships that are not tied to specific contexts. "The relationships transcend the level at which the knowledge currently is represented, pull out the common features of different-looking pieces of knowledge, and tie them together" (Hiebert & Lefevre, 1986, p. 5). For the purposes of this study, only one level of conceptual knowledge, namely the primary level, was considered.

A counterpart to conceptual knowledge is procedural knowledge. According to Hiebert and Lefevre, there are two kinds of procedural knowledge. "One kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems" (Hiebert & Lefevre, 1986, p. 8). Procedural knowledge is structured in that many of the algorithms utilized are dependent upon other algorithms (Hiebert & Lefevre, 1986, p. 7). "Perhaps the biggest difference between procedural and conceptual knowledge is that the primary relationships in procedural knowledge is "after," which is used to sequence subprocedures and superprocedures linearly. In contrast, conceptual knowledge is saturated with relationships of many kinds" (Hiebert & Lefevre, 1986, p. 8).

In order to develop an understanding of mathematics it is necessary for students (and teachers) to possess both procedural and conceptual knowledge (Hiebert & Lefevre, 1986, p. 22). This study investigated to what extent elementary school teachers possess both types of knowledge.

The work of Jon Star has expanded upon the developments of Hiebert & Lefevre in relation to the terms conceptual and procedural knowledge (2005). Star indicates that conceptual knowledge is not defined in the literature as "knowledge of concepts or procedures…rather it is defined in terms of the quality of one's knowledge of the concepts – particularly the richness of the connections inherent in such knowledge" (2005, p. 407). Star indicates that there are two levels of depth in conceptual knowledge. He also argues that Hiebert and Lefevre overlooked the multiple levels of conceptual knowledge. According to Star, "mathematics educators who strictly adhere to Hiebert and Lefevre's (1986) definition implicitly refer only to a particular subset of conceptual knowledge: that which is richly connected or deep" (2005, p. 407). This statement is a misrepresentation of Hiebert and Lefevre's work as they clearly defined two levels of conceptual knowledge: primary and reflective.

Despite this oversight regarding conceptual knowledge, Star does clarify multiple levels of procedural knowledge. By Hiebert and Lefevre's definition, "procedural knowledge is superficial: it is not rich in connections" (Star, 2005, p. 407). When exploring how students solve linear equations, Star hypothesizes that there may be more than one level for procedural knowledge. Star argues that "skilled equation solvers have the ability to use the equation-solving actions flexibly, so that a maximally efficient solution can be generated for any problem type" whereas a student with a lesser degree of procedural knowledge has a limited set of skills to apply to any problem type. Star presented the following example:

> Consider three relatively simple (and superficially quite similar) linear equations:
> (a) 2(x + 1) + 3(x + 1) = 10;
> (b) 2(x + 1) + 3(x + 1) = 11; AND
> (c) 2(x + 1) + 3(x + 2) = 10.
> Although each of these equations can be solved with the same sequence of steps (using a standard algorithm for solving linear equations), the most efficient strategy may not be the standard algorithm. Furthermore, what is meant by the most efficient strategy is quite nuanced. Is the most efficient strategy the one that is the quickest or easiest to do, the one with the fewest steps, the one that avoids the use of fractions, or the one the solver likes best? There are subtle interactions among the problem's characteristics, one's knowledge of procedures, and one's problem-solving goals that might lead a solver to implement a particular series of procedural actions. Someone with only a superficial knowledge of procedures likely has no recourse but to use a standard technique, which may lead to less efficient solutions or even an inability to solve unfamiliar problems. But a more flexible solver – one with a deep knowledge of procedures – can navigate his or her way through this procedural domain, using techniques other than ones that are overpracticed, to produce solutions that best match problem conditions or solving goals. I consider this kind of flexible knowledge to be both procedural and deep. (Star, 2005, p. 409)

The previous example helps to determine two levels of procedural knowledge – one that is superficial and one that is deep. These terms relate to a previous piece of work by Star which presented the information presented in Table 2.1 (Star, 2000, p. 84):

Table 2.1 Procedural Versus Conceptual Knowledge

Knowledge Type	Knowledge quality at endpoint of acquisition	
	Superficial	Deep
Procedural Knowledge	Fully compiled: Automatized	????
Conceptual Knowledge	????	Understood

Based on the example presented by Star and also an accurate description of the varying levels of conceptual knowledge as introduced by Hiebert and Lefevre, the information may be modified as in Table 2.2.

Table 2.2 Revised Procedural versus Conceptual Knowledge

Knowledge Type	Knowledge quality at endpoint of acquisition	
	Superficial	Deep
Procedural Knowledge	Fully compiled: Automatized	Flexible and Adaptable
Conceptual Knowledge	Primary Understanding	Reflective Understanding

Summary

The work of Star has contributed to the continued discussion of types of knowledge. Table 2.2, which distinguishes between types of knowledge and the depths of those types of knowledge, is useful in summarizing the work of Ryle, Skemp, and Hiebert and Lefevre (Scheffler is removed from this discussion as Hiebert and Lefevre's definitions related the terms directly to mathematics). Table 2.3 provides a summary of knowledge types.

Table 2.3 Summary of Knowledge Types

Knowledge Type	Knowledge quality at endpoint of acquisition	
Ryle	Superficial	Deep
Knowing How	Knowledge of Procedures	????
Knowing That	????	Knowledge of Propositions or Facts and Where they Come From
Skemp	Superficial	Deep
Instrumental Understanding	Automatized Procedures	????
Relational Understanding	????	Understanding
Hiebert and Lefevre	Superficial	Deep
Procedural Knowledge	Fully compiled: Automatized	Flexible and Adaptable
Conceptual Knowledge	Primary Understanding	Reflective Understanding

Selecting Terms for the Purpose of this Study

For purposes of this study, the terms originally introduced by Scheffler but expanded upon by the work or Hiebert, Lefevre, and Star are used. As explained, only primary conceptual knowledge was considered as part of the study. The reason for such an exclusion was that the questions involved in the study that assess concepts at Levels A and B of the GAISE document do not reach the reflective level of conceptual knowledge. However, two levels of procedural knowledge were considered in this study. These two levels are referred to as Procedural Low (PL) and Procedural High (PH) and correspond to a superficial and deep knowledge quality at the endpoint of acquisition, respectively.

Conclusion

The terms identified are used to facilitate the discussion of what elementary school teachers "understand" in relation to essential topics introduced in the GAISE document at Levels A and B. The method by which the understanding of elementary school teachers was examined is presented in the next chapter.

CHAPTER THREE

METHOD

This chapter presents the methods that were used to answer the following questions:

1. What is the understanding of elementary school teachers in the following areas of statistics: data displays, measures of central tendency, and measures of variation?
2. Does the implementation of the curricula materials and exposure to advanced assessment instruments influence elementary school teachers' awareness of their understanding of statistics?

This chapter is divided into six sections. The first section describes the setting of the study. The second section describes the rationale for a qualitative design, the type of design used, the role of the researcher, the selection of the participants, and the participants. The third section provides a discussion of the *Math Out of the Box* materials and how the topics covered compare to the GAISE document in relation to data displays, measures of central tendency, and measures of variation. The fourth section describes the phases of data collection. The fifth section discusses the methods of verification used in this study. The concluding section contains a discussion of the limitations of the study.

Setting

This study was conducted in a middle- to upper-middle class school district located in central New Jersey. Demographic data for the district are provided from the New Jersey School Report Cards for each of the seven schools in the district. Two of the seven schools were involved in this study. This information was provided by the New Jersey Department of Education.

In the 2005-06 academic year, a total of 4,221 students attended the seven schools in the district. Of the two schools involved in this study, one served students in grades K-3 (School X) and the other served students in grades 4-6 (School Y). School X had a total of 220 students with 53 in kindergarten, 53 in first-grade, 44 in second-grade, 46 in third-grade, and 24 in special education programs. School X had a total of 20 teachers to serve the 220 students creating a student to teacher ratio of 11 to 1. School Y had a total of 898 students with 249 in fourth-grade, 266 in fifth-grade, 248 in sixth-grade, and 135 in special education programs. There were a total of 75 teachers at School Y creating a student to teacher ratio of approximately 12 to 1.

In the state of New Jersey, an annual assessment is given entitled the NJASK. In 2005-06 the percent of third- and fourth-grade students at schools X and Y that were judged to be either proficient or advanced on the mathematics assessment were 84.7% and 76.6%, respectively. Third- and fourth-graders throughout the state were 82.4% and 82.3% proficient or advanced, respectively. Scores are reported for third- and fourth-grade students since those are the grade levels taught by the three participants.

Qualitative Design, Role of Researcher, and Participants

Assumption and Rationale for a Qualitative Design

The questions of interest required analysis of three particular elementary school teachers' understanding of various statistical topics. "Understanding of statistical topics" cannot be easily identified or described. A qualitative design shed light on the status of elementary school teachers' understanding of various statistical topics and how their awareness of their understanding changed. Classroom observations provided a vehicle to develop a clearer picture of elementary school teachers' understanding of various statistical topics. The researcher spent more than 100 hours over the course of 14 months in the field conducting observations and interviews.

The Type of Design Used

A qualitative approach was appropriate for this study because it offered methods that were more suitable for collecting evidence to answer the research questions. There was ample time for the collection of data from a number of sources. A case study was used to describe the influences of implementing *Math Out of the Box*. The "case" for this study was three particular elementary school teachers as they implemented this new curriculum with ten lessons focused on data analysis and statistics. The case was a "bounded system," bounded by the amount of time it took the teachers to introduce the concepts contained in the ten lessons during the spring and fall of 2006. There were extensive, multiple sources of information. The data collected was triangulated to help "tell the story" of these particular teachers.

The Role of the Researcher

The researcher served as a passive observer during classroom visits and as an active participant in interviews with the teachers. Because of the rapport developed by the researcher with the participants, open lines of communication were established. The participants opened their classrooms and readily discussed their practices and understanding of statistical ideas with the researcher.

Selection of Participants

Initial contact was made with the three teachers through a gatekeeper, the district supervisor for mathematics and science education. She recommended a group of teachers to participate in the study based on their qualifications and high standing in the district. Several teachers were observed, interviewed, and surveyed before deciding on three teachers as the focus of this case study. The study was limited to three participants because of convenience sampling and the length of time necessary to visit, interview, and assess the teachers at the depth involved in this study. The three participants were selected because they were more willing to dedicate the necessary time involved with the study than the other teachers and were open regarding their understanding of statistics.

Despite a self-reported dislike for the discipline of mathematics (discussion forthcoming), the participants enjoyed teaching mathematics and looked forward to the opportunity of teaching the statistical lessons contained in the *Math Out of the Box* materials. All three teachers were highly recommended for this study as they were viewed as exemplary teachers of mathematics by their principals and district supervisor. The teachers were open regarding their misconceptions concerning the material. This openness helped inform the researcher on the status of elementary school teachers' understanding of various statistical topics.

Participants

This study involved two third-grade teachers, Ms. Brown and Ms. Clark, and one fourth-grade teacher, Ms. Alvin (all pseudonyms). Ms. Brown and Ms. Clark were teachers at School X and Ms. Alvin was a teacher at School Y. Ms. Alvin was the most senior of the three with 9 years experience. Ms. Brown and Ms. Clark had 5 and 4 years experience, respectively. Ms. Alvin was the only teacher with a Master's degree (in general education).

Ms. Alvin. Ms. Alvin had taught all nine years at the fourth-grade level. She readily admitted mathematics was not her strong point. In college, she intentionally avoided a mathematics course for as long as possible. In fact, she was able to take physics to fulfill the mathematics requirement. Consequently, Ms. Alvin did not take any undergraduate courses for teaching mathematics specifically at the elementary school level, and only a single graduate mathematics teaching course.

Aside from the professional development (discussion forthcoming) related to the implementation of *Math Out of the Box*, Ms. Alvin had not participated in any hours of professional development related to mathematics education in the past six months. In the past three years, she had participated in fewer than six hours of professional development related to mathematics education. These values were self-reported and a description of these professional development experiences was not available at the time of this study.

Ms. Alvin was a member of the National Educational Association and the New Jersey Educational Association. She was not a member of any professional organizations related to the teaching of mathematics. She was familiar with the New Jersey Core

Curriculum Content Standards, but, before the baseline interview (discussion forthcoming) was unaware of the NCTM Standards, even though the New Jersey Core Curriculum Content Standards were based on them.

Ms. Brown. Ms. Brown had taught all five years at the third-grade level. Like Ms. Alvin, she readily admitted to not being particularly good at mathematics. Ms. Brown took a college algebra course as a freshman to satisfy the mathematics requirement as soon as possible. She also took a course in mathematics for elementary school teachers in her junior year.

Aside from the professional development related to the implementation of *Math Out of the Box*, Ms. Brown had participated in fewer than six hours of professional development related to mathematics education in the past six months. In the past three years, she had participated in 16-35 hours related to mathematics education. These values were self-reported and a description of these professional development experiences was not available at the time of this study.

Ms. Brown was not a member of any professional organizations. She remembered hearing about the NCTM Standards when she was in college, but did not remember them well. However, she was familiar with the New Jersey Core Curriculum Content Standards.

Ms. Clark. Ms. Clark had taught all four years at the third-grade level. She too readily admitted to an aversion to mathematics. In college she took an algebra class that she remembered quite well due to her struggles with the subject. She took this course as a senior after putting off the mathematics requirement for as long as possible. Like Ms. Brown, she too took a course in mathematics for elementary school teachers in her junior year.

Aside from the professional development related to the implementation of *Math Out of the Box*, Ms. Alvin had participated in fewer than 6 hours of professional development related to mathematics education in the past six months. In the past three years, she had participated in between 6-15 hours of professional development. These values were self-reported and a description of these professional development experiences was not available at the time of this study.

Ms. Clark was a member of the National Educational Association and the New Jersey Educational Association. Like the other two teachers, she was familiar with the New Jersey Core Curriculum Content Standards but unfamiliar with the NCTM Standards.

Math Out of the Box Materials

The following description of the materials was verified by an independent reviewer who is an expert in curriculum and assessment. The curriculum materials serving as the medium in this study build the content around real data collection activities. The activities address many of the objectives identified at Level A in the GAISE. Specifically, in grades 3 and 4 the first four objectives at Level A are touched upon. The content at grades 3 and 4 is similar and the tasks guide students in formulating questions, understanding that data are more than just numbers, and using tools (including data displays, measures of central tendency, and measures of variation) to analyze data.

Data Displays

Similar data displays are utilized in grades 3 and 4. The students learn to construct and interpret tally and frequency tables, line plots (similar to dotplots with X's instead of dots), and line graphs (time plots). The teacher manual in grades 3 and 4 encourages the teachers to ask students to represent data in multiple ways (e.g. tally tables and line plots) so that they may be able to realize the advantages and disadvantages of each display. However, those advantages or disadvantages are not discussed in the materials themselves.

Measures of Central Tendency

The mode and median are the only measures of central tendency introduced in grades 3 and 4. The mode is defined as the value that occurs most often. It is mentioned on multiple occasions that a data set can have one mode, more than one mode, or no mode. However, an example is not provided where a data set has more than one mode or no mode. The median is defined as the middle value. The materials discuss finding a median in a data set with an odd and even number of values. The mode is described as the only acceptable measure of central tendency for a categorical variable. However, the advantages and disadvantages of each measure of central tendency are not discussed.

Measures of Variation

The range is the only measure of variation introduced in grades 3 and 4. The range is defined as the difference between the highest and lowest values. The materials do not specifically indicate that the range is an inappropriate measure of variation for categorical variables. The teacher manual indicates that the range provides information about the spread of the data. It also indicates that a larger range generally implies the data are more spread out.

Outliers (a topic related to measures of variation) are also introduced in grades 3 and 4. The teacher manual defines an outlier as "a value that falls far removed from others in a data set." The calculations for finding such a value are not discussed at either grade level. The manual also indicates that often "an outlier is the result of a mistake in the data collection process…in statistical analysis, an outlier is discarded when a whole data set is examined" (Moss et al. 2005a, Grade 3, p. 157).

Summary

The data lessons in the *Math Out of the Box* materials involve some topics at the level of sophistication described in the GAISE. The fact that the materials build the content around real data encourages students to grasp the true essence of statistics. However, the materials do not discuss the advantages and disadvantages of various data displays, measures of central tendency, or measures of variation. These topics are discussed during professional development provided by the creators of the curriculum (discussion forthcoming). This study sheds light on whether or not elementary school teachers possess sufficient understanding of the statistical topics described above. The *Math Out of the Box* materials served as a powerful medium for such an exploration.

Three Phases of Data Collection

There were three phases of data collection. In Phase One, during the fall of 2005, baseline data were collected on the teachers. In Phase Two, during the spring of 2006, data were collected on the teachers as they implemented the *Math Out of the Box* materials. In the case of Ms. Alvin, the implementation of the materials was for the second time. However, during her first implementation she did not have time to complete all of the lessons of the unit. The study had not begun prior to her implementation of the materials for the first time. For Ms. Brown and Ms. Clark the spring of 2006 represented their first use of the materials. In Phase Three, during the fall of 2006, data were collected as the teachers implemented the materials for the second or third time. A table summarizing the instruments used during each phase of data collection is provided at the end of this section.

Phase One

Baseline data were collected during Phase One. This phase involved meeting with the teachers and conducting a baseline interview before the first, or in the case of Ms. Alvin second, implementation of the *Math Out of the Box* materials.

The baseline interview provided baseline data regarding the teachers' educational background, experience, level of participation in professional development, and comfort level with teaching data analysis and statistics. (Results were used in the above discussion of the participants.) The 45-minute interview was conducted prior to use of *Math Out of the Box* for Ms. Brown and Ms. Clark and after one implementation for Ms.

Alvin. One-on-one interviews were conducted with each teacher involved in the study. See Appendix A for a copy of this interview.

Phase Two

Phase Two began with the professional development training focused on statistics which was offered by the creators of the *Math Out of the Box* materials. The key component of data collection during this phase involved observations of the teachers as they implemented the statistical lessons of the *Math Out of the Box Materials*. There were also two formal interviews conducted after the implementation of the materials.

Researcher Presence During Professional Development Activities. The researcher was present during one eight-hour professional development training which focused on statistical topics. The training was provided by the developers of *Math Out of the Box* materials. During this one eight-hour session, the researcher sat with the teachers and took notes. The following description of the professional development activities was provided by the main professional development facilitator (Diaz, E-mail Correspondence, 4/16/2007). All of the teachers participated in sample activities for the K-5 curriculum regardless of which grade they were teaching.

Description of Professional Development Activities. The professional development is a K-5 overview of topics that provides teachers with a "big picture" of how some of the topics in data analysis are developed through the *Math Out of the Box* materials.

K-1 Focus. The day begins with a sorting activity and teachers sort the same collection a couple of different ways. In the first sorting activity one group of teachers sort a collection of objects and another group of teachers have to try to figure out the sort. In the second sorting activity, they have to sort using a new rule and to create a display that communicates the rule of sort they used. Teachers begin to learn that the purpose of displaying data is to share information and discussions are facilitated to help teachers understand how conventions used in data displays help communicate the information about the data.

2-3 Focus. The second part of the data professional development focuses on the importance of a "fair test." Teachers collect data about their wrist sizes. This is an actual lesson in the grade 3 materials and the session models for the teachers some of the issues that will arise in their own classrooms. Teachers are asked to provide some questions about their wrists. After these questions are charted, a discussion is facilitated about how the question determines the type of data that is collected. Teachers collect data about their wrists (e.g. the circumference, width on top, width on bottom) and the importance of describing specific details about data collection is discussed based on the outcomes of the varied data that are collected without any specific directions. Various means of organizing the data are discussed, particularly tabular methods. The group eventually decides on collecting data regarding each participant's wrist circumference and a line plot is constructed to display these measurements.

4-5 Focus. The line plot from the wrist data is used to discuss statistical information including the range, mode, and median. An investigation is carried out by the group about counting the number of steps over a certain distance. Teachers set up specifications for data collection, collect and organize the data and display their results. Statistical information about the data is figured and discussed within the context of the data set. The meaning of the mode, range, and median for that particular sample is discussed, so that the definitions (which teachers tend to know "by heart") are given meaning in the context of something they have experienced. Mode is not just the "most" but it is the number of steps taken over a certain length by the most number of teachers in the group. The range is not just the "most minus least" but the difference between the most steps taken and the least steps taken, so one can know if there is really a big range of difference in the group. The median is not just "the middle" but it means that the same number of people in the group walked fewer steps than the median as walked more steps than the median.

General. Teachers also do a leveling activity to examine the mean. The mean is included in the professional development materials so teachers may begin to understand the differences between the various measures of center. As discussed previously, the only measures of center in the student materials are the median and the mode. In this activity, the mean, median, and mode for teachers' shoe sizes is found and their meanings and differences are discussed in the context of the data set. (Note: If there are males present in the group, participants decide how to deal with that issue in regard to appropriate methods of data collection.)

Observations. The researcher served as a passive observer during five classroom visits per teacher. Transcripts of the lessons were composed. These transcripts were used to analyze the statistical content covered in each class as well as to determine if any misconceptions were exhibited by the teachers during instruction or in responses to student questions. Appendix B is an example of the observational protocol.

Statistical Content Interview. This 30-minute interview asked teachers to define common statistical terms that appear in the *Math Out of the Box* materials, discuss how these measurements can be calculated, and why they may be useful. In relation to these topics, teachers were asked to indicate whether or not their level of understanding of these topics changed as a result of the use of the materials. This interview assessed the teachers' possible change in understanding during Phase Three of data collection. Appendix C is the form used for this interview.

Math Out of the Box Interview. This 45-minute interview provided feedback during implementation of the *Math Out of the Box* curriculum. The questions contained in this interview assessed teacher learning after the implementation of the *Math Out of the Box* materials. The questions in this interview were based on the work of Diaz (2004). Appendix D is the form used for this interview.

