DIDACTICAL PHENOMENOLOGY OF PROBABILITY

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This research is inspired by the first book in the series launched by Reidel on mathematical education. Freudenthal (1983) expounds his philosophical approach in great detail considering many topics in mathematics but excluding (perhaps surprisingly) probability. The paper is presented in three parts which are kept succinct. After showing the relevance of didactical phenomenology, a perspective on approaches to probability is given as a framework for the experimental research which has been undertaken.

1. Didactical Phenomenology

Mathematical concepts, structures and ideas serve to organise phenomena from the concrete world as well as from mathematics. For example, triangles organise the world of contour phenomena and numbers organise quantity; later, geometrical figures are organised by constructions and proofs, while numbers are viewed within the decimal system. Continuing abstraction unites similar looking mathematical phenomena under a global concept like a group or a topological space.

The phenomenology of a mathematical thought object means describing it (a "nooumenon") in its relation to the phenomena of which it is a means of organising. In this relation the didactical element is stressed, how the relation is acquired in the teaching-learning process: hence the term didactical phenomenology.

Probability organises the world of chance phenomena and idealised chance phenomena. One needs to find a posteriori constructed relation between the mathematical concept of probability and the world of chance objects. There are a variety of means to discover the underlying didactical phenomenology. One must use one's own knowledge of probability, its applications and its history. Textbook analysis has a role to play.

Finally, and perhaps most importantly, one must observe the learning process to understand the process of the constitution of mathematical objects and the attainment of concepts. But, with regards to learning, one must always remember that, in no way do we realise all the things children need to learn (as shown most startlingly by Piaget's work on conservation).

Rather than looking for materials to concretise a particular concept one should look for phenomena that might compel the learner to constitute the mental object that is being mathematised. This constitution of mental objects precedes concept attainment. As Freudenthal says, first applications then concepts is the best order.

In probability a starting point is the language used to describe chance events – likely, probably, certain, impossible etc. Assessment of likelihood through some form of betting might be a way to a more formalised approach. Might this be an approach which would "compel" the learner to constitute the mental objects encompassing probability? It is not possible to answer this question yet but it provides a focus to the work described below. Others may postulate a different approach and hence stimulate a fruitful discussion.

2. Common Philosophical Approaches

There are three basic philosophical positions on the nature of probability.

(a) Classical, Laplacian or a priori view – the probability of a simple event is obtained by making an assumption of equal likelihood.

(b) Frequentist view – the probability of a simple event is obtained from the observed relative frequency of that event in repeated trials.

(c) Subjective view – the probability of a simple event is obtained by a personal assignment.

Perhaps one does not need to make a preference between the three positions initially (just as one might like both chips and mashed potato). Problems may arise later depending on the situation being considered (just as certain foods may blend better with chips or with mash). But it is important, pedagogically, to sample each possibility so that one can make one's own choice later (some may still prefer chips to the traditional sausage and mash).

The main controversy is between the subjective viewpoint on the one hand and the symmetric and frequentist approaches on the other side. For a subjectivist, probability is inherent in the mind, while for the other approaches probability is inherent in the object.

The first view asserts that physical considerations of symmetry lead directly to a primitive notion of"equally likely cases". However, any uncertain situation typically possesses many plausible symmetries, so a truly objective theory would therefore require a procedure for choosing a particular symmetry and justifying the choice.

The frequentist view of probability is also contentious. For the measure of uncertainty is assigned to an individual event by embedding it in a collective – an infinite class of "similar" events which are assumed to have certain "randomness" properties; then probability is the limit towards which the relative frequency tends. However, an individual event can be embedded in many different collectives, with no guarantee of the same resulting limiting frequences: one requires a procedure to justify the choice of a particular embedding sequence. Further there are obvious difficulties in defining what is meant by "similar" or by "randomness", indeed there is an element of circularity involved. Even the notion of settling down presents difficulties – how many trials are involved in long term frequency?

The subjectivist recognises that regarding a specific symmetry as probabilistically significant is itself, inescapably, an act of personal judgement; it is equally true of an assertion of similarity amongst different, individual events as required in the frequentist formulation.

This analysis needs to be seen within an appropriate educational context. The curriculum issues are summarised in three basic questions:

(a) What intuitive ideas of probability do children have?