Phase Three

Phase Three served as the major component of data collection. This phase represented a time when all the teachers had already implemented the statistical lessons at least one time. During Phase Three, Observations, the Statistical Content Interview, and the *Math Out of the Box* Interview were used as described in Phase Two. In addition, the teachers' content knowledge was assessed as described below..

Assessments. In addition to the repeated administration of the Statistical Content Interview, a more in-depth analysis of teachers' understanding of statistics was conducted during Phase Three. This level of analysis involved topics related to the concepts introduced at their particular grade level. However, the content of these assessments also involved a more sophisticated level of understanding of statistics as proposed by the GAISE. In particular these questions involved material related to data displays, measures of central tendency, and measures of variation. Questions were taken from established assessments such as the NAEP and Diagnostic Teacher Assessments in Mathematics and Science developed at the University of Louisville. These assessments have established high levels of reliability and validity (Bush, et al, under review). These assessments also contained questions developed by the researcher who was a mathematics assessment specialist. One of these questions was related to a question posed on the 2006 AP® Statistics exam. This particular question was chosen because it pulled together how measures of central tendency and variation can be used to make a decision and was judged to be accessible by elementary school teachers. The decision to include this particular question, based on the entry level available to elementary school teachers, was confirmed by an independent reviewer who was also an assessment specialist in mathematics. Appendices E, F, G, H, and I contain the questions used in these assessments.

Summary. Table 3.1 summarizes the instruments used during each phase of data collection.

Table 3.1 Overview of Data Collection Process

Phase	Phase One	Phase Two	Phase Three
Time	Fall 2005	Spring 2006	Fall 2006
Instruments			
Baseline Interview	X		
Researcher Presence at Training		X	
Observations		X (5)	X (5)
Statistical Content Interview		X	X
Math Out of the Box Interview		X	X
Content Assessments:			
Appendix E			X
Appendix F			X
Appendix G			X
Appendix H			X
Appendix I			X

Methods of Verification

There were four methods of verification, as described by Creswell (2006), used throughout the course of this case study. These consisted of prolonged engagement, triangulation, identification of researcher biases, and in-member checks. Each of these is discussed in the paragraphs that follow.

Prolonged Engagement and persistent observations were used as a means of validating the data collected. By the middle of Phase Two, the participants were becoming more comfortable discussing their misconceptions with the researcher. This openness led to more valid reporting of the results as the teachers did not hold back any information during interviews.

Triangulation was one of the primary sources for validation in this case study. Nine different instruments were used during data collection. Each instrument involved in the study was analyzed separately and then compared with other instruments to verify the data provided an accurate view of the teachers' understanding. The process involved corroborating evidence from the multiple sources in order to provide insight into the status of elementary school teachers' statistical understanding.

Researcher biases are clarified later in this chapter. The audience must understand the position of the researcher. The influences of past experiences that may have biased the researcher are discussed in order to ensure an accurate interpretation can be taken.

In-member checks were another major source of reliability. Lincoln and Guba (1985) state that in-member checks are "the most critical technique of establishing credibility" (p. 314). At several points during the collection of data and especially at the conclusion of the study, participants were asked to verify reports that resulted from their participation. In particular, the participants were presented with transcripts of the observations and interviews and asked to ensure that the transcriptions were accurate descriptions of what had transpired. During these in-member checks, there was not a single area of disagreement between the researcher and the participants. This method of verification was crucial to establishing reliability in this study.

Limitations

There are many advantages to conducting a qualitative case study; however there are also limitations. First of all, this study was limited by the sample size. By examining only three teachers, generalizations to all elementary school teachers are not appropriate. However, since these teachers were highly recommended by their district supervisor and principals as exemplary teachers of mathematics, they provide a snapshot of what may be expected of exemplary elementary school teachers' understanding of certain statistical topics.

Secondly, this study was influenced by the biases of the researcher. The researcher believed elementary school teachers should possess an in-depth understanding of statistics (and mathematics). It is only with an in-depth understanding that teachers can appropriately respond to students' questions and push their thinking in new directions. The researcher also speculated that most elementary school teachers (and teachers at all levels) do not possess the understanding the authors of the GAISE document suggest. If teachers do not possess the understanding expected of students then it is difficult to create learning environments where students' understanding can reach an adequate level. The researcher believed that most teachers, especially elementary school teachers, view mathematics in a negative fashion.

Inherently, these acknowledged biases may have had an impact on the analysis of this study. These biases may have caused the researcher to examine teachers' understanding from a deficit rather than constructivist view. In other words, the study focused more on the identification of misconceptions and shortcomings in the teachers' understanding rather than viewing the understanding the teachers possessed as a building block for future learning. These biases also influenced the selection of the research questions. However, the realization of these biases caused the researcher to pay close attention to the data collected and reduce the influence of these biases in assessing the understanding of the teachers.

Conclusion

The *Math Out of the Box* materials served as a powerful medium to investigate three elementary school teachers' understanding of essential statistical topics. Based on the rationale for a qualitative design and the methods of verification, the results presented in the following chapter are both reliable and valid.

CHAPTER FOUR

ANALYSIS AND RESULTS

This chapter presents the analysis and results of the study. The first section categorizes the questions contained in each instrument based on the content (i.e. data displays, measures of central tendency, and measures of variation) and the type of knowledge assessed (i.e. procedural low, procedural high, and conceptual). The results and data analysis are contained in the next five sections. The second section is an analysis of the teachers' performance in the area of data displays, the third is an analysis of the teachers' performance in the area of measures of central tendency, and the fourth is an analysis of the teachers' performance in the area of measures of variation. The fifth section makes connections between the teachers' understanding and the recommendations contained in the GAISE document. The concluding section discusses the teachers' awareness of their understanding.

Categorization of the Questions

This section organizes questions in the following categories: data displays, measures of central tendency, and measures of variation. Tables 4.1, 4.2, and 4.3 show the categorization of each question contained in the content knowledge instruments (Appendices E, F, G, H, and I). Each question was identified as either multiple-choice (MC) or open-ended (OE) and categorized as assessing procedural-low (PL), procedural-high (PH), or conceptual knowledge (C). The categorizations indicated in the table have been verified by two independent reviewers who are mathematics assessment specialists.

For questions where reviewers did not initially agree, a meeting was held to reach agreement on the categorization of the question.

Data Displays

Table 4.1 Categorization of Data Display Questions

Appendix	Question Number	Type of Display	Type of Question	Type of Knowledge
E	1	Bargraph	MC	PL
E	2	Table	MC	PL
E	3	Bargraph	MC	PL
E	5	Line Plot	MC	PL
E	7a	Line Graph	OE	PH
E	7b	Line Graph	OE	PH
E	8a	Bargraph	OE	PL
E	8b	Bargraph	OE	PL
F	1	Modified Bargraph	MC	PL
F	3	Bargraph	MC	PH
F	4	Line Plot	MC	PH
F	6	Pictogram	MC	PL
F	7a	Line Graph	OE	PL
F	7b	Line Graph	OE	PH
F	8a	Pictogram	OE	PH
F	8b	Pictogram	OE	PH
F	8c	Pictogram	OE	PH
G	2	Stemplot	MC	PL
G	4	Circle Graph	MC	PL
G	6	Boxplot	OE	PH
G	6a	Line Graph	OE	PH
G	6b	Line Graph	OE	C
G	6c	Line Graph	OE	C
G	7	Circle Graph	OE	PH
H	2	Dotplot	OE	PL
I	1	Histogram	OE	PL
I	4	Bargraph/Histogram	OE	PL
I	5	Bargraph/Histogram	OE	C

Measures of Central Tendency

Table 4.2 Categorization of Measures of Central Tendency Questions

Appendix	Question Number	Type of Measure	Type of Question	Type of Knowledge
C	2a	Mean	OE	PL
C	2b	Mean	OE	PH
C	2c	Mean	OE	C
C	3	Average	OE	C
C	4a	Median	OE	PL
C	4b	Median	OE	PH
C	4c	Median	OE	C
C	5	Mean/Median	OE	C
C	6a	Mode	OE	PL
C	6b	Mode	OE	PH
C	6c	Mode	OE	C
E	4	Mode	MC	PL
E	6	Median	MC	PL
F	2	Median	MC	PL
F	5	Mode	MC	PL
G	1	Median	MC	PL
G	3	Mean/Median/Mode	MC	PL
G	5	Average	OE	PH
G	6	Average	OE	C
H	3	Mean	OE	PL
H	3	Median	OE	PL
H	3	Mode	OE	PL
H	5	Mean/Median/Mode	OE	C
I	2	Mean	OE	PH
I	2	Median	OE	PH
I	2	Mode	OE	PH
I	3(1)	Mean	OE	C
I	3(2)	Median	OE	C
I	3(3)	Mode	OE	C

Measures of Variation

Table 4.3 Categorization of Measures of Variation Questions

Appendix	Question Number	Type of Measure	Type of Question	Type of Knowledge
C	7a	Range	OE	PL
C	7b	Range	OE	PH
C	7c	Range	OE	C
C	8a	Outlier	OE	PL
C	8b	Outlier	OE	PH
C	8c	Outlier	OE	C
H	3	Range	OE	PL
H	3	Outlier	OE	PL
H	4	Range/St. Dev/Outlier	OE	C
I	2	Range	OE	PH
I	2	Standard Deviation	OE	PH
I	3(4)	Range	OE	C
I	3(5)	Standard Deviation	OE	C

Summary

Tables 4.4, 4.5 and 4.6 summarize the categorizations for the three topic areas.

Table 4.4 Summary of Data Display Questions

Data Displays	**Total**	**MC**	**OE**	**PL**	**PH**	**C**
Bargraph/ Histogram	10	4	6	7	2	1
Table	1	1	0	1	0	0
Line Graph	7	0	7	1	4	2
Line Plot	2	2	0	1	1	0
Circle Graph	2	1	1	1	1	0
Boxplot	1	0	1	0	1	0
Stemplot	1	1	0	1	0	0
Pictogram	4	1	3	1	3	0
Dotplot	1	0	1	1	0	0
Total	29	10	19	14	12	3

Table 4.5 Summary of Measures of Central Tendency Questions

Measures of Central Tendency	Total	MC	OE	PL	PH	C
Mean	6	0	6	2	2	2
Median	9	3	6	5	2	2
Mode	8	2	6	4	2	2
Average/General	3	0	3	0	1	2
Mean/Median	1	0	1	0	0	1
Mean/Median/Mode	2	1	1	0	1	1
Total	29	6	23	11	8	10

Table 4.6 Summary of Measures of Variation Questions

Measures of Variation	Total	MC	OE	PL	PH	C
Range	6	0	6	2	2	2
Standard Deviation	2	0	2	0	1	1
Outlier	4	0	4	2	1	1
Range/Std.Dev./Outlier	1	0	1	0	0	1
Total	13	0	13	4	4	5

Teachers Performance in the Area of Data Displays

The three teachers performed similarly on the assessments in the area of data displays. Table 4.7 shows their performance on the questions contained in Appendices E, F, and G. These assessments were from the NAEP and University of Louisville assessments. As indicated in the previous section, many of these questions were multiple-choice and thus were evaluated as either right or wrong. In Table 4.7, an R indicates a right answer and a W indicates a wrong answer. For many of these questions, especially the open-ended questions, the responses from the teachers are elaborated upon further.

Table 4.7 Performance in Area of Data Displays (Appendices E, F, and G)

Appendix	Number	Type	Level	Ms. Alvin	Ms. Brown	Ms. Clark
E	1	Bargraph	PL	R	R	R
E	2	Table	PL	R	R	R
E	3	Bargraph	PL	R	R	R
E	5	Lineplot	PL	R	R	R
E	7	Linegraph	PH	W	R	R
E	8	Bargraph	PL	R	R	R
F	1	Bargraph	PL	R	R	R
F	3	Bargraph	PH	W	R	R
F	4	Lineplot	PH	R	R	R
F	6	Pictogram	PL	R	R	R
F	7a	LineGraph	PL	R	R	R
F	7b	LineGraph	PH	R	R	R
F	8	Pictogram	PH	W	W	R
G	2	Stemplot	PL	W	W	W
G	4	Circle Graph	PL	R	R	R
G	6	Boxplot	PH	W	W	W
G	7a	Linegraph	PH	R	R	R
G	8	Circle Graph	PH	W	W	W

Table 4.8 shows the distribution of right and wrong answers on questions involving data displays for each level of knowledge assessed in Appendices E, F, and G.

Table 4.8 Summary of Individual Teacher's Performance in the area of Data Displays (Appendices E, F, and G)

Teacher	Type of Display	Procedural Low		Procedural High	
		Right	Wrong	Right	Wrong
Ms. Alvin	Bargraph/ Histogram	4	0	0	1
	Table	1	0	0	0
	Line Graph	1	0	2	1
	Line Plot	1	0	1	0
	Circle Graph	1	0	0	1
	Boxplot	0	0	0	1
	Stemplot	0	1	0	0
	Pictogram	1	0	0	1
Ms. Brown	Bargraph/ Histogram	4	0	1	0
	Table	1	0	0	0
	Line Graph	1	0	3	0
	Line Plot	1	0	1	0
	Circle Graph	1	0	0	1
	Boxplot	0	0	0	1
	Stemplot	0	1	0	0
	Pictogram	1	0	0	1
Ms. Clark	Bargraph/ Histogram	4	0	1	0
	Table	1	0	0	0
	Line Graph	1	0	3	0
	Line Plot	1	0	1	0
	Circle Graph	1	0	0	1
	Boxplot	0	0	0	1
	Stemplot	0	1	0	0
	Pictogram	1	0	1	0

Based on the results listed in Table 4.8, Ms. Alvin, Ms. Brown, and Ms. Clark each correctly answered 90% of the procedural low questions. Ms. Alvin, Ms. Brown, and Ms. Clark correctly answered 37.5%, 62.5%, and 75% of the procedural high questions, respectively.

Comments on Specific Questions

All three teachers were unsuccessful in their attempts to answer the questions involving stemplots and boxplots. The question involving a stemplot that appears in Appendix G is shown in Figure 4.1.

Students in a sixth-grade class were timed to the nearest second to see how long they could stand on one foot with their eyes closed. The times for the class are listed below in a stem-and-leaf plot. Which of the following is true?

2	7 8 9
4	7 7 8 9
6	1 2 3 4 5 6 7
7	2 2 2 6 7 8

a. The shortest time was 28 seconds
b. Half the class had times under 58 seconds
c. The longest time was 77 seconds
d. 50% of the class had times over 63 seconds

Figure 4.1 Stemplot Question

The question involving a boxplot that appears in Appendix G is shown in Figure 4.2.

The box-and-whiskers plot below represents the test scores of three classes on the same test.

a. Which class performed the best and which class performed the worst?
b. Provide justifications for your choices with data from the box-and-whiskers plots.

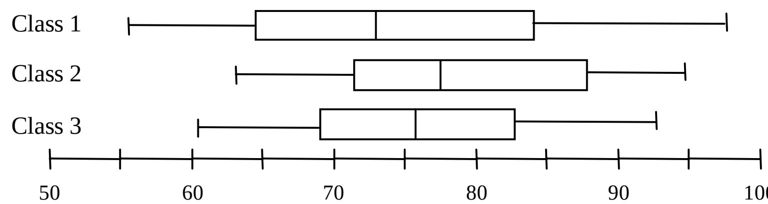


Figure 4.2 Boxplot Question

After taking this assessment, all three teachers commented that they had never seen such a graphical display and had "no clue" how to answer such a question.

By examining the difference in performance by the three teachers in regard to the two questions involving circle graphs, one may begin to distinguish between procedural low (PL) knowledge and procedural high (PH) knowledge. Figure 4.3 is the fourth question on Appendix G and involved the teachers selecting the most appropriate graphical display when the data were represented as percentages.

Which graph or plot below would best represent this data on solid waste; 42% paper, 7% glass, 19% plastic, 11% wood, 15% food and 6% miscellaneous?

a. scatter plot
b. stem-and-leaf plot
c. box-and-whiskers plot (box plot)
d. circle graph

Figure 4.3 Circle Graph Question 4, Appendix G

Figure 4.4 is the eighth question on Appendix G and required the teachers to consider how the central angles of a circle graph should be constructed to represent the data appropriately.

A survey of middle school students resulted in data about the quantity of soft drinks they consumed in a week. The data is displayed in the table below:

# of drinks	2 or fewer	3	5	6	over 7
# of students	4	6	7	5	3

The students were asked to construct a circle graph for the data. One student determined the size of the angles for each section of the graph by determining the size of each angle, such that ($4/25 = 16/100 = 16^\circ$). The student drew the angles with a protractor and had space left over. (a) What error is this student making, and (b) how would you help her?

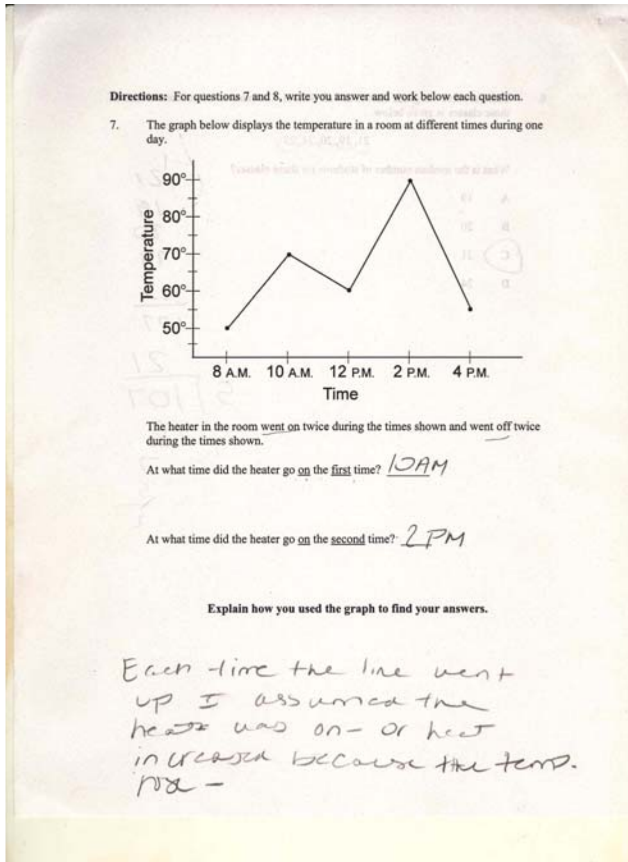
Figure 4.4 Circle Graph Question 8, Appendix G

The fourth question on Appendix G was considered a PL question whereas the eighth question was considered a PH question. All three teachers were successful at answering the PL question; however all three teachers were unsuccessful at answering the PH question. Again, all three teachers indicated they had "no clue" how to help the student correct their error.

In addition to the tables which show performance on each question, it is helpful to examine some of the responses to the open-ended tasks. These responses help inform what the teachers know and what they do not know.

Sample Responses

The following example in Figure 4.5 sheds light on why the response by Ms. Alvin to question number 7 on Appendix E was viewed as incorrect. Ms. Brown's response is also shown in order to indicate what was considered a correct response to this task.



Directions: For questions 7 and 8, write you answer and work below each question.

7. The graph below displays the temperature in a room at different times during one day.

The heater in the room went on twice during the times shown and went off twice during the times shown.

At what time did the heater go on the first time? 10AM

At what time did the heater go on the second time? 2PM

Explain how you used the graph to find your answers.

Each time the line went up I assumed the heater was on – or heat increased because the temp. rose –

Figure 4.5 Ms. Alvin's Response, Question 7, Appendix E

Ms. Alvin's response was incorrect in that she selected the points on the line graph that correspond to when the temperature was at a peak rather than when the direction in temperature change shifted from a decrease to an increase. This error may have been due to carelessness. An example of a correct response can be seen by examining Ms. Brown's Response in Figure 4.6.

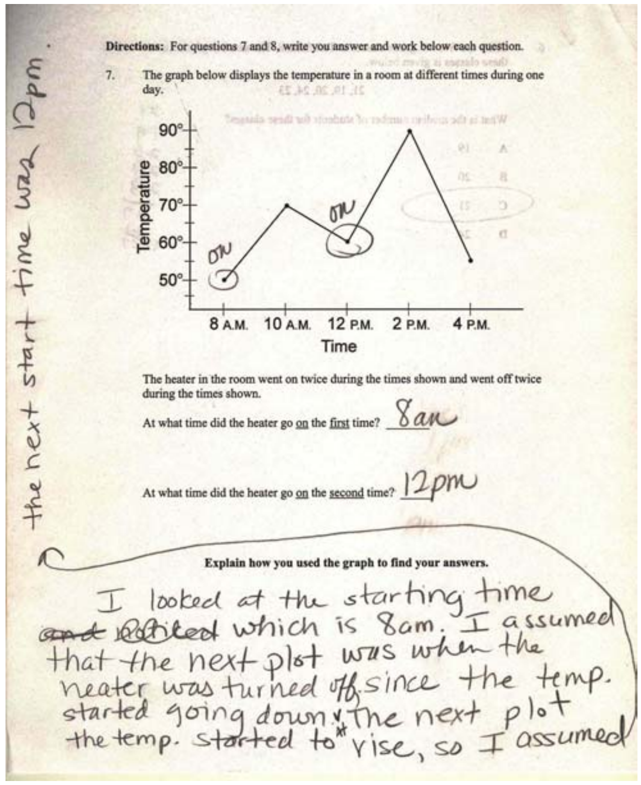


Figure 4.6 Ms. Brown's Response, Question 7, Appendix E

Ms. Alvin, Ms. Brown, and Ms. Clark showed the greatest disparity on question 8 in Appendix F. This question involved choosing the appropriate pictogram to model what was given in the stimulus of the question. The only teacher that answered this question correctly was Ms. Clark. The following figures provide the stimulus and responses of Ms. Alvin, Ms. Brown, and Ms. Clark, in Figures 4.7, 4.8, 4.9, and 4.10 respectively.

The Stimulus:

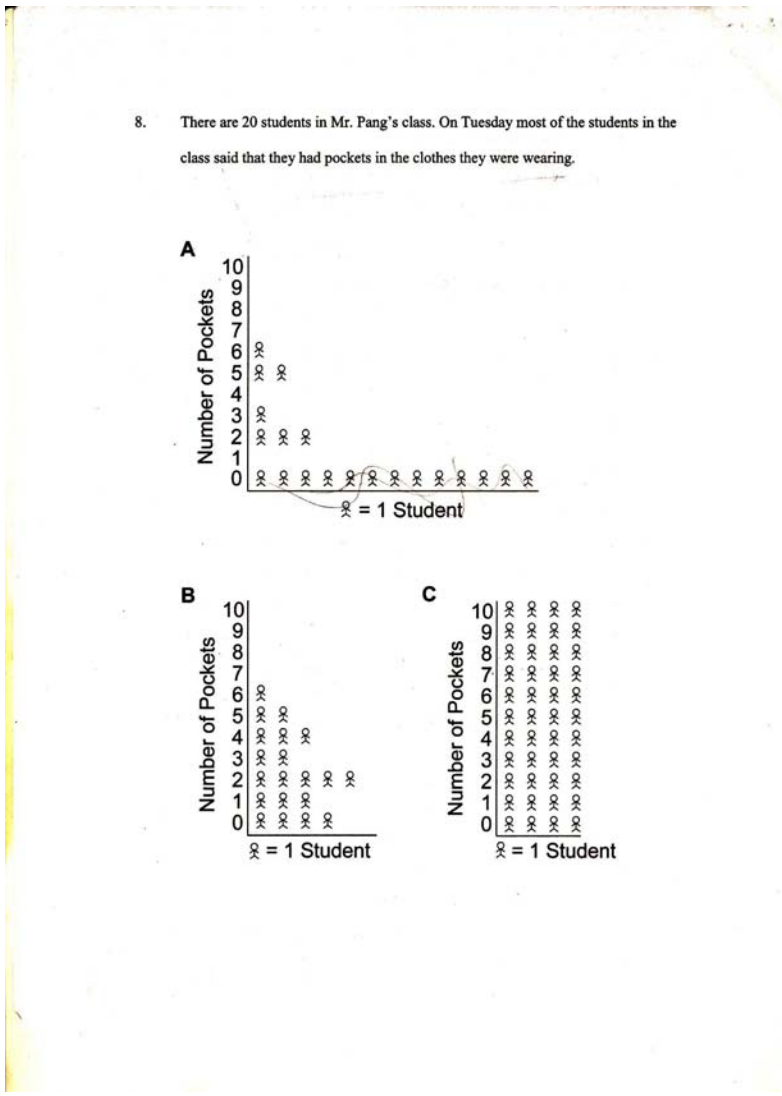
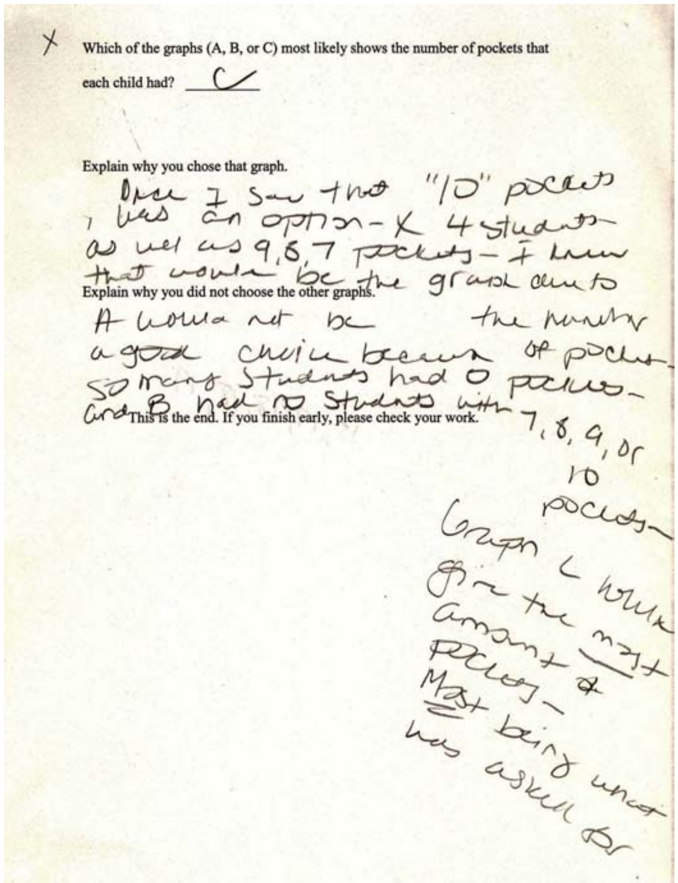


Figure 4.7 Stimulus, Question 8, Appendix F



X Which of the graphs (A, B, or C) most likely shows the number of pockets that each child had? C

Explain why you chose that graph.

Once I saw that "10" pockets was an option - X 4 students as well as 9, 8, 7 pockets - I knew that would be the graph due to

Explain why you did not choose the other graphs.

A would not be a good choice because so many students had 0 pockets - the number of pockets - and B had no students with 7, 8, 9, or 10 pockets - Graph C would give the amount of pockets & the most being what was asked for.

This is the end. If you finish early, please check your work.

Figure 4.8 Ms. Alvin's Response, Question 8, Appendix F

Which of the graphs (A, B, or C) most likely shows the number of pockets that each child had? ~~B~~ A

Explain why you chose that graph.

Because the (problem) question stated that most, not all 20 had pockets.

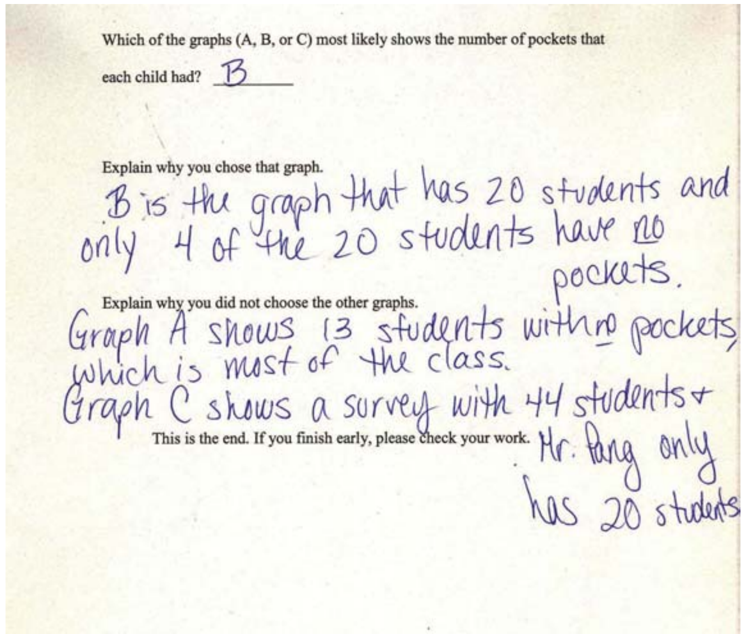
Explain why you did not choose the other graphs.

Graph C has too many students

Graph B has too little.

This is the end. If you finish early, please check your work.

Figure 4.9 Ms. Brown's Response, Question 8, Appendix F



Which of the graphs (A, B, or C) most likely shows the number of pockets that each child had? B

Explain why you chose that graph.

B is the graph that has 20 students and only 4 of the 20 students have no pockets.

Explain why you did not choose the other graphs.

Graph A shows 13 students with no pockets, which is most of the class.
Graph C shows a survey with 44 students + Mr. Pang only has 20 students

This is the end. If you finish early, please check your work.

Figure 4.10 Ms. Clark's Response, Question 8, Appendix F

<u>Performance on More Extended Tasks</u>
<u>Related to Data Displays</u>

The teachers were asked to respond to more extended tasks in Appendices H and I. On question number 2 in Appendix H, teachers were asked to identify the graphical displays involved in the task. This was classified as a procedural low question. There were several questions involving data displays on Appendix I. Question 1 asked the teachers to identify the type of graphical display. Questions 4 and 5 involved the teachers providing a data set that may resemble the distribution in the bargraph and histogram, respectively. The goal of this question was to see if the teachers would realize that a bargraph is associated with categorical data whereas a histogram is associated with numerical data.

All three teachers were successful in their ability to identify the graphical displays in Question 1 on Appendix H. For this question, the teachers indicated the dotplots were lineplots. Since the two displays are virtually identical with the only exception that a dotplot has dots and a lineplot has X's, this was viewed as an acceptable response. Upon further investigation the teachers had never been exposed to the terminology of dotplots.

On question 2 in Appendix I, all three teachers identified every data display as a bargraph. It was clear they did not realize that these displays were histograms. The ability to recognize that a particular data display is a histogram was considered procedural low knowledge. There was a more important concept that was assessed by asking participants to provide an example of a data set that could be displayed using the display in Data Display 1 and the display in Data Display 2. Participants were expected to associate categorical data with the bargraph and numerical data with the histogram. The displays in Appendix I appeared as shown in Figure 4.11 and 4.12.

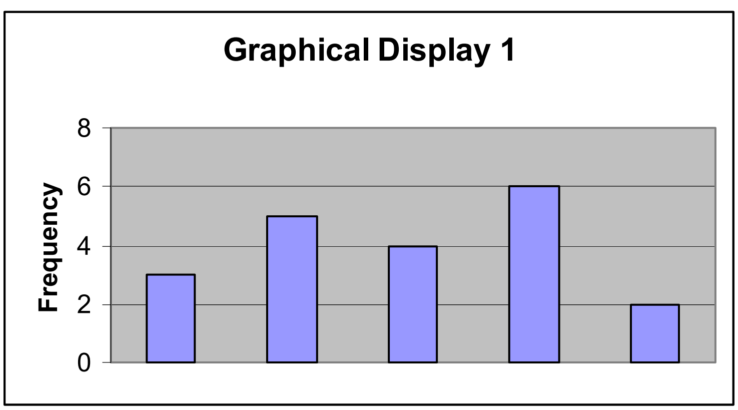


Figure 4.11 Graphical Display 1 from Appendix I

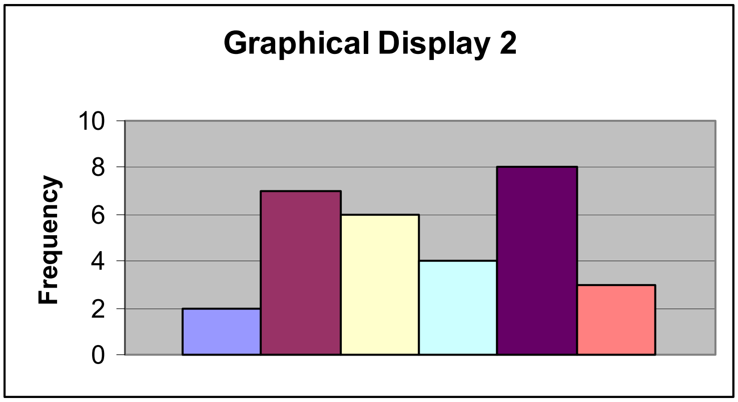


Figure 4.12 Graphical Display 2 from Appendix I

The two questions on Appendix I helped inform the researcher of the teachers' lack of understanding between categorical and numerical data. Transcript comments 1 to 8 are a conversation between the researcher and Ms. Clark during the assessment of the questions contained in Appendix I.

Number	Person	Comment
1.	Researcher	What type of display is display 1 and what type of display is display 2?
2.	Ms. Clark	They both look like bars. I don't believe bars are always supposed to be touching, but that doesn't always seem to be true. So I would call them both bargraphs.
3.	Researcher	Can you give me an example of a set of data that could have these types of graphical displays? So start with display 1, what is an example of a set of data that could be represented with such a data display?
4.	Ms. Clark	I guess like maybe a game day. Let's say we were playing kickball and in Game 1 my team scored 3 points, Game 2 we scored 5 points, Game 3 we scored 4 points, Game 4 we scored 6 points, and in Game 5 we scored 2 points.

		Or maybe the different times up at bat. So in the first in my team scored 3, in the second we scored 5, and so on.
5.	Researcher	What is an example of a set of data that could be represented with display 2?
6.	Ms. Clark	Uhm, I could probably use the same kickball example. Maybe on day 2 of game day, the times we were up at bat we scored this number of runs in each inning.
7.	Researcher	So is there a difference between the graphical displays? Would you use one to represent a certain type of information?
8.	Ms. Clark	I would use both for the same information. Obviously the bars don't reach the same heights on both graphs so they couldn't be the exact same information, but the displays could be used for similar situations. So using my same scenario, the first day when we were first up at bat we got 3 runs and the second day when we were first up at bat we got 2 runs. I guess if I were going to be more detailed, in graphical display 1 all the bars are the same color and in graphical display 2 the bars are different colors.

From the example(s) provided by Ms. Clark in transcript comments 1 to 8, it is apparent that she did not have an understanding that bargraphs are associated with categorical data whereas histograms are associated with numerical data. The responses provided by Ms. Alvin and Ms. Brown were similar to the response provided by Ms. Clark. This lack of understanding is also apparent in transcript comments 9 to 14 of a conversation that took place between Ms. Alvin and one of her students.

The transcript comments 9 through 14 were from a conversation that took place during a lesson out of the fourth grade *Math Out of the Box* materials in regard to the difference between bargraphs and histograms. This conversation occurred as a result of a direction of the teacher to have students construct bargraphs based on their sorting algorithm.

Number	Person	Comment
9.	Ms. Alvin	If you feel good about your table, I want you to start making your bargraph.
10.	Ms. Alvin	For an example of a bargraph look on your desks.
The data display on the desks the teacher was referring to was a histogram of numerical data.		
After providing some time for the students to work on this, the teacher created a bargraph on the board based on the table she already had as an example.		
11.	Student	Ms. Alvin, the bargraph you drew on the board does not have any space between the bars. Should ours have space between the bars?
12.	Ms. Alvin	Yes, they should
13.	Student	But the ones on our desks have the bars touching each other.
The teacher walked around the room and noticed that most of the students had constructed bargraphs with the bars touching.		
14.	Ms. Alvin	You know, I don't think it matters whether the bars are touching or not. There is not a difference between these graphs. You can make your bargraphs either with the bars touching or without the bars touching.

The lesson presented in transcript comments 9 to 14 represented one of the only occasions in the third and fourth grade materials where students were asked to construct bargraphs. It is not a critical component for teachers to know whether the bars should or should not touch. However, it is crucial that teachers understand how to distinguish between categorical and numerical data.

The primary data display utilized in the *Math Out of the Box* materials is the lineplot. All three teachers were successful in their ability to teach students to construct lineplots. The discussion of the teachers' ability to utilize lineplots and other data displays for various descriptive statistics is delayed until later discussions in this chapter. The transcript comments 15 to 38 is an example of Ms. Brown teaching her students how to construct a lineplot in lesson 13 of the *Math Out of the Box* materials. In this lesson the teacher was working with students to change the data display representing students' wrist sizes from a tally table to a lineplot.

Number	Person	Comment
15.	Ms. Brown	I think we should organize the information in another way. I saved your post-it notes and I'm going to give them back to you.
16.	Student	It's a line plot
17.	Ms. Brown	How do you know?
18.	Student	I remember from doing Homeruns (a previous example that was used and displayed in a lineplot)
19.	Ms. Brown	What would we use this for?
20.	Student	So we can get information from it by putting things in groups.
21.	Ms. Brown	Anyone else?
22.	Student	To put it on a graph
23.	Ms. Brown	Anyone else?
24.	Student	To organize data or to arrange it.

25.	Ms. Brown	Let's try it. You got your post-it notes. How can we use them?
26.	Student	We can put them on the graph where our wrist sizes should go.

Ms. Brown called upon students, one at a time, to put their post-it notes on the line plot that represented their wrist size. In the end the line plot had 2 post-it notes above 5, 10 post-it notes above 6, and 3 post-it notes above 7. The plot itself was drawn from 3 inches to 15 inches and labeled Wrist Measurements.]

27.	Ms. Brown	What if I took these [post-it notes] off; what could I replace them with?
28.	Student	X's
29.	Ms. Brown	What would each X represent?
30.	Student	Post-It Notes
31.	Ms. Brown	What else?
32.	Student	One person
33.	Ms. Brown	Yes, one student.

Ms. Brown took off post-it notes and put up X's in spots where post-it notes were.

34.	Ms. Brown	As I'm doing this I'm going to make sure this X and this X are at the same level. Why is that important?
35.	Student	If not, then you couldn't tell where it is and...
36.	Ms. Brown	So it would be hard to read.
37.	Student	If it was up, some people might think the X was at 2 (2 values up instead of just 1)
38.	Ms. Brown	We could have left the post-it notes, but you almost always see X's so it's good for you to see it that way.

The transcript comments 15 to 38 are representative of the way all three teachers introduced the concept of constructing lineplots to their students. This type of display eventually serves as the context for discussions regarding student and teacher interactions with measures of central tendency and variation.

Summary

Tables 4.9 and 4.10 provide overall performance for all three teachers in each area of data display in the procedural low, procedural high, and conceptual knowledge types. Table 4.9 contains summary data for all three teachers combined for the procedural low and procedural high knowledge types. Table 4.10 contains summary data for all three teachers combined for the procedural low and conceptual knowledge types from Appendices H and I that were not included in the original tables.

Table 4.9 Summary of Teachers' Performance on Appendices E, F, and G

Teachers	Type of Display	Procedural Low		Procedural High	
		Right	Wrong	Right	Wrong
Ms. Alvin, Ms. Brown, Ms. Clark	Bargraph/ Histogram	12	0	2	1
	Table	3	0	0	0
	Line Graph	3	0	8	1
	Line Plot	3	0	3	0
	Circle Graph	3	0	0	3
	Boxplot	0	0	0	3
	Stemplot	0	3	0	0
	Pictogram	3	0	1	2

Collectively, Ms. Alvin, Ms. Brown, and Ms. Clark correctly answered 90% of the procedural low questions and 58% of the procedural high questions.

Table 4.10 Summary of Teachers' Performance on Appendices H and I

Teachers	Type of Display	Procedural Low		Conceptual	
		Right	Wrong	Right	Wrong
Ms. Alvin, Ms. Brown, Ms. Clark	Dotplot	3	0	0	3
	Histogram	3	0	0	3
	Bargraph vs. Histogram	0	3	0	3

Collectively, Ms. Alvin, Ms. Brown, and Ms. Clark correctly answered 67% of the procedural low questions and 0% of the conceptual questions.

The results from Appendices E, F, and G show that Ms. Alvin, Ms. Brown, and Ms. Clark were capable of reading information from most types of data displays. As pointed out, all three teachers had difficulty with boxplots and stemplots. However this difficulty is likely due to a lack of exposure rather than a lack of understanding with these particular data displays. The teachers were generally successful at answering procedural low questions. The difficulties arose in the procedural high circle graph question and the questions involving the distinction between bargraphs and histograms. Based on these results, it appears these elementary school teachers possess a procedural low level of knowledge of most data displays; however they did not possess the advanced knowledge necessary to understand the complexities of choosing among various graphical displays nor did they understand the process of creating some graphical displays (e.g. bargraphs, histograms, and circle graphs).

Teachers Performance in the Area of
Measures of Central Tendency

The three teachers performed similarly on the content knowledge assessments in the area of measures of central tendency. Table 4.11 shows their performance on the questions contained in Appendices E, F, and G. These assessments were from the NAEP and University of Louisville assessments. As indicated in the first section of this chapter, many of these questions were multiple-choice and thus were evaluated as either right or wrong. In the table, an R indicates a right answer and a W indicates a wrong answer.

Table 4.11 Performance in Area of Measures of Central Tendency
(Appendices E, F, and G)

Appendix	Number	Type	Level	Ms. Alvin	Ms. Brown	Ms. Clark
E	4	Mode	PL	R	R	R
E	6	Median	PL	R	R	R
F	2	Median	PL	W	W	R
F	5	Mode	PL	R	R	R
G	1	Median	PL	R	R	R
G	3	Mean/Median/Mode	PL	R	R	R
G	5	Average	PH	W	W	W
G	6	Average	C	W	W	W

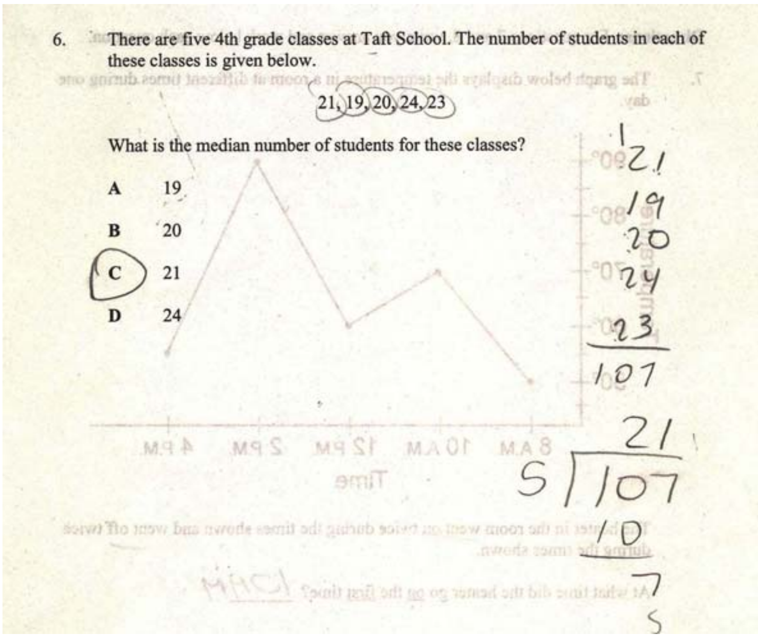
Comments on Specific Questions

All three teachers were successful in answering the median question which appears on Appendix E. The question was number 6 as shown in Figure 4.13.