(b) What conceptual difficulties are there in the teaching of probability?

(c) How can intuitive conceptions of probability be socially mediated in the classroom towards more formal probabilistic ideas?

3. Experimental Work

A test instrument was devised to address the first question directly and provides indicators for the second question. These results are now being used in devising appropriate classroom materials.

The questions were selected according to their capacity to reveal intuitive notions of chance and their relationship to the approaches. Obviously the influence of teaching or other experiences will mediate the findings; but the 120 children (aged 12-13 years or grade 8) tested had not met probability as a formal part of their curriculum prior to testing. It will not be possible to classify intuitive notions into either a classical, frequentist or subjective viewpoint, but the responses may reveal links. Rather than talk about errors and mistakes, children's conceptions or misconceptions are highlighted.

Thus to indicate the approach only one question (which is perhaps the most basic one) is analysed, in some detail. It is hoped that this will provoke and encourage critical comment.

Question: Write a sentence which ends: "is something that happens by chance", using your own words to start it.

This free response question was included to see the range of phenomena considered to be chance events – and the variety was impressive. Between them the children covered the range of meanings of chance as given by the dictionary. The responses were grouped into several categories – the largest of which involved references to games of chance either directly or by implication. The most noticeable feature of these 38 replies was that all except two children referred to winning or obtaining a particular result. Four boys wrote "Getting a 6 . . . " with another suggesting "Getting a 2" – all presumably with reference to dice. A girl said "Your name pulled out of a hat . . . " and four children referred to "winning a competition" while others were more specific: "winning at bingo"; "sometimes if you go to a fair you can win a great big toy". A subcategory of this group were the replies which seemed to view a very unlikely or unexpected event as due to chance.

Typical of these replies were:

"A coin that landed on its side . . ." – boy.

"Winning the pools 5 times" – boy.

This is a sharp contrast with the view that it is precisely these sorts of rare events which cannot be chance – that there must be some kind of causal explanation.

Ten children used examples of accidents such as:

"Car breaks down" – girl.

"A house falling on you" – boy.

"Being run over by 2 buses on the same day" – girl.

Another group of seven children referred to natural phenomena such as rain, hurricane, earthquake, thunderstorm and eclipse. Coincidences were mentioned by four children:

"Meeting a teacher in Basingstoke" – girl.

"When you say something together" – girl.

Some children were concerned with their futures and offered examples which others might consider were not primarily chance events:

"If I get into set one for maths" – girl.

"Love", wrote one girl,"Marriage", wrote another.

But for these children the reasons for these events might well be unknown, making the events appear arbitrary and so down to chance. In these examples there undoubtedly are causal factors but ultimately the questions – why me? why now? are seen as unanswerable except in terms of chance.

For the majority of the children chance represents a measure of their knowledge and information. The cases where an attribution of chance might be disputed are just those where another person has more information or understanding about that situation. Chance is often equated with opportunity rather than with inherent randomness. It is perhaps easier to develop a subjective approach from such notions than the classical or frequentist approaches.

4. Concluding Comments

The results of the whole test did not give a unique answer to children's underlying intuitions of chance but there are strong indications of the basis from which probabilistic concepts can be developed.

Children are familiar with the probabilistic words such as likely, certain and impossible, which are in everyday usage. However their intuitions of such terms are not very precise. Certainty is equated with high likelihood, while impossibility is linked to physical situations rather than logical events. The word "chance" is used in a number of different contexts, often being equated with seeming arbitrariness or superstition rather than randomness.

The overall responses show that equal likelihood is not a particularly intuitive idea; nor is the idea of probabilities settling down common. However, all children are happy to make probabilistic assessments of single, unrepeatable events as a subjectivist might. One could not claim that children would make their assessments coherently. But children do believe that such judgements can be appropriately made. In this rather primitive sense children are subjectivist.

However one also needs a means of evaluating probabilities; initially this can be developed from notions of equal likelihood or limiting frequencies. Equal likelihood is the common starting point, reflecting in pedagogy what happened historically. However, this is not an idea which can be simply taken for granted. Time needs to be spent on justifying and discussing how symmetries might be utilized to make an assumption of equal likelihood: particularly important is the need for such an assumption. Similar comments apply to the introduction of frequentist ideas. A subjective viewpoint offers a framework within which children's notions of chance can be developed.

References

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