6.	There are five 4th grade classes at Taft School. The number of students in each of these classes is given below. 21, 19, 20, 24, 23 What is the median number of students for these classes? **A** 19 **B** 20 **C** 21 **D** 24

Figure 4.13 Question 6, Appendix E

Two of the three teachers did not solve the problem by arranging the values from least to greatest and finding the middle number. Ms. Alvin and Ms. Brown computed the mean of the numbers in the data set. The mean of the numbers is 21.4 and the median of the numbers is 21. Figure 4.14 provides Ms. Alvin's calculation.



6. There are five 4th grade classes at Taft School. The number of students in each of these classes is given below.

21, 19, 20, 24, 23

What is the median number of students for these classes?

A 19

B 20

C 21

D 24

21
19
20
24
23
107

21
5)107
10
7
5

Figure 4.14 Ms. Alvin's Response, Question 6, Appendix E

Ms. Alvin and Ms. Brown were able to select the right answer among the choices provided; however Ms. Clark had a better understanding of the median in the context of this question.

It is interesting to contrast Ms. Alvin and Ms. Brown's performance on a similar question which appeared in Appendix F. The question was number 2 as shown in Figure 4.15.

2. The high temperatures in degrees for 7 days last week are shown below.

70, 71, 68, 71, 62, 73, 68

What is the median temperature?

A 68

B 69

C 70

D 71

Figure 4.15 Question 2, Appendix F

Ms Alvin made the same error; however Ms. Brown made a different error in solving this problem. Figures 4.16 and 4.17 provide the work of Ms. Alvin and Ms. Brown, respectively.

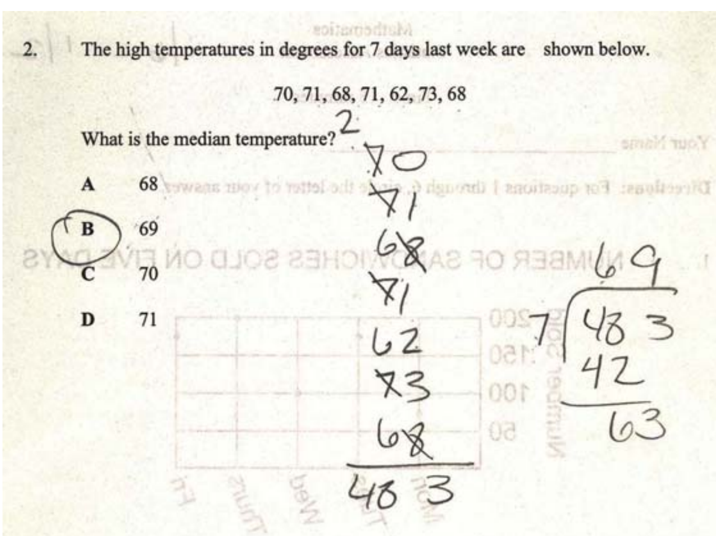
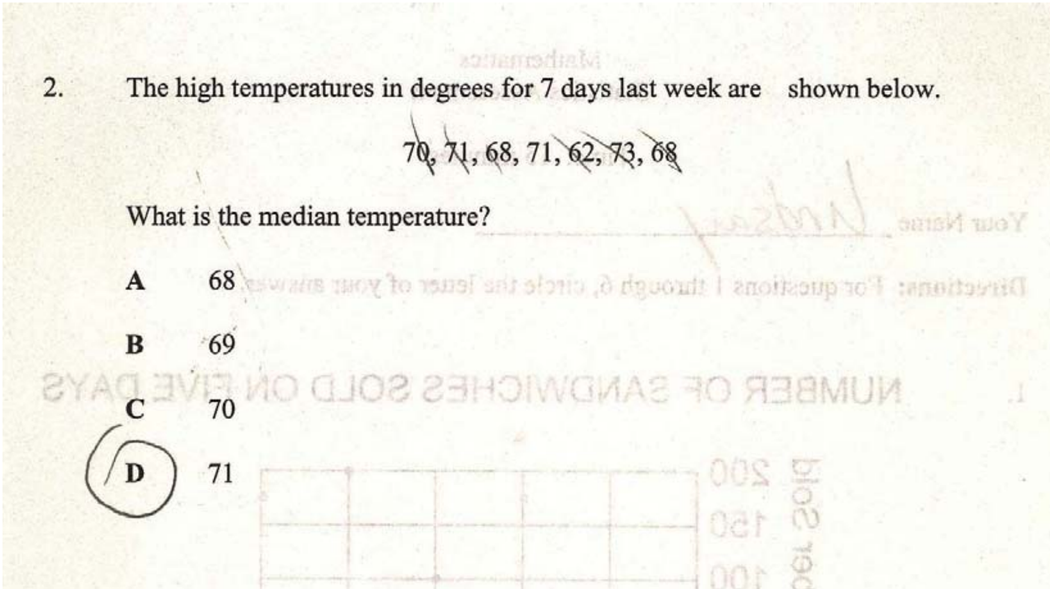


Figure 4.16 Ms. Alvin's Response, Question 2, Appendix F



2. The high temperatures in degrees for 7 days last week are shown below.

70, 71, 68, 71, 62, 73, 68

What is the median temperature?

A 68

B 69

C 70

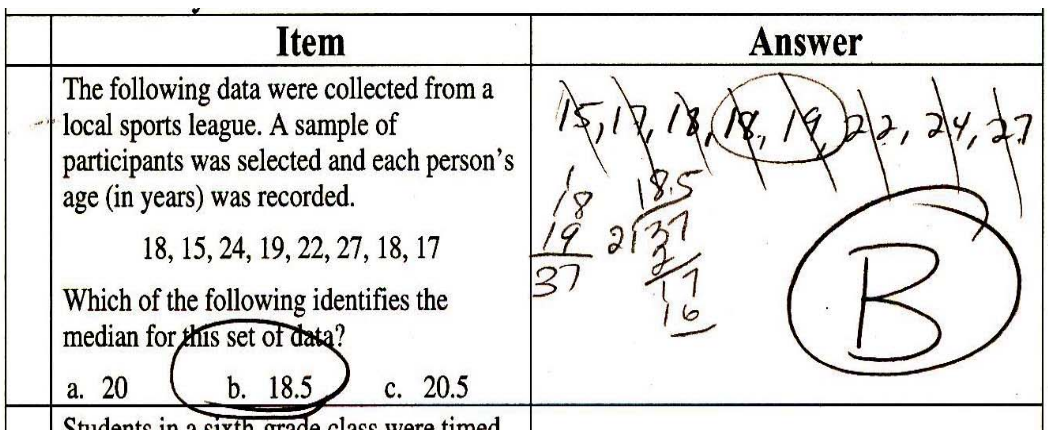
D 71

Figure 4.17 Ms. Brown's Response, Question 2, Appendix F

What is interesting about Ms. Brown's error is it appears as though her understanding of the procedure of finding the median was changing. In this particular problem, Ms. Brown knew that the median involved the middle number; however she neglected to order the numbers from least to greatest.

It should be noted that researchers and assessment specialists must exercise extreme caution in creating questions. Ms. Alvin and Ms. Brown were able to answer question 6 on Appendix E correctly while performing an incorrect calculation. The same method of solving the problem, as displayed by Ms. Alvin, on question 2 of Appendix F did not lead to the correct answer. Questions should be constructed carefully so common errors do not result in obtaining the correct answer. This error became apparent by studying the teachers' work rather than by merely checking their answers.

Ms. Brown's understanding of the procedure for finding a median was evolving. This is evident in Figure 4.18 which provides her response to question 1 on Appendix G.



Item	Answer
The following data were collected from a local sports league. A sample of participants was selected and each person's age (in years) was recorded. 18, 15, 24, 19, 22, 27, 18, 17 Which of the following identifies the median for this set of data? a. 20 b. 18.5 c. 20.5	15, 17, 18, 18, 19, 22, 24, 27 18 19 37 18.5 2)37 B

Figure 4.18 Ms. Brown's Response, Question 1, Appendix G

On this particular question, Ms. Brown correctly ordered the numbers from least to greatest and calculated the mean of the middle two numbers. This particular progression of Ms. Brown's understanding may be attributed to her learning with the *Math Out of the Box* materials. It should be noted that Ms. Alvin made the same error of calculating a mean for this question.

A closer look at the teachers' responses to the more extended questions which appear in Appendices C, H, and I help further determine the understanding these teachers possess in regard to measures of central tendency.

Calculating the Mean, Median, and Mode

All three teachers were able to describe an appropriate method for determining the mean, median, and mode. It was somewhat surprising that Ms. Alvin was able to provide an appropriate description for finding the median, as it contrasted with her approach to solving Questions 6, 2, and 1 on Appendices E, F, and G, respectively.

What is the difference between the
Mean and the Median?

Two of the three teachers had difficulty explaining what these measures of center actually represent. Ms. Clark indicated how to calculate such measures rather than explaining what the measurements represent. An example of her response in regard to the mean and median is shown in statements 39 to 42.

Number	Person	Comment
39.	Researcher	What does the mean represent?
40.	Ms. Clark	The mean is another word for average. So if you had the numbers, 1,2,3,4,5, then the mean would be 1+2+3+4+5 divided by 5.
41.	Researcher	What does the median represent?
42.	Ms. Clark	The median is the middle number, so if the numbers were 1 through 5 the median would be 3.

Ms. Brown was confused regarding the difference between the two measures of center. An example of her response in regard to the mean and median is shown in statements 43 to 50.

Number	Person	Comment
43.	Researcher	What does the mean represent?
44.	Ms. Brown	The average.
45.	Researcher	What is an average?
46.	Ms. Brown	The usual amount over a range of numbers.
47.	Researcher	What does the median represent?
48.	Ms. Brown	The number that is in the middle…but wait…they are the same. No, not really
49.	Researcher	So, what is the difference between the mean and the median?
50.	Ms. Brown	I don't know the difference.

The responses 43 to 50 show that Ms. Clark possessed a procedural low knowledge of the mean and the median. Although she could calculate the measures of center, she had difficulty conveying what the values represent. Ms. Brown also possessed a procedural low knowledge of the mean and the median. As one can see from her responses to the questions on Appendices E, F, and G, Ms. Brown was able to calculate the measures of center; however it is clear from responses 43-50 that she does not possess a conceptual understanding of these measures of center.

The response to these questions that provided the most conceptual response came from Ms. Alvin. The initial response indicates that Ms. Alvin could calculate the measures of center, but could not provide more of an explanation. However, the response represented by Comments 51 through 56 was especially surprising due to her inability to answer the procedural low questions in an appropriate manner on Appendices E, F, and G.

Number	Person	Comment
51.	Researcher	What does the mean represent?
52.	Ms. Alvin	I don’t know.
53.	Researcher	What does the median represent?
54.	Ms. Alvin	I don’t know… I don’t know how to explain it. I guess I just know how to do it. I think that explaining it is difficult. It is easier just to show.
55.	Researcher	What is the difference between the mean and the median?
56.	Ms. Alvin	The median is finding the middle of all the data you have collected. You are not…I do not know…you have all the information there, but you do not manipulate the numbers to get one number. I don’t know. All I can say is…the difference is when you are finding the average you are taking all of the numbers and manipulating them to get one number that represents the whole group and to find the median you still have that information, you’re just finding the one that falls in the middle.

Of all the responses to this sequence of questions 51 to 56, Ms. Alvin provided the most conceptual description of the difference between the mean and the median. This response is interesting in light of her attempts to solve the questions in Appendices E, F, and G. This sequence of questions and responses shows that it is possible for someone to possess conceptual knowledge of a particular topic without possessing the procedural knowledge to perform the associated calculations.

Another interesting response was provided by Ms. Clark in regard to the usefulness of the median and the mean. The conversation 57 to 60 displays Ms. Clark's lack of conceptual knowledge.

Number	Person	Comment
57.	Researcher	What is the median useful for?
58.	Ms. Clark	The median, being the middle number, would be useful…I don't really know. I guess just to know what a median is. I don't know why you would really need to know what the middle number is, but I guess to know how many times something is done or halfway.
59.	Researcher	Could you give me an example of a set of data where the mean would be more useful than the median?
60.	Ms. Clark	Since I really don't know what the purpose of the median is, the mean would be more important to me in any situation.

<u>Performance on Card Sorting Task in Relation to Measures of Central Tendency</u>

The card sorting tasks involved three distributions – one normal distribution and two skewed distributions. In regard to measures of central tendency, the teachers were asked to examine the histograms and: 1) indicate which measures of center could be calculated based on the information in the histogram and 2) arrange the measures of center for the distributions from least to greatest based on the value of the mean, median, and mode. Recall from the previous section that all three teachers called the data displays bargraphs rather than histograms. The distributions were displayed as in Figure 4.19, 4.20, and 4.21.

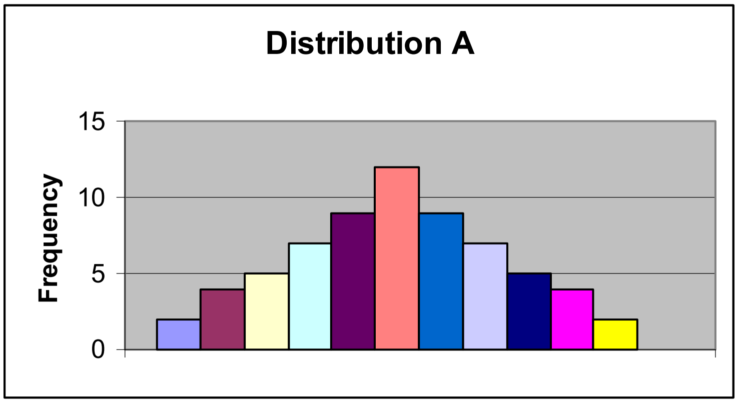


Figure 4.19 Distribution A from Appendix I

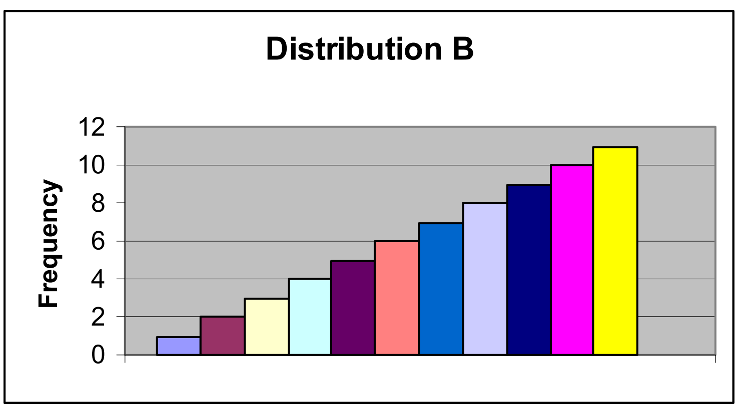


Figure 4.20 Distribution B from Appendix I

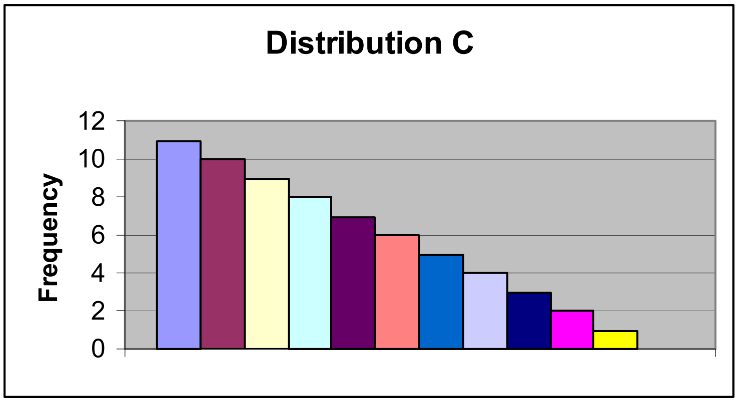


Figure 4.21 Distribution C from Appendix I

All three teachers indicated that measures of center could be found using these distributions. However, only Ms. Clark was able to describe how to find the measures. Comments 61 to 68 represent Ms. Clark's description for finding the mean, median, and mode.

Number	Person	Comment
61.	Researcher	Could you use the information in the displays to determine the mean?
62.	Ms. Clark	I think I could do it; it might take a piece of paper. You had said they were on the same scale. If I knew these numbers I could calculate the mean by adding up all the values and dividing by the total amount.
63.	Researcher	Could you use the information in the displays to determine the median?
64.	Ms. Clark	Yes, I would list all the numbers and find the middle number.
65.	Researcher	Could you use the information in the displays to determine the mode?

66.	Ms. Clark	Yes
67.	Researcher	How would you do that?
68.	Ms. Clark	The mode, meaning what occurs the most often, so I would say the highest bar.

Notice, in comment responses 61 to 68, Ms. Clark always reverted back to the procedures she knew to calculate these measures of central tendency. It should also be noted that, based on simply a histogram, one cannot determine the mean, median, or mode exactly. This is due to the fact that each bar of a histogram represents a range of values. For example, if one was presenting test scores in a histogram and scores ranged from 50 to 100, the first bar of the histogram may be used to display the frequency of scores that fell between 50 and 60. A histogram is used to display data and also give a sense of the overall shape of the distribution. Not one of the three teachers mentioned the shapes of the distributions in their responses. The teachers also were unable to relate the mean to the median (i.e. noting that the median would be equal, greater than, and less than the mean in Distributions A, B, and C, respectively). Although this is a difficult question, none of the teachers had a strategy for exploring it. However, the teachers were able to use the shapes of the distributions to order the cards from least to greatest for measures of central tendency. Comments 69 to 71 represent Ms. Alvin's description of how to organize the distributions from least to greatest for the means.

Number	Person	Comment
69.	Researcher	Assuming these displays are on the same scale, place the cards

		from least to greatest according to their means. So place these cards based upon which distribution would have the smallest mean, which would have a mean in the middle of the other two, and which would have the greatest mean.
Ms. Alvin arranged the cards as follows: C, A, B		
70.	Researcher	Why did you order the cards this way?
71.	Ms. Alvin	If these represent scores on a test from 50 to 100, then the 100 (pointing at Distribution C) would have the fewest amount. In Distribution B, there are more 100's than any other score. Since Distribution A seems to have them equally spaced throughout, then I am leaving that one in the middle.

Ms. Alvin was able to correctly utilize the shapes of the distributions to order the cards from least to greatest according to the mean. Ms. Alvin and Ms. Clark were also able to successfully order the cards according to the median (i.e. Distribution C, Distribution A, Distribution B). However, Ms. Brown was unable to order the cards in this manner. Comments 72 through 80 represent a conversation between the researcher and Ms. Brown.

Number	Person	Comment
72.	Researcher	Can you order the distributions according to which distribution would have the smallest median, which would have the median in between the other two distributions, and which would have the largest median?
73.	Ms. Brown	I don't really know. I am looking at this right here (Pointing to the orange bar in Distribution A). The median is the middle number. So the median for Distribution A would be in the orange bar.
74.	Researcher	What about for the other ones?

75.	Ms. Brown	These on the ends (Pointing to the highest bars). Oh, oh, oh! No these in the middle (pointing to the orange bars in both distributions B and C). But they are the same. I got it. I am trying to rearrange them.
76.	Researcher	How are you trying to rearrange them?
77.	Ms. Brown	By putting the highest bar in the middle for B and C, like it is in A.
78.	Ms. Brown	They would all be the same?
79.	Researcher	All the medians would be the same? Where would they be?
80.	Ms. Brown	Well, I am looking at it. The middle number is in the orange bar. So all the medians would be the same.

Ms. Brown did not acknowledge that the median of the distributions would be influenced by the way the data were distributed. In other words, Distribution C would have the smallest median since there are more values on the lower end of the scale, Distribution B would have the largest median since there are more values on the higher end of the scale, and the median of Distribution A would be in between the medians of Distributions C and B. Again, it should be noted that these values cannot be determined exactly, but it is possible to use the shapes of the distributions to determine in which bar the medians are located. The confusion between the variable and the frequency manifested itself in other ways that are discussed later.

All three teachers were able to order the distributions from least to greatest according to their modes. In order to do this, the teachers assumed that each bar represented a distinct test score. In other words the cards were ordered from Distribution

C to Distribution A to Distribution B, since the highest bar in those distributions was from lowest to highest on the scale. Again, this technically is incorrect because in a histogram one cannot determine what those scores represent. For instance, in Distribution C, although the highest bar appears on the lowest end of the scale, the mode could actually be in one of the lower bars since what the mode is concerned with is the specific value which occurs most often. It does not deal with which range of scores (i.e. bar) that had the highest frequency. The teachers were unable to respond in this manner, since they did not understand the differences between numerical and categorical data nor did they understand the grouping that typically takes place in a histogram. Given their understanding, their responses were viewed as if they understood the concept of the mode in this particular context.

Summary

Based on the results from questions contained in Appendices E, F, G, and I, the teachers demonstrated a procedural knowledge of measures of central tendency. However, the teachers also showed weak conceptual knowledge in some areas of measures of central tendency (e.g. Ms. Alvin's description of the difference between the mean and the median represented by comment number 56; all three teachers' ability to order the distributions from least to greatest according to the means represented by Ms. Alvin's comments numbered 69-71). Some concepts involving measures of central tendency follow in the next section focused on measures of variation. This is due to the fact that a particular lesson that is used as an example involved more than just measures of central tendency.

In order to begin discussion of these issues, the context of students' birth lengths is introduced as it was in a lesson that was observed. This context serves as a point of discussion because it relates the concepts of the mode and range, thus bridging the areas of measures of central tendency and variation. Furthermore, this particular context relates to the professional development training focused on statistics offered by the creators of the *Math Out of the Box* materials.

<u>A Context For Discussion</u>

Comment responses 81 through 106 represent how Ms. Brown worked with her students to help them generate a lineplot to represent their birth lengths. It should be noted that the inclusion of the concepts of the median, mode, and range was due to a choice made by Ms. Brown. These concepts had come up in a previous lesson (discussion forthcoming) after Ms. Brown attended the professional development focused on statistics (discussion forthcoming) that was offered by the creators of the *Math Out of the Box* materials.

Number	Person	Comment
81.	Ms. Brown	Ok, I had you bring in your birth lengths. I want you to come up to the board and put your birth lengths up.

Teacher had students put birth lengths on the board. They were as follows:

25 inches
20 1/2 inches
22 1/2 inches
21 inches
21 inches
21 inches
21 inches

22 inches
21 inches
19 1/2 inches
21 1/2 inches
21 inches
20 inches
19 1/2 inches
19 1/2 inches
20 inches
21 inches

82.	Ms. Brown	Take a look at the values, what do you notice?
83.	Student	Mostly all of them start with 2.
84.	Student	Most common one was 21.
85.	Student	Most common one was 20 or 21.
86.	Ms. Brown	Anyone else, I see that some of us have 1/2's and others do not. Do you think it would be easier or more difficult if we had whole numbers or halves?
87.	Ms. Brown	What could we do to make all the 1/2's a whole? I think we should find a way to make 1/2's all whole.
88.	Student	We could round 20 1/2 to 21.
89.	Ms. Brown	Or we could round down, we have to decide together. Do we want to round up or keep the number and just drop the 1/2?

Ms. Brown started to round up to the next number. Students eventually helped as she was going through.

90.	Ms. Brown	Now you're going to analyze the data. You're not going to do it alone. I'll be here to help and your group members will be here to help. You all might be stronger in a certain area so work together.

Students worked on a lesson that appeared in the *Math Out of the Box* materials. The lesson involved the students creating a tally table of the measurements and then creating a lineplot based on their tally tables.

Ms. Brown arranged the students in groups of four and then had one student read the directions out loud. These directions included reading the added tasks on the back of the page which read: Try This! Find the median, mode, and range. After she had the student read the directions she began to help the students construct their tally tables.

91.	Ms. Brown	Are students' names important in the tally table?
92.	Student	No
93.	Ms. Brown	What do you need here (pointing to the right side of the tally table)?
94.	Student	The length
95.	Ms. Brown	Where will you put that?
96.	Student	Where it says length.
97.	Ms. Brown	And you need to put tally marks to show how many students had each length. I am going to let you go ahead and get started. On the bottom you need to make a line plot. What do you need to remember?
98.	Student	The X's need to be the same height.

As students were working, one student was first filling in the line plot. The teacher was telling her that might not be the best way to do it, implying that the tally table needed to be filled in first.

The students worked well on moving information from the tally table to the line plot. The tally table was as follows:

Birth Length	Count
20	5
21	8
22	2
23	1
24	0
25	1

The students used this information to correctly construct lineplots with 5 X's above 20, 8 X's above 21, 2 X's above 22, 1 X above 23, 0 X's above 24, and 1 X above 25.

Everything went fine until one of the students asked a question regarding the mode and the range.

99.	Student	I can figure out how to find the median, but we disagree how to find the mode and range.

It should be noted here that prior to the class starting (during circle time at the beginning of school), Ms. Brown introduced a data set displayed in a lineplot with the following information:

3 X's above 3, 6 X's above 4, 2 X's above 5, 7 X's above 6, and 3 X's above 7

Ms. Brown told the students that the mode of this data was 7, and that the range of this data set was 7 – 2 = 5. This was based on the fact that the highest frequency was 7 and the difference between the highest frequency (7) and the lowest frequency (2) was 5.

100.	Ms. Brown	How are you trying to find the mode?
101.	Student	I think you should look at the bottom of the line plot rather than the number of X's. Well I guess you look at the number of X's to see which birth length happened most often. So I think the mode should be 21.
102.	Ms. Brown	How are other people in your group finding the mode?
103.	Student	They are saying that the mode is the most number of X's, so they say it is 8.
104.	Ms. Brown	The mode is the most number of X's, so it is 8. I like your thinking in trying to get 21, but the mode is 8 here. What were you thinking about with the range?
105.	Student	Well, I am probably wrong. I was thinking that the range was the difference between the smallest birth length and the biggest birth length. I thought it was 6 (this was from 25 – 19), but they are saying that it is 7 (this was from 8 – 1).
106.	Ms. Brown	Yes, it is 7.

What is most interesting about this observation is that the concepts of the mode and range came up during the professional development offered by creators of the *Math Out of the Box* materials eight days prior to the teaching of this lesson. Even more interesting is that when these concepts were introduced by a student during a lesson involving wrist sizes only two days after the professional development, Ms. Brown corrected the students' misconceptions.

<u>Professional Development Training</u>

Professional development training and what transpired in the class two days after the training is described. Consider these descriptions in light of what occurred during the lesson on birth lengths.

The context for the lesson introduced during the professional development was the circumference of individuals' wrists from the materials. During this training, the researcher sat at the table with Ms. Brown as she was participating in the professional development. A line plot was constructed with four X's above 6, eight X's above 7, 9 X's above 8, and 2 X's above 9. During Ms. Brown's exploration with the teachers, she looked over at the researcher and indicated, "I know this stuff." She proceeded to say that the mode of the data was 9 and the range was 9 – 2 or 7. During the professional development training, the correct method to find the mode and the range were discussed. Ms. Brown clearly realized her error (i.e. using the frequencies rather than the values of the variables) and by the end of the training focused on this topic indicated that the mode was 8 inches and the range was 3 inches.

The Wrist Sizes Lesson Two Days Later

Recall the creation of the lineplot for students' wrist sizes as introduced by Ms. Brown (comment numbers 15-38). Recall that the lineplot created during the wrist size lesson had 2 X's above 5, 10 X's above 6, and 3 X's above 7. During a discussion one of the students in Ms. Brown's class introduced the terminology of the range. Comments 107 through 150 are from a discussion that occurred between Ms. Brown and one of her students following the construction of that lineplot.

Number	Person	Comment
107.	Ms. Brown	[To a particular student] You brought up a good word earlier. What was that word?
108.	Student	Range
109.	Ms. Brown	What is the range?
110.	Student	Range is the difference between the biggest number of X's and the smallest number of X's.
111.	Ms. Brown	Anyone else?
112.	Student	Difference between the most popular and least popular.
113.	Ms. Brown	Can you show me?
The student showed that there were 10 X's at 6 and 2 X's at 7		
114.	Student	So you take the 10 minus 2 to get a range of 8.
115.	Ms. Brown	Anyone else? How many number choices did we have when we recorded the information?
116.	Student	15
117.	Ms Brown	How many measurements, in inches, did we actually have?

118.	Student	3
119.	Ms. Brown	What measurement is the highest we used?
120.	Student	6 inches
121.	Ms. Brown	Was that the highest number of inches that we used?
122.	Student	No. We used 7 as the highest.
123.	Ms. Brown	What was the smallest length?
124.	Student	5

Ms. Brown wrote the word "Range" on the board with the numbers 7 and 5 beneath the word.

125.	Ms. Brown	I want to know the difference between these two numbers. What do I do?
126.	Student	Add them
127.	Ms. Brown	Close
128.	Student	Subtract them
129.	Ms. Brown	So, 7 – 5 = 2. Guess what that is.
130.	Student	The range.
131.	Ms. Brown	I made a mistake. Two what?
132.	Student	Inches
133.	Ms. Brown	So it is not the number of X's. You just look down here (pointing to the numbers below the line in the lineplot).
134.	Student	I do not get how it is 7 and 5.
135.	Ms. Brown	We took the greatest number we had and the least.
136.	Student	We took the most before.
137.	Student	What about the mode?

138.	Ms. Brown	Let's talk about the mode.
139.	Student	I think I know the mode. I think the mode is the opposite of the range, so it is 7 + 5 = 12.
140.	Student	I think when you do the number of X's, the highest amount of X's is 10, so the mode would be 10.
141.	Ms. Brown	(Calling on a particular student) What did you say about the mode?
142.	Student	It would be the measurement that has the most X's above it.
143.	Ms. Brown	What would that be?
144.	Student	6 (The student went up to the board and pointed to it.)
145.	Ms. Brown	So what is the mode?
146.	Student	6
147.	Ms. Brown	Did I have to add anything?
148.	Student	No
149.	Ms. Brown	I just have to say 6 inches. Did I have to count X's?
150.	Student	You could just look at it.

During this sequence of events, Ms. Brown corrected the misconceptions introduced by the students. This was the same misconception (comment number 110 for the range and 141 for the mode) that Ms. Brown exhibited during the professional development training. However, just 6 days after this lesson was introduced and she corrected the students' misconceptions, Ms. Brown reverted to her own misconceptions. It should be noted that the student who went up to the board to correctly show that the mode of the wrist sizes was 6 inches (comment number 144) was the same student that questioned finding the range and mode in the birth lengths lesson (comment number 99). The following table provides a timeline for what transpired. In Table 4.12, day 0 corresponds to the professional development training, day 2 corresponds to the wrist sizes lesson, and day 8 corresponds to the birth lengths lesson.

Table 4.12 Timeline for Ms. Brown's Misconception Regarding the Range

Day	Event
0	Professional Development Training introduced the concepts of mode and range. During this training, Ms. Brown originally exhibited the misconceptions regarding the concepts of mode and range.
0	Ms. Brown's misconceptions were corrected by her experiences with the lesson in the professional development training.
2	Ms. Brown was able to correct the same misconceptions she exhibited during the professional development training when they were introduced by her students.
8	Ms. Brown reverted to her original misconceptions as more time had passed between the professional development training and her coverage of the mode and range. Ms. Brown taught the students during the "morning circle" how to find the mode and range of a data set incorrectly. This teaching contradicted the way she originally introduced the concepts to the students on day 2.
8	Ms. Brown, in error, informed a student that he was incorrect in the manner he found the mode and range which contradicted the responses she praised 6 days earlier.

Based on this sequence of events, Ms. Brown did not exhibit even a procedural low knowledge of the mode nor the range. More importantly, these events have implications for the importance of sustained professional development.

These events bridge the areas of measures of central tendency and variation. By examining the teachers' performance in the area measurements of variation, insight may be gained into whether the teachers possess sufficient knowledge of these topics.

Teachers Performance in the Area of Measures of Variation

The three teachers performed similarly on the content knowledge assessments in the area of variation. The responses to the prompts involving measures of variation are reported in anecdotal fashion since all of the tasks involved more in-depth responses. The teachers were not asked to respond to any stimuli involving measures of variation on Appendices E, F, and G. Continuing upon the misconception identified in the sequence of events outlined above with Ms. Brown, the teachers' performance on the card sorting task is discussed.

Performance on Card Sorting Task in Relation to Measures of Variation

Recall the distributions from the Card Sorting Task were displayed as in Figures 4.22, 4.23, and 4.24.

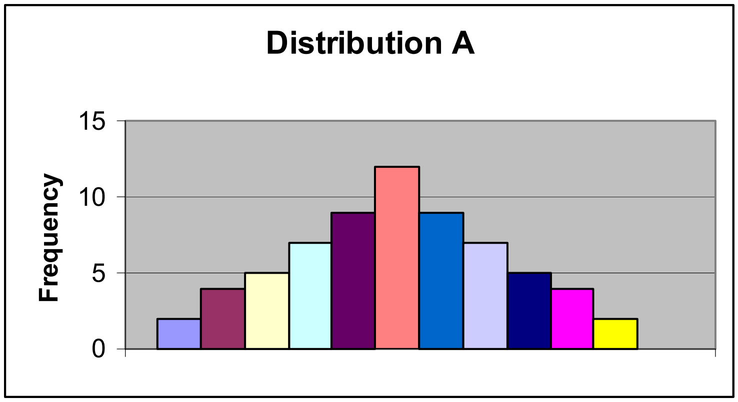


Figure 4.22 Distribution A from Appendix I

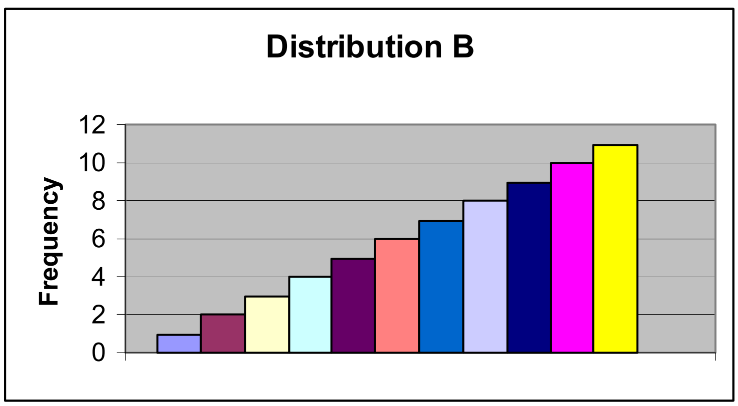


Figure 4.23 Distribution B from Appendix I

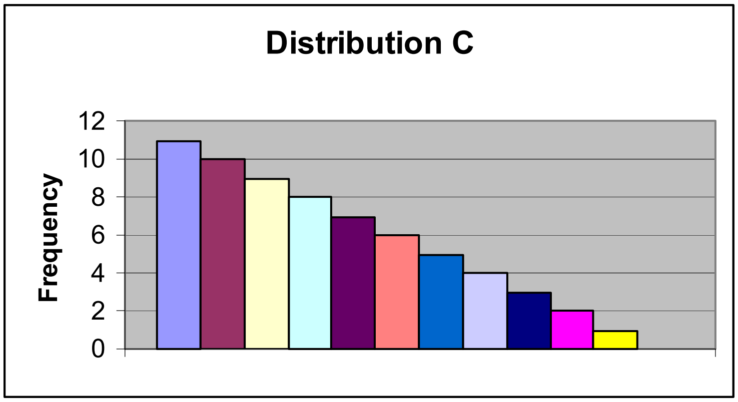


Figure 4.24 Distribution C from Appendix I

In regard to measures of variation, the teachers were asked to examine the histograms and: 1) say which measures of variation could be calculated based on the information in the histogram and 2) arrange the measures of variation for the distributions from least to greatest based on the value of the range and standard deviation.

Ms. Alvin and Ms. Clark had not displayed the misconception involving the range revealed by Ms. Brown in the previous sequence of events. However, during the Card Sorting Task, all three teachers introduced the misconception of taking the difference between the highest frequency and the lowest frequency in order to find the range. The transcript comments 151 to 168 show how Ms. Clark revealed the misconception in the context of arranging the distributions from least to greatest according to the range. The initial lines in the transcript were from question 2 in Appendix I where Ms. Clark was asked if she could use the data displays to find the range.

Number	Person	Comment
151.	Researcher	How would you use the information in the distributions to find the range?
152.	Ms. Clark	I would subtract the lowest number from the highest number.
153.	Researcher	Can you arrange the distributions from least to greatest according to their range?
154.	Ms. Clark	The range would be the difference between the highest and the lowest results so 11 – 1 would be 10 (for Distribution C). But it would be the same for distributions A and B.
155.	Researcher	How are you getting that 11-1 in Distribution C?
156.	Ms. Clark	The highest score (pointing to the highest bar which is on the left side of the distribution) minus the lowest score (pointing to the lowest bar which is on the right side of the distribution). But again when it comes to A and B, 11 (pointing to the highest bar which is on the right side of distribution B) minus 1 (pointing to the lowest bar which is on the left side of the distribution). The range of the data is a range of 10. I do not know how I would figure it out without the scores though, because what I want to do is just subtract 11 minus 1 and get a range of 10 for the bars, but if we're talking about the scores and I don't know the scores… Even if I knew the scores, it would still be the same score. Let's say I gave a value to the yellow bar and it was 100% and I was subtracting it to get the range from the lowest value. The lowest value is still, let's say the purple bar is 50%. I don't think that is right. I think it is the range of the bars. 11 – 1 = 10. The range of the information is a range of 10. I do not know.
157.	Researcher	So how do you find the range?
158.	Ms. Clark	I am trying to find the range by subtracting the highest result from the lowest result. So the range of a piece of information, so for example if I had like 12 different pieces of information and it started with 30 and ended with 50, then the range would be 20. I am just confused by the bars, so I am looking at the bars as pieces of information. So on B and C there is a bar with

		11 and there is a bar on each of these with 1, 11 minus 1 is 10, so I think the range would be the same. Distribution A is difficult to say, but I think it is about 12 (pointing to the highest bar in the center of the distribution) minus 2 (pointing to bars on both the left and right side of the distribution), so again that would be a range of 10.
159.	Researcher	You had mentioned the test scores: where you said this was 100 and that was 50. If that was the case, how would you figure out the range? Would you do it the same way?
160.	Ms. Clark	Between 100 and 50?
161.	Researcher	Well if, like you said, the purple bar represented 50% and the yellow bar represented 100%, what would the range be then? Would it still be 10?
162.	Ms. Clark	Pointing at Distribution A: Well if I was using the numbers 100 and 50, the middle would be 75. So no, the range I guess would be 25.
163.	Researcher	How do you get that 25?
164.	Ms. Clark	The highest to the lowest score (pointing to the highest score being in the middle with the highest bar and the lowest score being on the right with lowest bar).
165.	Researcher	So where is the highest score and where is the lowest score?
166.	Ms. Clark	The highest is in the middle minus the lowest which is at the ends, either end. The difference between 100 and 75 is 25 and the difference between 75 and 50 is 25.
167.	Researcher	What about for Distributions B and C?
168.	Ms. Clark	Well the highest score in Distribution B is 100 and the lowest score is 50, so the range is 50. For Distribution C…the highest score is 50 and the lowest score is 100, so the range is 50.

Ms. Clark revealed a somewhat different misconception than Ms. Brown. Here, Ms. Clark initially seemed to have the right idea (comment number 153) and again later (comment number 159). However, what she meant as the highest score was not the same as what is typically thought of as the highest score. In reference to Distribution A (the approximately normal distribution), Ms. Clark assigned the score of 75 to the bar in the middle with scores of 50 and 100 on the ends. Here by highest, Ms. Clark meant the mode (represented by the tallest bar), so she calculated 75 – 50 or 100 – 75 to obtain a range of 25 rather than 50. When the same line of reasoning was used in reference to Distributions B and C, Ms. Clark obtained the correct range of 50. Ms. Alvin exhibited a similar misconception in her description of how to find the range.

All three teachers were unable to determine the standard deviations of the distributions nor could they hypothesize which distribution would have a larger or smaller standard deviation (the term spread was also used in an attempt to stimulate a response). The teachers all indicated they had never been exposed to standard deviation nor did they understand the concept. However, Ms. Clark correctly used the concept of spread in responding to the catapult question as discussed in the following section.

Performance on the Catapult Question

The catapult question involved the teachers examining comparative dotplots in order to choose the best catapult for landing ping pong balls within a certain band of a target line for a game. The context was thoroughly explained to the teachers and they were asked to explain the context back to the researcher, prior to their choosing a catapult. The dotplots were displayed as in Figure 4.25.

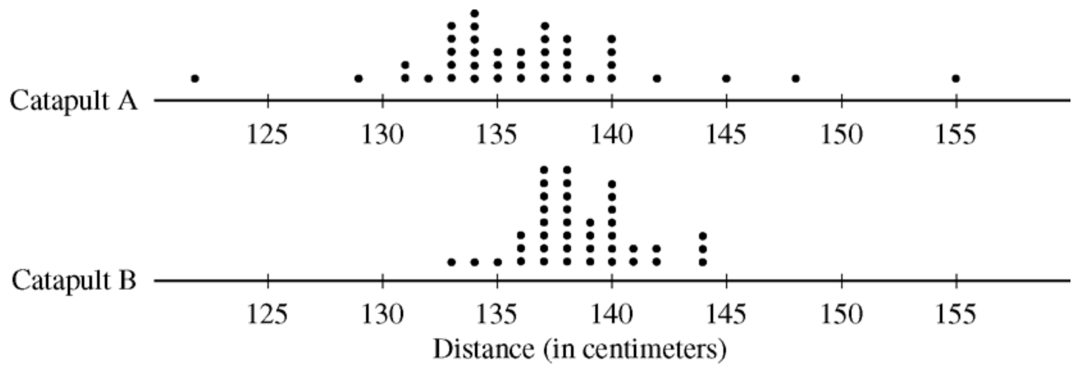


Figure 4.25 Dotplots from Appendix H

This question attempted to get at whether or not the teachers realized that in order to win the game, one would want to reduce the variability (or spread).

Ms. Alvin and Ms. Clark chose catapult B because the range was smaller and the ping pong balls were less spread out. Ms. Brown chose catapult A and could not fully explain her reasoning for such a choice. She did not realize the importance of trying to minimize variability. The transcript comments 169 and 170 display Ms. Clark's response to this question.

Number	Person	Comment
169.	Researcher	If the parents want to maximize the probability of having the Ping-Pong balls land within the band, which one of the two catapults, A or B, would be better to use than the other? Catapult A or B can be placed anywhere parents desire to maximize their chances of landing balls within the 5 cm band. Justify your choice.
170.	Ms. Clark	I think I would use catapult B because there is not a lot of, what I call, outliers. It is more consistent than catapult A. So for example, the range for catapult A is approximately 25 centimeters and the range for catapult

	B is about 10 centimeters. So I guess catapult B is more reliable.

In relation to the misconception Ms. Clark revealed during the Card Sorting Task, it is interesting to see that Ms. Clark was fairly successful in her ability to calculate the range. The catapult question was introduced prior to the Card Sorting Task.

In relation to measures of central tendency, Ms. Alvin and Ms. Clark utilized the mode as their method of deciding where to place the catapult. Ms. Clark's response (comment 172) reflects this line of thinking.

Number	Person	Comment
171.	Researcher	Using the catapult that you chose [before], how many centimeters from the target line should this catapult be placed? Explain why you chose this distance.
172.	Ms. Clark	I would place it at either 136 or 137 centimeters from the target line. The majority of the ping pong balls…I guess I am looking for accuracy…so the majority of the ping pong balls would land…looking at catapult B most of the ping pong balls landed at 137 so give or take five centimeters…I think that placing it about 137 or 138 centimeters away from the target line would be appropriate.

Ms. Clark and Ms. Alvin were able to choose the more appropriate catapult and successfully place the catapult in an appropriate position based upon measures of variation and central tendency, respectively. These responses again show some level of conceptual knowledge in relation to these overarching areas. Ms. Brown's response was misguided as she chose catapult A and could not justify a reason for such a choice. However, she did indicate that she would place the catapult 134 centimeters from the target line since that distance occurred the most often.

During her response regarding which catapult to choose (comment number 171), Ms. Clark mentioned the term outliers. An outlier is defined as “a value that falls far removed from others in a data set.” Calculations for finding such a value are not discussed at either grade level. The manual also indicates that often “an outlier is the result of a mistake in the data collection process…in statistical analysis, an outlier is discarded when a whole data set is examined” (Grade 3, p. 157).

The three teachers seemed to understand the concept of outliers within the definition provided by the materials. In comment number 174 Ms. Clark provided the definition for an outlier.

Number	Person	Comment
173.	Researcher	What is an outlier?
174.	Ms. Clark	It’s the number that is significantly further away from the other pieces of information. This is usually due to a mistake.

Ms. Brown followed up with a more accurate definition of an outlier (comment number 176).

Number	Person	Comment

175.	Researcher	What is an outlier?
176.	Ms. Brown	A piece of data that doesn't fit into your normal shape of a lineplot. It is way out in nowhere land. My kids asked me a question about that. They asked if it is incorrect data, but I don't think that it is, I mean I know that it is not, but it could be, anything could be incorrect, but I just told them that it's just kind of out there hanging out over here and it's just not fitting into this but it is still data and it is still part of our line plot.

Ms. Brown's description of an outlier is actually more appropriate than what is provided by the *Math Out of the Box* materials. Outliers are often one of the most interesting things to consider when exploring data sets. If an outlier occurs naturally rather than due to a recording error, then further questions should be asked to investigate why such a situation occurred.

All three teachers were unable to describe how to find an outlier beyond simply saying "look at the distribution and if it is way out there, then that is an outlier." The *Math Out of the Box* materials did not introduce how to find an outlier (there are several definitions and algorithms); they just introduced the concept. It appeared as though all three teachers understood outliers. This is another example of conceptual knowledge without procedural knowledge. In other words, the teachers knew what an outlier was but they could not use a procedure to find one.

Summary

The three teachers involved in this study had difficulty understanding measures of variation. It is clear that all three teachers possessed misconceptions regarding the concept of the range. Ms. Brown's misconception in this area seems particularly deeply rooted as exhibited by the reappearance of the misconception even after its correction. In order to gauge what the teachers know and do not know it is important to return to what the GAISE document recommends for students at Levels A and B. The GAISE recommendations are compared to the results outlined above.

Connecting the Teachers' Understanding to the GAISE

Based upon the recommendations outlined by the GAISE document, it would appear that the three teachers involved in this study were not prepared to help students realize the objectives of Levels A and B. The teachers did not possess an adequate level of knowledge of the topics expected of students at Levels A and B. In order to elaborate upon this statement, topics in the areas of data displays, measures of central tendency, and measures of variation are discussed.

Data Displays

The GAISE document indicates that students at Level A should understand how to construct and use frequency tables, bargraphs, stem-and-leaf plots, dotplots, scatterplots, and timeplots. The teachers in this study had not been exposed to stem-and-leaf plots or scatterplots. This was exhibited by their performance on questions 2 and 5 on Appendix G. Dotplots are considered extremely similar to lineplots and the teachers seemed to have a good understanding of those data displays based on their experience with the *Math Out of the Box* materials.

The GAISE document indicates that it is important for students to realize the difference between a bargraph and histogram at Level A. Based on the teachers' responses to various questions (e.g. comment numbers 1-8), it appears that they did not possess such knowledge. This type of knowledge would be considered as conceptual. Teachers should understand the difference between categorical and numerical data and should realize what is an appropriate data display for each. Based on the results of this study, these three elementary school teachers were not prepared to help their students make these realizations.

At Level B, students should understand how to construct and use histograms, frequency tables, boxplots, grouped frequency tables, and time series plots. Based on the results of this study, these teachers were unfamiliar with histograms and boxplots. They were unable to answer question 6 on Appendix G based on their lack of familiarity with boxplots. Students at this level should also be introduced to the misuses of graphs. Possessing the high level of procedural knowledge necessary to construct a circle graph (as appears on question 8 of Appendix G) is related to the ability to determine if graphs are misused. If one cannot appropriately construct a data display, then it may be assumed that same person would have difficulty in determining if a data display misrepresents the information. It is important when constructing any data display to make sure the display is representative of the data involved. An inability to comprehend what appears in various data displays influences students' abilities to find appropriate measures of central tendency.

Measures of Central Tendency

At Level A, students are expected to understand the mean, median, and mode as a fair share, middle point, and value that occurs most often, respectively. Based on the results of this study, these three teachers did not understand the mean and mode. The teachers were able to calculate the mean; however two of the three teachers were unable to realize the mean represents a fair share. Ms. Alvin was the only teacher that was able to provide a description of the mean that included the idea that the mean is a fair share (comment numbers 51-56). This is exhibited by their inability to determine when a mean was more useful than a median.

Students at Level A should be able to determine when certain measures of central tendency are inappropriate for different data sets. A lack of understanding of the appropriateness of various measures of central tendency is apparent in Ms. Clark's response to when a median may be more useful than a mean (comment numbers 57-60).

Measures of Variation

These three teachers seemed most ill-prepared in the area of variation. At Level A, students should understand what maximum and minimum values represent and how these values relate to the range for a set of data. It is clear from the results of the study that the teachers did not understand the concept of the range, or at least were not able to apply what they knew to a histogram with an undefined independent variable. In fact, all three teachers revealed misconceptions in regard to finding the range for a set of data. At Level A, students should also begin to question why variations occur in data (e.g. outliers). Based on their interaction with the *Math Out of the Box* materials, it appears that the teachers knew what an outlier is, but they did not realize the importance of investigating outliers further. This is related to the definition presented in the materials themselves. However, Ms. Brown seemed to have the beginnings of such an understanding.

At Level B, students should begin to realize variation involves comparing values to a central value. Based on the results of this study, these three teachers had not made such a realization. According to the results involving the determination of the spread for the card sorting tasks, the teachers were unable to relate how these distributions may be spread about a central value. The GAISE document also recommends that students at Level B be introduced to the interquartile range. Given the teachers' lack of knowledge in reference to the range, it is safe to assume they did not possess the knowledge of the interquartile range.

Summary

Based on the results of this study it would appear that the three teachers involved in this study were not prepared to help students realize the objectives of Levels A and B as outlined in the GAISE document. It is unreasonable to expect teachers to teach students material at a level which exceeds their own level of knowledge. In the next section of this chapter, the influence these experiences had on the teachers' awareness of their knowledge in the area of statistics is discussed.

Teachers' Awareness of Their Understanding

All three teachers had some reservations about teaching mathematics in general. In relation to data analysis and statistical topics, the teachers felt fairly comfortable teaching material at their grade level. However, they were hesitant to consider themselves ready to teach anything beyond their grade level. This feeling is evident in the transcript comment numbers 177 and 178.

Number	Person	Comment
177.	Researcher	How would you assess your level of understanding of data analysis and statistical topics covered at your grade level?
178.	Ms. Clark	I feel comfortable teaching the material at the third grade level, but I wouldn't want to go up to the fourth grade level. Students have difficulty with the concepts. I understand data at this level and feel comfortable teaching the students.

These feelings were echoed by Ms. Alvin and Ms. Brown. They all felt comfortable teaching the material at their particular level, but not beyond.

Through their experiences with the *Math Out of the Box* materials, the teachers became exposed to more data analysis and statistical topics than they had in the past. As a result, they had to learn or re-learn some of the material. This is evident in the transcript comment numbers 170 to 180.

Number	Person	Comment
179.	Researcher	What concepts did you have to review prior to teaching the data analysis and statistical lessons?

180.	Ms. Alvin	I had to review all of the material: mean, median, mode, range. I had never heard of an outlier before. We never covered this kind of stuff before these materials. I mean…in science, we may have done an activity where we counted the number of seeds in a bean pod. We may have talked about what number occurred most often. We didn't teach enough of it, so I hadn't thought about this stuff for a long time.

It is clear that Ms. Alvin was not the only teacher who had a change in view regarding the knowledge necessary to teach data analysis and statistics. Transcript comments 181 through 182 represent comments from a conversation between the researcher and Ms. Clark.

Number	Person	Comment
181.	Researcher	Has your thinking about data analysis and statistics changed as a result of the *Math Out of the Box* lessons?
182.	Ms. Clark	Definitely, because I didn't even know most of these terms. I knew what a mean or a mode was, but there were things that I didn't even understand. I had only been given a shallow amount of data before. This presented it within a context so it wasn't just numbers. It had more meaning to it, than just doing it. There were a lot of levels that you could go all over with.

Ms. Brown also developed a realization that there was a lot more to learn when it came to mathematics as shown in transcript comments 183 through 184.

Number	Person	Comment

183.	Researcher	What have you learned about data analysis and statistics as a result of your work with the *Math Out of the Box* materials?
184.	Ms. Brown	That I'm not that good at it, but it's OK because my kids can help me through it and it's OK that if I don't know something I can learn it with my students. We can all learn it together.

This quote has two views that can be taken. On the one hand, it can be seen as a negative that a teacher would think of entering a classroom without knowing the material. On the other hand, it can be seen as a positive, that at least this teacher is willing to learn new things with her students. As the sophistication level involved in standards are increased and teachers have not experienced the learning of concepts firsthand, they will need time to learn the material.

Ms. Clark made a similar comment as shown in comments 185 through 186.

Number	Person	Comment
185.	Researcher	What have you learned about yourself as a learner?
186.	Ms. Brown	That I am capable too. I learned a lot that I didn't know before. I mean it is third grade math but I still didn't necessary know all of it or experience all of it and do it.

Based on these limited results, it does appear that the teachers gained more of an awareness of where their knowledge was in regard to statistics. It should be noted that their feelings were also influenced by the questions that the researcher posed. Based on these questions, the teachers felt as though they were not prepared at the level suggested in the GAISE document.

The most telling result in regard to the teachers' awareness of their knowledge comes from the results of a question which was taken from the Begg and Edwards (1999) study. At the beginning and end of the study, the teachers were asked the following question:

If you could receive a day of professional development focused on data analysis and statistical topics, would you prefer to have the professional development focused on: (a) activities at your grade level; or (b) data analysis and statistical content? At the beginning of the study, all three teachers chose professional development focused on activities. At the end of the study, all three teachers chose professional development focused on data analysis and statistical content. As a follow-up question to this change in opinion, all the teachers were asked whether their change was due more to their interaction with the *Math Out of the Box* materials or with their interaction with the questions posed on various assessments that were part of this study. All three teachers indicated their change was due more to their interaction with the questions and their self-discovered lack of familiarity with many data analysis and statistical topics.

Conclusion

The results of this study indicate that these elementary school teachers were not prepared to teach the level of detail associated with the GAISE document. Based on these results, these teachers primarily had a procedural low level of knowledge in the area of statistics. There are several implications to this study and further questions for consideration that are discussed in the next chapter.

CHAPTER FIVE

IMPLICATIONS

This chapter discusses the implications that may be drawn from this study. The chapter is divided into six sections. The first section provides a short synopsis of the answers to the research questions. The second section focuses on the difference between procedural and conceptual knowledge in the area of statistics. The third section discusses the implications for teacher training. The fourth section focuses on the importance of sustained professional development. The fifth section discusses curricular implications. The chapter concludes with the identification of topics and questions for future research.

Answering the Research Questions

This research study attempted to answer the following research questions:

1. What is the understanding of elementary school teachers in the following areas of statistics: data displays, measures of central tendency, and measures of variation?
2. Does the implementation of the curricula materials and exposure to advanced assessment instruments influence elementary school teachers' awareness of their understanding of statistics?

Based on the results of this study, it is possible to provide answers to the research questions. The answers provided in this section are brief but are expanded upon throughout the chapter. The elementary school teachers involved in this study generally had a low-level (procedural low) understanding of essential topics in statistics. Their understanding was lacking in each area explored during the course of this study. The teachers' interaction with the *Math Out of the Box* materials and exposure to advanced assessment instruments influenced their awareness of their understanding. By the end of the study, the teachers had realized they lacked the sophisticated level of understanding that is necessary to teach statistics effectively. The answer to question 1 is expanded upon in Section 2 and the answer to question 2 is expanded upon in Section 4.

Procedural versus Conceptual Knowledge of Essential Topics in Statistics

As described previously, it is difficult to define "understanding of statistical content." The results of this study help shape a distinction between what it means to understand certain statistical topics at the procedural versus the conceptual level. Based on this study, these distinctions can be made in the areas of data displays, measures of central tendency, and measures of variation.

In the area of data displays, teachers should, according to the GAISE document, recognize the advantages and disadvantages of various data displays. This recognition represents conceptual knowledge; however the teachers in this study rarely exhibited this type of knowledge. The ability to read information from a particular display involves a procedural knowledge of that particular display. The teachers involved in this study were all able to use the information in tables, bargraphs, lineplots, dotplots, linegraphs, and pictograms to answer particular questions. However, not one of the teachers was able to understand the conceptual basis for the appropriateness of certain data displays for particular sets of data.

The appropriateness of various data displays for particular sets of data is dependent on whether the variable is categorical or numerical. The teachers involved in this study were misguided in their focus. In the birth lengths setting, for example, Ms. Brown focused on the frequency for each birth length rather than the birth lengths themselves. She did not distinguish between the value of the variable (the actual length) and the number of times that particular value occurred. This inability to identify what the variable was led to her misconception regarding the mode and range. In order for teachers to develop conceptual knowledge in regard to data displays, they must understand that the variable of interest should determine the appropriate types of data displays. Elementary school teachers with a conceptual understanding of data displays should realize that there are appropriate displays for categorical data (e.g. frequency tables, bargraphs, and circle graphs) and appropriate, but different, displays for numerical data (e.g. histograms, stemplots, boxplots). The elementary school teachers in this study did not possess such an understanding. While they had procedural knowledge in that they could answer questions that required them to read information from graphs, they did not possess conceptual knowledge that would allow them to realize the appropriateness of different data displays for certain situations.

In the area of measures of central tendency, there is also a distinction between procedural and conceptual knowledge. Although Ms. Alvin had difficulty with the procedure of finding a median at times, overall these three teachers were successful in applying the procedures to find the mean, median, and mode. However, Ms. Alvin was the only teacher that was able to describe some of the properties that make the mean and the median different.

For conceptual knowledge in the area of measures of central tendency, teachers should understand the appropriateness and usefulness of the mean, median, and mode. They should also realize that the appropriateness and usefulness of the mean, median, and mode are connected to whether the data is categorical or numerical. Of the three, the mode is the only useful measure of central tendency for categorical data. For example, when considering the religion associated with a particular area, it would only be appropriate to report the mode. The mode for this particular situation would be the religion that was represented most often. It would not be appropriate to calculate the mean or find the median of Buddhism, Christianity, Hinduism, Islam, Judaism, and the many other religions around the world.

In order for elementary school teachers to possess conceptual knowledge of measures of central tendency related to numerical data, they should realize that the appropriateness and usefulness of the median and mean are connected to measures of variation. In regard to the median, teachers' understanding should begin with the realization that the median is the middle value when all values are arranged from least to greatest. Their understanding of the median should extend to their realization that it is the value that has the smallest absolute distance from all other values in the data set as compared with any other data point. Consider the following data set to examine this property.

$$1, 4, 5, 9, 14$$

The median of this data set is 5. This value has the smallest absolute distance (18) from all other values. The absolute distance for 1 is $|1-1|+|1-4|+|1-5|+|1-9|+|1-14| = 28$. The absolute distance for 4 is $|4-1|+|4-4|+|4-5|+|4-9|+|4-14| = 19$. The absolute distance for 5 is $|5-1|+|5-4|+|5-5|+|5-9|+|5-14| = 18$. The absolute distance for 9 is $|9-1|+|9-4|+|9-5|+|9-9|+|9-14| = 22$. The absolute distance for 14 is $|14-1|+|14-4|+|14-5|+|14-9|+|14-14| = 37$. The smallest absolute distance (18 for this example) occurs at the median. Elementary school teachers who possess a conceptual understanding of the median should realize that the median has this property.

In regard to the mean, elementary school teachers should realize the mean is the balance point of the data. Because of this the mean is influenced by extreme values. Teachers should realize the reason for this is that the mean involves all values in a data set where as the median only involves the one particular value that falls in the middle. To illustrate this, consider a slightly modified data set from the one provided in the previous example.

$$1, 4, 5, 9, 140$$

The median of this data set is 5, which is the same as the median in the previous example. The mean of the original data set (1, 4, 5, 9, 14) is 6.6 whereas the mean of this new data set is 31.8. The influence of one extreme value on the mean may be drastic. As a result, in order for elementary school teachers to possess conceptual understanding of measures of central tendency they need to realize this property. Ms. Alvin was the only teacher among the three involved in this study that revealed the beginnings of such an understanding.

Measures of variation are appropriate for numerical variables only. Since the teachers were not able to distinguish variable types, they were necessarily limited in their understanding. In regard to measures of variation, elementary school teachers should realize that measures of variation (e.g. range, mean absolute deviation, and standard deviation) help inform whether the mean or the median would be more representative of the center for a data set. If the spread of a data set is large, then the median may be more representative of the data (e.g. a skewed distribution). It should be noted that measures of variation do not directly determine which measure of center is more appropriate for a given data set. However, the influence variation has on the measures of central tendency should be acknowledged.

The teachers involved in this study did not possess sufficient knowledge in the area of measures of variation. Due to their lack of ability to apply various procedures to describe measures of variations, it is difficult to draw implications regarding the difference between procedural and conceptual knowledge in relation to measures of variation. The possession of procedural knowledge in this area involves the ability to calculate certain measures. For instance, teachers with a procedural knowledge of the range would be able to calculate this descriptive statistic.

All three teaches displayed misconceptions regarding the calculation of the range. The misconceptions were all related to the teachers' confusion regarding whether the variable was categorical or numerical. This misconception could also be attributed to whether or not the teachers had sufficient experience with histograms as a data display. A lack of understanding with the display may have resulted in the teachers focusing on the frequency rather than the appropriate variable. Ms. Brown calculated the difference between the maximum and the minimum frequency for the birth lengths rather than the difference between the maximum and minimum birth length. All three teachers calculated the difference between the maximum and the minimum frequency of the "test scores" on the Card Sorting Task rather than the difference between the maximum and minimum "test score." Since this is the primary measure of variation dealt with at the elementary level according to the GAISE, it is paramount that elementary school teachers possess a conceptual understanding of the topic.

The example above illustrates how conceptual knowledge, or the lack thereof, informs procedural knowledge, or the lack thereof. The teachers in this study did not realize the difference between categorical and numerical data and how this notion relates to data displays and measures of central tendency. This lack of conceptual knowledge informed their misconceptions regarding the calculation of the range. These teachers viewed the numerical data as categorical data and were unable to find the range.

Teachers with a conceptual knowledge of the range would realize that the measurement represents the amount the data is spread out. Knowledge of measures of variation informs the teachers' understanding of advantages and disadvantages for various measures of central tendency. It is clear that understanding essential aspects of statistics is dependent upon understanding other essential aspects of statistics. Without this fundamental knowledge in the areas of data displays, measures of central tendency, and measures of variation, elementary school teachers will have difficulty teaching statistics at the level proposed by the authors of the GAISE document.

The Status of Elementary School Teachers' Knowledge

The three teachers involved in this study were recommended by their district supervisor for mathematics and science education and principals as exemplary teachers of mathematics. These teachers had extreme difficulty with the calculation of one of the most straightforward measurements in statistics – the range. They exhibited misconceptions that had been addressed during their professional development training. Although the sample of teachers for this study was small, the fact that these teachers were considered exemplary would lead one to wonder what may be expected of teachers who are not considered exemplary. In order to prepare teachers for teaching statistics at the level proposed by the authors of the GAISE, they must be exposed to experiences with statistical content during their preservice teacher training or through sustained professional development. The evidence found here through the example of Ms. Brown suggests that one experience, even though it initially facilitated greater understanding, is not sufficient as teachers may lapse back to their original misconceptions.

Implications for Teacher Training

The three teachers involved in this study do not possess knowledge of essential statistical concepts as outlined at Levels A and B of the GAISE document. If the sophisticated level of understanding described by the authors of the GAISE document is to be realized by K-12 students, it is important that teachers are prepared to teach statistics at this level. Since these expectations are relatively new, most preservice teachers likely have not had sufficient experiences during their K-12 schooling to develop such an understanding. As a result, teacher preparation programs might develop courses to ensure the objectives identified in the GAISE document are realized by the teachers who will be called upon to educate students in such a manner. Since the results of this study suggest one exposure to the content is insufficient, perhaps multiple exposures to statistical ideas during teacher preparation programs are necessary.

As discussed in the previous section, learning statistical content within a context that emphasizes the advantages, disadvantages, and appropriateness of various data displays, measures of central tendency, and measures of variation would be a good place to start the training. They must first learn the concepts in depth and be exposed to contexts which emphasize the advantages and appropriateness of the various data displays, measures of central tendency, and measures of variation.

The discipline of statistics goes beyond the focus of this study. Statistics also involves the formulation of questions, the design of studies to answer those questions, and the use of statistical tools to make inferences about populations based on samples. An entire course might be dedicated to statistics education during teacher preparation programs for elementary teachers in order to help them develop the experiences described in the GAISE Document.

As discussed previously, students must progress through the experiences suitable for Level A before moving onto Level B, and thus must progress through appropriate experiences at Level B before moving onto Level C. If K-12 students are expected to make such a progression, then so should teachers. Without this progression, teachers may have difficulty developing the sophistication expected in order to be prepared to teach students at such an advanced level. A course dedicated entirely to statistics education might help teachers; however this implication will only have an impact on teachers that are not yet in the field. If inservice teachers have not been exposed to this type of progression, then professional development opportunities should be provided to expose them to the necessary experiences.

Sustained Professional Development Experiences

These three inservice teachers could likely benefit from professional development training that focuses on the development of statistical content knowledge. By the end of the study, all three teachers had acknowledged an awareness of their lack of content knowledge in the area of statistics and a desire to receive professional development focused on this particular content strand. Although the teachers' interaction with the curricula materials and the assessments implemented by the researcher did not sufficiently influence the teachers' understanding of essential topics in statistics, it caused them to reconsider the suitability of their own content knowledge. In other words, this interaction problematized the awareness of their knowledge.

Once teachers recognize new viewpoints or what may be lacking in their own understanding, problematization occurs. Problematizing teachers' knowledge is essential for professional development to be successful in changing teachers' preparedness for teaching statistics (Cobb & Bauersfled, 2005). Teachers that realize they have a lack of understanding in a particular area are more likely to benefit from professional development focused on content. The three teachers involved in this study provide an example that illustrates the importance of such a realization. At the beginning of the study, the teachers preferred activities and more than likely would have avoided or half-heartedly participated in professional development focused on content. At the end of the study, they preferred professional development focused on content. With the problematization they experienced, the teachers would be more likely to absorb the content introduced during professional development.

In order for the professional development to be effective it should be delivered to inservice teachers several times over several years. As the misconceptions exhibited by Ms. Brown before and after the professional development training offered by the creators of the *Math Out of the Box* indicate, it is imperative that sustained professional development be provided. Based on the results of the study, it can be concluded that a single effort at correcting teachers' misconceptions may not be successful. Professional development in the area of statistics might be more successful if it were offered on a more sustained basis. Further studies are necessary to investigate the effect of various professional development training opportunities on the statistical content knowledge of elementary school teachers.

Implications for Curricular Development

Given the lack of content knowledge elementary school teachers possess, caution should be used when creating curricular materials. Curricular materials should include contextual examples for elementary school teachers to call upon to realize the advantages, disadvantages, and appropriateness of various data displays, measures of central tendency, and measures of variation. The *Math Out of the Box* materials provide an excellent example of how statistical content can be learned within a context. These materials provide opportunities for richer learning experiences than the students might otherwise encounter in their textbooks. In many instances, *Math Out of the Box* involves more than just a surface-level question to assess for student understanding. The materials build the content around real data collection activities. These experiences should help students develop an ability to make connections. In order for the students to realize these connections and get the most out of the contextual experiences, their teachers must possess sufficient knowledge of statistics to guide them along the way. This research suggests that even exemplary elementary school teachers may have difficulty helping their students get the most out of the curriculum.

In light of the possibility that teachers may lack sufficient knowledge of essential statistical concepts, care should be used in the construction of the definitions that are provided in the materials. The definitions presented in curricular materials may serve as the only exposure to the content the teachers have. For example, Ms. Clark's description of how to treat an outlier in a data set was similar to the description that was presented in the curriculum. This description was inaccurate in that it states that "an outlier is the result of a mistake in the data collection process…in statistical analysis, an outlier is discarded when a whole data set is examined" (Moss et al., 2007, p. 157).

Some curricular creators are reluctant to include sample answers to open-ended questions since teachers may look only for those "correct" answers. This may be especially the case when teachers lack the knowledge to expand upon the answers provided in the curriculum. However, it may be worse for teachers to accept an incorrect answer (or give one themselves) than for them to look for "correct" answers. Given the results of this study, both appropriate and inappropriate examples should be provided for the teacher to call upon. Sample solutions would help teachers make those distinctions. Professional development focused on the implementation of the materials should address the issue of teachers accepting more than the samples provided. The materials should also indicate that these are sample responses and they should encourage teachers to call upon other resources to verify whether students have provided an appropriate response.

The types of questions that appear in curricular materials need to assess students in identifying the advantages, disadvantages, and appropriateness of various data displays, measures of central tendency, and measures of variation in different contexts. Traditional textbooks tend to ask questions which focus on reading information from graphs. These types of questions are not appropriate to assess the level of understanding described by the authors of the GAISE document. However, textbooks are not the only medium in which low-level questions are posed to students.

Konold and Khalil (2003) examined the levels of questions posed on high-stakes tests in the area of data analysis. The results of their work shed light on why many students associate statistics with "doing something with numbers." They claim that test developers for these examinations interpret data analysis and statistics as taking information from a graph. Many questions, in fact, involve only examining a graph and finding a particular point. Konold and Khalil conclude that "current high-stakes assessments are virtually ignoring all but the most rudimentary skills in data analysis" (2003, p.6).

Asking questions which force students to look beyond the graphs helps them develop conceptual knowledge of essential topics in statistics. Furthermore, if the students are thinking beyond the graphs, then the teachers also have to think beyond the graphs. Inservice teachers who have not had sufficient experiences to develop sophisticated understanding of statistical content will rely upon their curricula materials for content, questions, and suitable answers to those questions.

Areas for Future Research

The results of this study begin to describe the status of elementary school teachers' understanding of essential topics in statistics. Further studies are needed to generalize to all elementary school teachers. This study investigated the understanding elementary school teachers possess and found this understanding to be lacking for these three elementary school teachers. Broader studies which contain a larger sample of teachers will help further examine the status of elementary school teachers' understanding of essential topics in statistics. Additionally, studies should explore more aspects of the discipline of statistics. For example, studies should explore teachers' understanding of the formulation of questions, design of studies to answer those questions, and making inferences based on the results of studies.

Whether all teachers are prepared to help their students develop the level of statistical understanding outlined in the GAISE document is a question that should also be examined. The objectives outlined in the GAISE document may be too far-reaching for the current status of elementary school teachers' knowledge of the discipline. The objectives at Levels A and B exceed the current level of understanding possessed by the three teachers involved in this study.

If teacher preparation programs are modified to include a course on teaching statistics, then the influence of such a course should be examined. This influence needs to be examined on an immediate and a longitudinal basis. Simply exposing preservice teachers to more advanced statistical concepts during their teacher preparation programs does not mean that this knowledge will transfer into practice.

In addition to examining teacher preparation programs, the effect of professional development, specifically sustained professional development, on inservice teachers should be examined. Particular attention should be given to activities which help the teachers develop understanding of the advantages, disadvantages, and appropriateness of various topics in statistics. Examples of some activities that may be attempted are contained in the GAISE document.

Research should also be conducted on the types of learning experiences students are exposed to in "traditional" curricular materials. The types of questions posed along with the way the subject of data is introduced to students may squelch their opportunity to develop more advanced ways to think about the process of statistics. Performance on assessments that examine procedural and conceptual knowledge should be compared between students taught with "traditional" materials and those taught with materials similar to *Math Out of the Box* that present statistical concepts within a context.

These areas for future research lead to the identification of some specific questions that may be explored. Based on the results and implications of this study, there are many more questions that could be posed.

Some Specific Questions

Based on the discussion above, the identification of the following questions may help the research community continue this endeavor.

1. What is the understanding of elementary, middle, and high school teachers in the area of statistical methodology (e.g. formulating questions, designing a study to answer those questions, and making inferences)?
2. What is the impact of a statistics course focused on the realization of the objectives outlines in the GAISE document on preservice teachers' content knowledge upon the completion of such a course? One year after the course? Three years after the course?
3. What is the impact of sustained professional development emphasizing statistical content on teachers' content knowledge?
4. If teachers are presented with a data display, what types of questions will be posed? Do these types of questions reflect: a) the type of understanding they possess in regard to statistics? And b) the experiences they have been exposed to in their own training?

Conclusion

The elementary school teachers involved in this study were considered to be exemplary teachers of mathematics by their district supervisor and principals. If teachers who are considered exemplary do not possess the necessary content knowledge to help students realize the objectives of the GAISE document, then it can be assumed that teachers who are not considered exemplary also do not possess such knowledge. In order to prepare elementary school teachers to teach statistics as described in the GAISE document, teachers should progress through the levels of statistical understanding themselves. The effect of a statistics course in teacher preparation programs and sustained professional development on teachers' preparedness to teach statistics should be further explored. Teachers who are unprepared to teach material at the level the GAISE document now prescribes face an unfair situation where they have not mastered the material they are now called upon to teach. The expectations for students cannot be increased without addressing teachers' preparation to meet those expectations.

APPENDICES

Appendix A

Baseline Teacher Interview Protocol

Name: ______________________________ Date: ____________________

Please answer each question as accurately as possible. There are no right or wrong answers.

1. What grade do you teach?

2. How many years have you taught (through last year)?

3. Describe your educational background:

    a. What mathematics education courses have you taken?
        1. Mathematics for elementary school teachers
        2. Mathematics for middle school teachers
        3. Geometry for elementary/middle school teachers
        4. Methods of teaching mathematics
        5. Other

    b. What mathematics courses have you taken?

4. When did you last complete a mathematics education course for credit? Describe the course

5. When did you last complete a mathematics course for credit? Describe the course

6. What is the total amount of time you have spent on professional development in mathematics or the teaching of mathematics in the last 12 months? in the last 3 years? (Include attendance at professional meetings, workshops, and conferences, but **do not** include formal courses for which you received college credit or time you spent **providing** professional development for other teachers.)

Appendix A (continued)

Baseline Teacher Interview Protocol

	None	< 6 hours	6-15 hours	16-35 hours	> 35 hours
Last 6 months					
Last 3 years					

7. Are you a member of any professional organizations? If so, which ones?

8. How familiar are you with the NCTM Standards?

9. How would you assess your level of understanding of data analysis and statistical topics covered at your grade level?

10. What does it mean to analyze data?

11. What is the importance of analyzing data?

12. What are some of the most important things to consider when analyzing data?

13. How would you describe your teaching approach to data analysis topics?

14. How do you feel about implementing Math Out of the Box?

15. Were you involved in the decision to use the curriculum? How do you feel about that?

16. What type of training did you undergo to prepare you for implementation?

17. Do you feel ready to use the curriculum?

Appendix B

Observational Protocol

Teacher:	Date:	Class Length:
Descriptive Notes	**Reflective Notes**	

Appendix C

Statistical Content Interview

Please define each concept in your own words. If you want to bring up an example that's fine:

1. What is data?
2. What is the mean?
    a. If successful to above:
        i. How do you find the mean?
        ii. Is there more than one way?
        iii. What information does this value tell you?
3. What does the word average represent?
4. What is a median?
    a. If successful to above:
        i. How do you find the median?
        ii. Is there more than one way?
        iii. What information does this value tell you?
5. What is the difference between the median and the mean?
6. What is a mode?
    a. If successful to above:
        i. How do you find the mode?
        ii. Is there more than one way?
        iii. What information does this value tell you?

Appendix C (Continued)

Statistical Content Interview

7. What is the range?
    a. If successful to above:
        i. How do you find the range?
        ii. Is there more than one way?
        iii. What information does this tell you?
8. What is an outlier?
    a. If successful to above:
        i. How do you find an outlier?
        ii. Is there more than one way?
        iii. What information does this tell you?

Appendix D

Math Out of the Box Interview*

1. What did you learn about mathematics when teaching with Math Out of the Box^{TM} that you had never learned from teaching with other textbooks?

2. What did you learn about how children learn mathematics when teaching with Math Out of the Box^{TM} that you had never learned from teaching with other textbooks?

3. How were the Math Out of the Box^{TM} lessons connected to each other? Did the way the lessons were connected help you learn mathematics in a new way?

4. What kind of data analysis and statistical thinking did your students do with Math Out of the Box^{TM}? What kind did you do?

5. What was the most important thing YOU LEARNED about:
   a. yourself as a teacher?
   b. yourself as a learner?
   c. your students?
   d. mathematics?

*Diaz, Donna Clemson University Dissertation. 2004.

Appendix E

Assessment 1 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

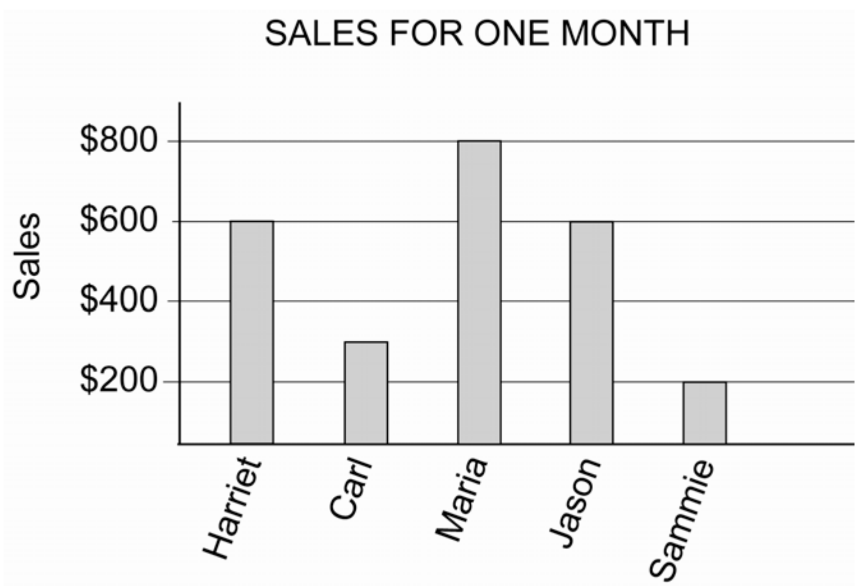
Mathematics
Data Assessment

Time: 15 minutes

Your Name ______________________________

Directions: For questions 1 through 6, circle the letter of your answer.

1.



According to the graph above, which person's sales were closest to $400 for the month?

A Harriet **B** Carl

C Jason **D** Sammie

Appendix E (Continued)

Assessment 1 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

2.

POINTS EARNED FROM SCHOOL EVENTS

Class	Mathathon	Readathon
Mr. Lopez	425	411
Ms. Chen	328	456
Mrs. Green	447	342

Which class scored the most points from the two events?

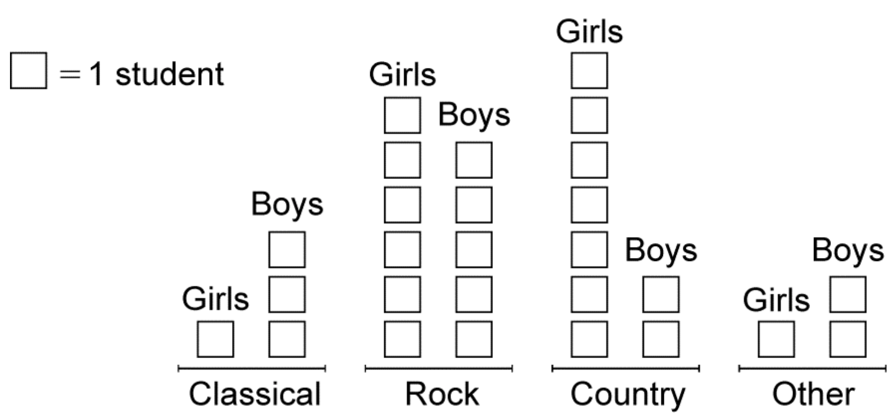
A Mr. Lopez' class

B Ms. Chen's class

C Mrs. Green's class

D All classes earned the same amount.

3. Each boy and girl in the class voted for his or her favorite kind of music. Here are the results.



Of the choices listed, which kind of music was most popular in this class?

A Classical **B** Rock

C Country **D** Other

Appendix E (Continued)

Assessment 1 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

4. The amounts in dollars that each of ten 4th grade students earn for a weekly allowance are given below.

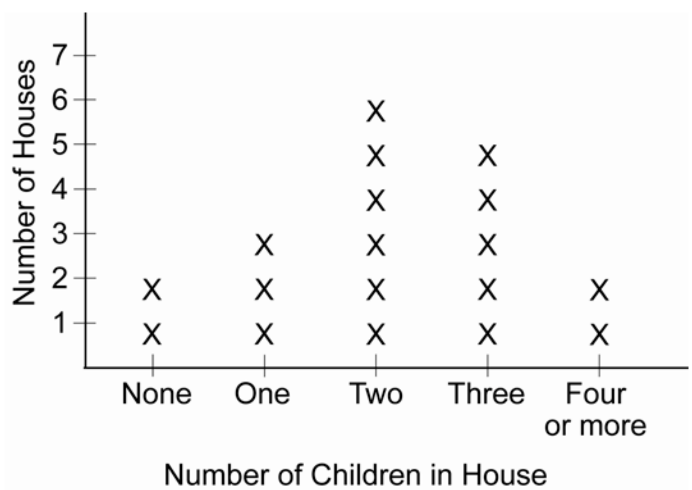
4, 5, 3, 3, 5, 2, 4, 5, 2, 6

What is the mode for these allowances?

A 2 **B** 3

C 4 **D** 5

5. The number of children living in each house on Elm Street is displayed in the graph below.



Which statement is true?

A There are 16 houses on Elm Street.

B Most houses on Elm Street have 3 children in them.

C There are five houses with 2 children in each house.

D There are five houses with 3 children in each house.

Assessment 1 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

6. There are five 4th grade classes at Taft School. The number of students in each of these classes is given below.

21, 19, 20, 24, 23

What is the median number of students for these classes?

A 19

B 20

C 21

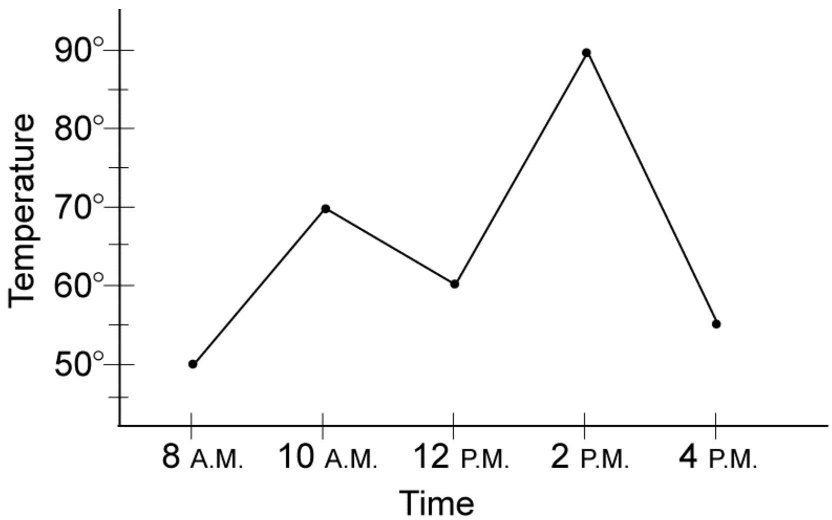
D 24

Appendix E

Assessment 1 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

Directions: For questions 7 and 8, write you answer and work below each question.

7. The graph below displays the temperature in a room at different times during one day.



The heater in the room went on twice during the times shown and went off twice during the times shown.

At what time did the heater go on the first time? _______

At what time did the heater go on the second time? _____

Explain how you used the graph to find your answers.

Appendix E (Continued)

Assessment 1 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

8. The table below shows the number of miles Mr. Parks drove on five different days.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Miles Driven	40	50	60	30	20

a. Make a bargraph below to display the data.

b. Between which two days was the biggest change in the number of miles driven?

Between ____________ and ____________

Was it an increase or a decrease? _______________

Appendix F

Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

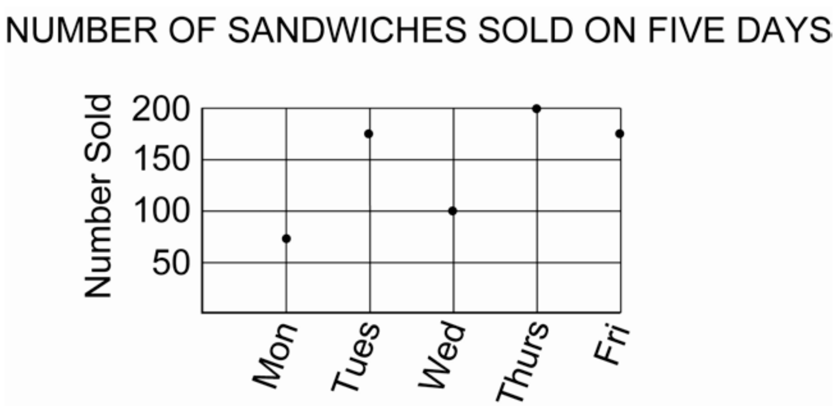
Mathematics
Statistical Assessment

Time: 15 minutes

Your Name ________________________________

Directions: For questions 1 through 6, circle the letter of your answer.

1.



According to the graph above, about how many sandwiches were sold on Tuesday?

A 100 **B** 150

C 180 **D** 200

Appendix F (Continued)

Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

2. The high temperatures in degrees for 7 days last week are shown below.

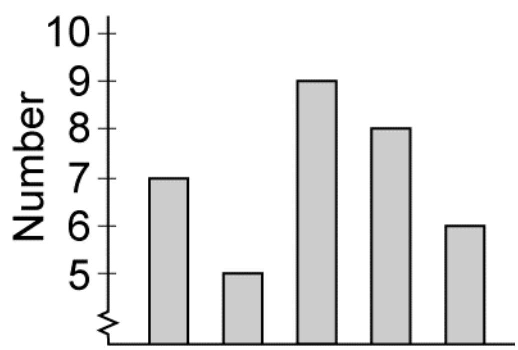
70, 71, 68, 71, 62, 73, 68

What is the median temperature?

A 68 **B** 69

C 70 **D** 71

3.



Jim made the graph above. Which of these could be the title for the graph?

A Number of students who walked to school on Monday through Friday

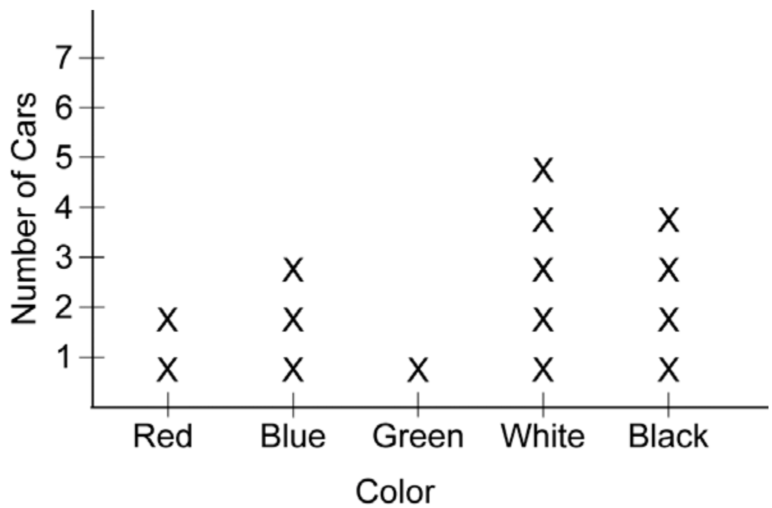
B Number of dogs in five states

C Number of bottles collected by three students

D Number of students in ten clubs

Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

4. The graph below displays the number of cars of each color in a parking lot.



Three of these cars leave the lot. Which could be the colors of the three cars that leave?

A All three cars could be red.

B All three cars could be green.

C Two cars could be green and one could be blue.

D One car could be green and two could be blue.

5. Nina walks one mile each day for 9 days. Her times, in minutes, are shown below.

19, 23, 21, 20, 21, 20, 22, 20, 19

What is the mode of her times?

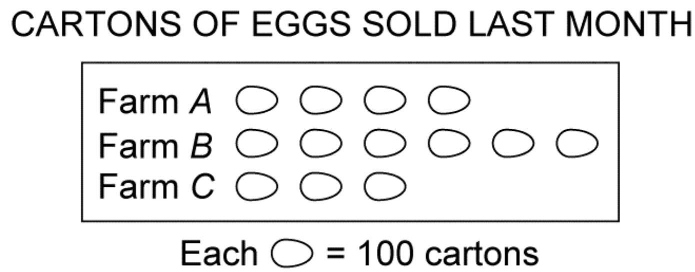
A 19 **B** 20

C 21 **D** 22

Appendix F(Continued)

Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

6.



According to the graph, how many cartons of eggs were sold altogether by farms A, B, and C last month?

A 13 **B** 130

C 1,300 **D** 13,000

Appendix F (Continued)

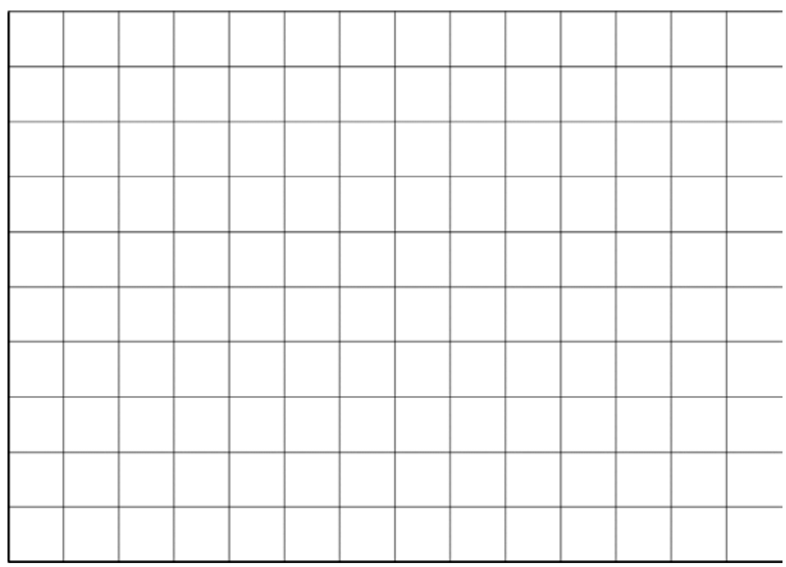
Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

Directions: For questions 7 and 8, write you answer and work below each question.

7. The table below shows Jill's scores on a weekly spelling test for 6 weeks.

Week Number	1	2	3	4	5	6
Score	80	90	85	75	80	70

a. Make a line graph to display the data.



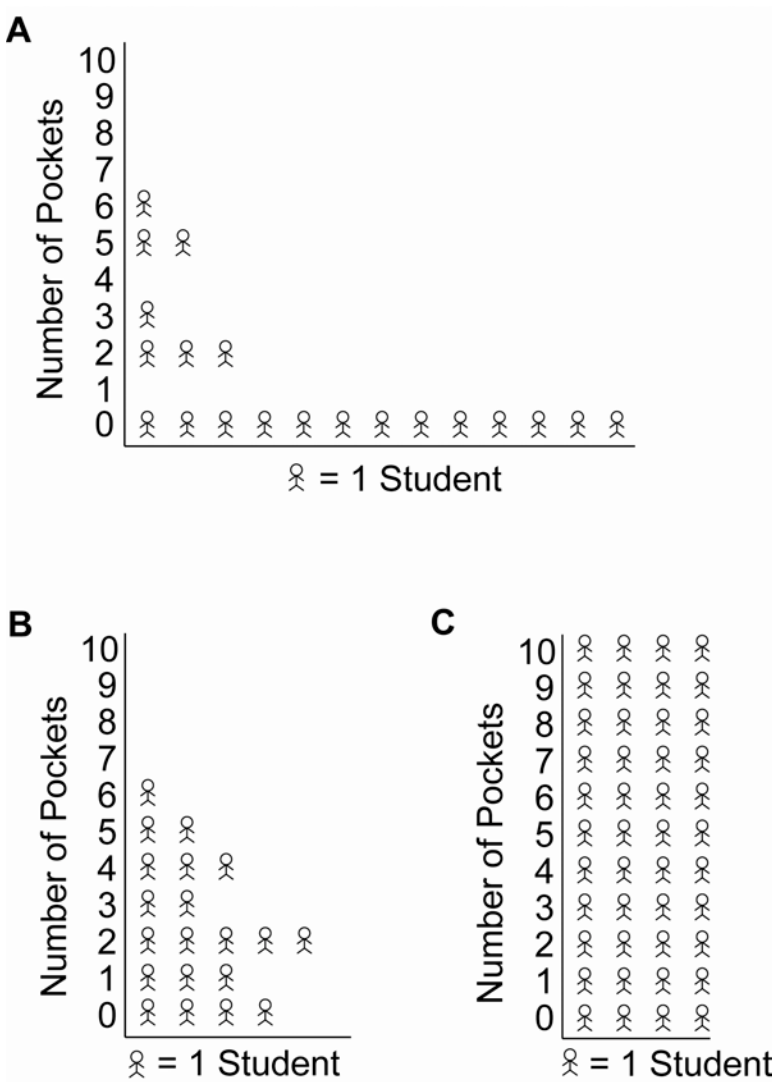
b. Jill thinks that her scores are always increasing as she moves from week 1 to week 6. Do you agree? _____

Explain how you used your graph to find your answer.

Appendix F (Continued)

Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

8. There are 20 students in Mr. Pang's class. On Tuesday most of the students in the class said that they had pockets in the clothes they were wearing.



Appendix F (Continued)

Assessment 2 (NAEP Items 4th Grade Level)
Used by ETS to assess Content Knowledge of Students

Which of the graphs (A, B, or C) most likely shows the number of pockets that each child had? __________

Explain why you chose that graph.

Explain why you did not choose the other graphs.

This is the end. If you finish early, please check your work.

Appendix G

Selected Statistical Items from University of Louisville

Diagnostic Teacher Assessments in Mathematics and Science	Name ____________________ Date ____________
	Start Time _________
	Finish Time _________

Directions for completing items: Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items. Let the test facilitator know when you are finished. Thank you very much for your time.		
	Item	**Answer**
	The following data were collected from a local sports league. A sample of participants was selected and each person's age (in years) was recorded. 18, 15, 24, 19, 22, 27, 18, 17 Which of the following identifies the median for this set of data? a. 20 b. 18.5 c. 20.5	

Appendix G

Selected Statistical Items from University of Louisville

<table>
<tr>
<td></td>
<td>Students in a sixth-grade class were timed to the nearest second to see how long they could stand on one foot with their eyes closed. The times for the class are listed below in a stem-and-leaf plot. Which of the following is true?

2 | 7 8 9
4 | 7 7 8 9
6 | 1 2 3 4 5 6 7
7 | 2 2 2 6 7 8

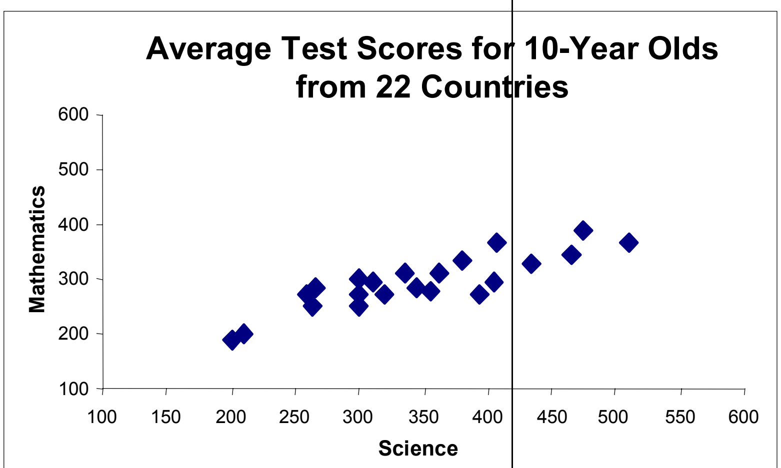
e. The shortest time was 28 seconds
f. Half the class had times under 58 seconds
g. The longest time was 77 seconds
h. 50% of the class had times over 63 seconds</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Which data set below has a mode of 8, a mean of 5, and a median of 4?

a. 8, 2, 8, 4, 3
b. 3, 8, 4, 3, 8
c. 10, 5, 9, 8, 5
d. 1, 8, 8, 5, 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Which graph or plot below would best represent this data on solid waste; 42% paper, 7% glass, 19% plastic, 11% wood, 15% food and 6% miscellaneous?

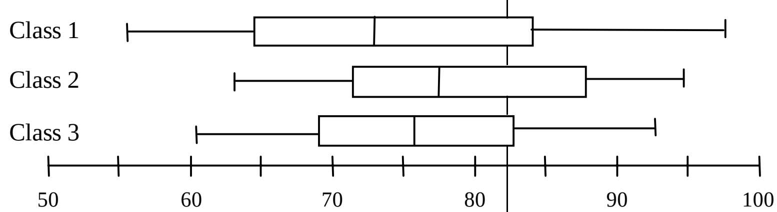
e. scatter plot
f. stem-and-leaf plot
g. box-and-whiskers plot (box plot)
h. circle graph</td>
<td></td>
</tr>
</table>

Appendix G (Continued)

Selected Statistical Items from University of Louisville

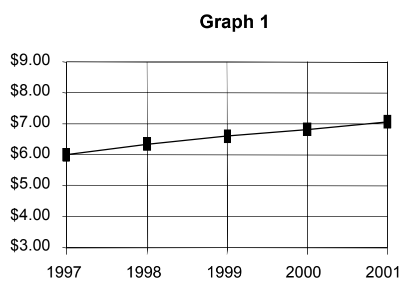
	The scatterplot below shows the average test scores on two different tests—one for Mathematics and one for Science—for 10-year-olds from 22 different countries. Which of the following would be the best approximation of the average test score on the Mathematics test for a country whose children averaged 550 on their Science tests?  a. between 490 and 520 b. between 380 and 410 c. between 570 and 600 d. between 320 and 350	

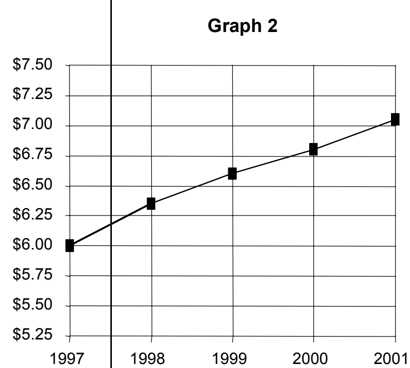
Selected Statistical Items from University of Louisville

	The box-and-whiskers plot below represents the test scores of three classes on the same test. a. Which class performed the best and which class performed the worst? b. Provide justifications for your choices with data from the box-and-whiskers plots.  	

Selected Statistical Items from University of Louisville

The same data can be represented in multiple, and sometimes biased, ways. The graphs below represent the same data on the average hourly wages of employees working at an amusement park. (a) How are the graphs different? (b) How could Graph 1 be used in an argument? (c) How could Graph 2 be used in an argument? Explain your responses.





Selected Statistical Items from University of Louisville

	A survey of middle school students resulted in data about the quantity of soft drinks they consumed in a week. The data is displayed in the table below:	

# of drinks	2 or fewer	3	5	6	over 7
# of students	4	6	7	5	3

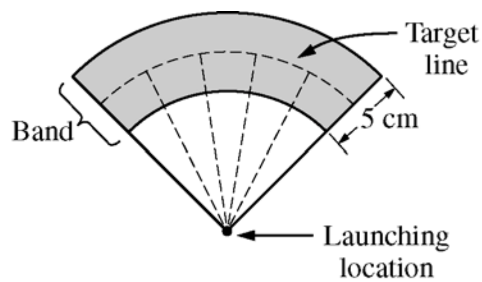
	The students were asked to construct a circle graph for the data. One student determined the size of the angles for each section of the graph by determining the size of each angle, such that (4/25 = 16/100 = 16^0). The student drew the angles with a protractor and had space left over. (a) What error is this student making, and (b) how would you help her?	
9	One of your students is collecting data on how many hours students spend doing homework each week night. The student asks for volunteers to answer a questionnaire. a. Explain why this sampling strategy is inappropriate. b. Describe an activity that would help this student understand the misconception.	

Appendix H

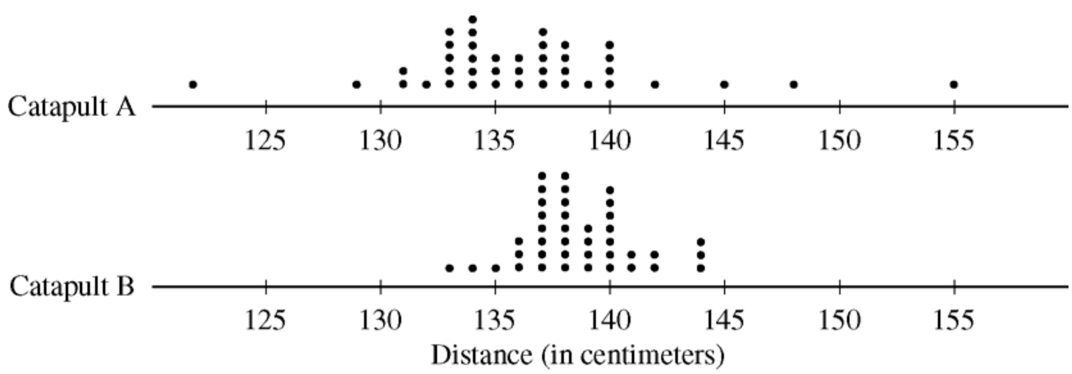
Content Knowledge Assessment
Using Center and Variation to Make Decisions
(from 2006 AP Statistics Exam)

The following description was read and explained to the participants. After the entire context was worked through, participants were asked if they understood the situation and were asked to explain it to the researcher.

> Two parents have each built a toy catapult for use in a game at an elementary school fair. To play the game, students will attempt to launch Ping-Pong balls from the catapults so that the balls land within a 5-centimeter band. A target line will be drawn through the middle of the band, as shown in the figure below. All points on the target line are equidistant from the launching location.



> If a ball lands within the shaded band, the student will win a prize.
>
> The parents have constructed the two catapults according to slightly different plans. They want to test these catapults before building additional ones. Under identical conditions, the parents launch 40 Ping-Pong balls from each catapult and measure the distance that the ball travels before landing. Distances to the nearest centimeter are graphed in the dotplots below.



1. Explain the context in your own words.
2. What type of graphical displays are these?

Appendix H (Continued)

Content Knowledge Assessment Using Center
and Variation to Make Decisions
(from 2006 AP Statistics Exam)

3. What information can you find based on what is presented (i.e. mean, median, mode, range, and standard deviation)?
4. If the parents want to maximize the probability of having the Ping-Pong balls land within the band, which one of the two catapults, A or B, would be better to use than the other? Catapult A or B can be placed anywhere parents desire to maximize their chances of landing balls within the 5 cm band. Justify your choice.
5. Using the catapult that you chose in question 4, how many centimeters from the target line should this catapult be placed? Explain why you chose this distance.

Appendix I

Card Sorting Tasks

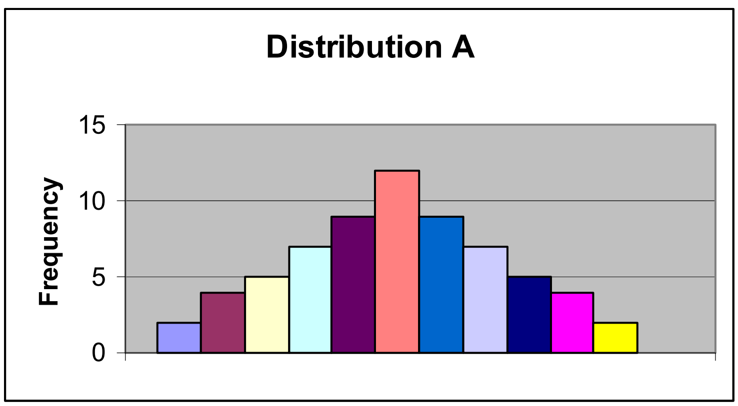
The distributions were cut out and placed on index cards.

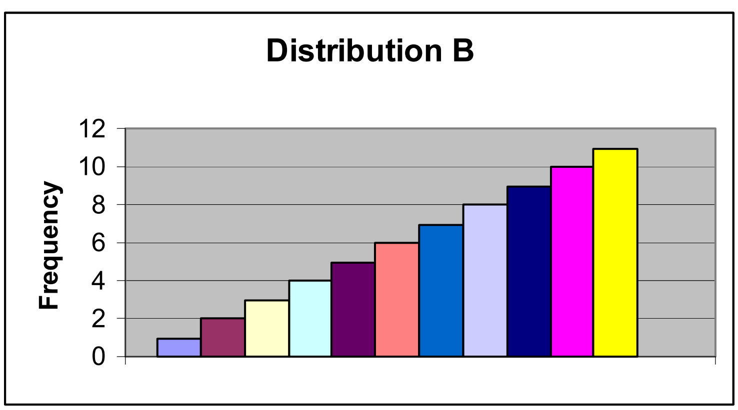
1. What type of graphical display is Distribution A? Distribution B? Distribution C?

2. Can you use the information in the displays to determine the mean? median? mode? range? standard deviation? How would you do this?

3. Assuming these distributions are graphed on the same horizontal scale, place the cards in order from least to greatest for the following values.

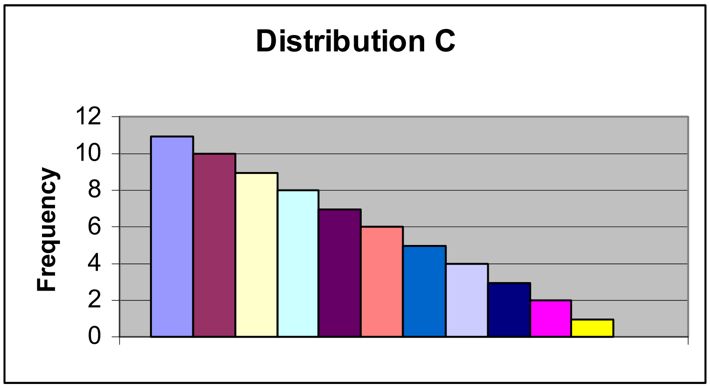
    1. mean
    2. median
    3. mode
    4. range
    5. variability (standard deviation)





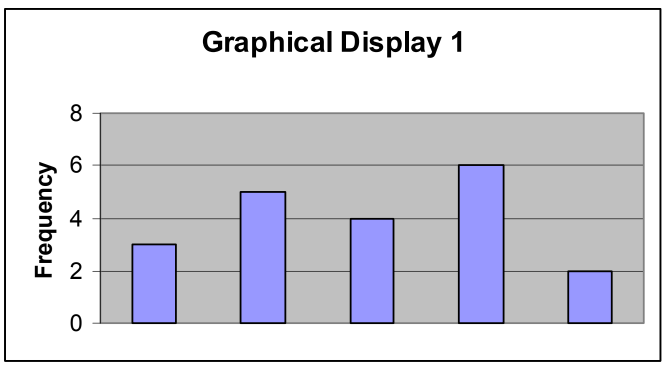
Appendix I (continued)

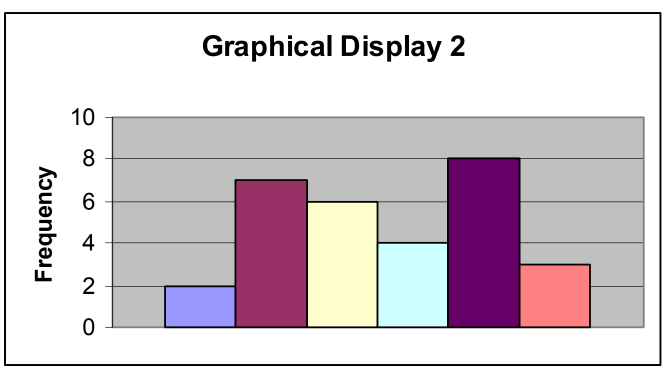
Card Sorting Tasks



4. What type of display is Graphical Display 1? Graphical Display 2?

5. Give an example of a set of data that could have each of these types of graphical displays.





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