TEACHER KNOWLEDGE FOR TEACHING STATISTICS THROUGH INVESTIGATIONS

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This report compares the teacher knowledge of two early career primary school teachers (drawn from a study of four teachers) as it was needed in the classroom during the teaching of statistics through investigations. The study involved video recording a sequence of four or five lessons and audio recoding post-lesson stimulated recall interviews with the teachers. These interviews were based on the teacher viewing selected episodes from the lesson videos. The results showed marked differences in the teacher knowledge of the two teachers, as analysed against a framework developed from the mathematics teacher knowledge domain and the statistical thinking domain. The conclusions and implications drawn from the results are discussed in relation to both initial (or preservice) teacher education and professional development for teachers.

INTRODUCTION

Internationally, there is a marked trend in school curricula away from a focus on statistical skills (such as graphing and finding measures to representing a set of data) towards reasoning and thinking statistically. New Zealand has been recognised as a world leader with respect to statistics in the school curriculum (Begg, Pfannkuch, Camden, Hughes, Noble & Wild, 2004), and investigations have been a component of statistics at all levels in the national curriculum since 1992. This is reinforced in the 2007 revised New Zealand curriculum in which the learning area is now named Mathematics and Statistics, and the investigative cycle and statistical thinking are explicitly referred to at all school levels.

For investigations to be implemented successfully in the classroom, teacher knowledge is required. However, the question that emerges is, just what teacher knowledge is needed? As the domain of statistics education research is relatively young in comparison with mathematics education research, the answer to this question is not known with any degree of certainty.

This paper reports on part of a study into what teacher knowledge is needed for teaching statistics through investigations at the primary school level. Two teachers (from four in the study) are compared in relation to an analysis of their knowledge.

LITERATURE REVIEW

Teacher knowledge has been extensively researched, particularly since Shulman's (1986) landmark paper in which he named and defined pedagogical content knowledge. Shulman described this as one of three categories of content knowledge and the one that differentiates the teacher from the mathematician.

Subsequent research on teacher knowledge has followed various lines of enquiry, few of which have investigated how such knowledge is used in the classroom. It has been recommended that questions regarding the mathematical knowledge that is used in the classroom and how that knowledge is used should be the focus of research (Ball, Lubienski & Mewborn, 2001; Mewborn, 2001). Some of the activities in which teachers regularly engage, such as "figuring out what students know; choosing and managing representations of mathematical ideas; appraising, selecting and modifying textbooks; deciding among alternative courses of action; and steering a productive discussion" (Ball et al., 2001, p. 453) involve mathematical reasoning and thinking. Consequently, various types of teacher knowledge in relation to mathematics are drawn upon regularly, even for teacher work that appears, in the first instance, to be more pedagogical than mathematical.

If a significant amount of the teacher's work is essentially mathematical, classifications of teacher knowledge must take this into account. One such classification has been developed in which teacher knowledge consists of four components, two related to content knowledge and two related to pedagogical content knowledge (Ball, Thames & Phelps, 2005; Hill, Schilling & Ball, 2004). The components are: common knowledge of content (what the educated person

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knows and can do), specialised knowledge of content (the extra aspects needed by a teacher, such as making sense of and evaluating students' unconventional methods), knowledge of content and students (such as students' common misconceptions, and concepts/skills that they will struggle with or find easy), and knowledge of content and teaching (such as the best representations and models to use and how to sequence the teaching).

The context of any particular classroom influences the learning that occurs within that classroom. Since a classroom consists of not only individual students and the teacher but also social practices that govern what happens within that classroom, Borko et al. (2000) suggest that any study of knowledge and learning in the classroom must occur within the classroom. Likewise, Cobb and McClain (2001) argue that the tasks of teaching (such as planning for learning, interacting with students, and evaluating classroom incidents) provide the primary contexts for teachers' learning, and consequently research needs to be situated within these contexts.

Various researchers have argued the case for statistics being different from mathematics (Cobb & Moore, 1997; Pereira-Mendoza, 2002; delMas, 2004; Ben-Zvi, Garfield & Zieffler, 2006; Scheaffer, 2006). Furthermore, statistical thinking is considered to be different from mathematical thinking. Wild and Pfannkuch (1999) describe the fundamental aspects of statistical thinking, namely (1) a recognition of the need for data (rather than relying on anecdotal evidence); (2) transnumeration – being able to capture appropriate data that represents the real situation and change representations of the data in order to gain further meaning from the data; (3) consideration of variation – which influences the making of judgments from data and involves looking for and describing patterns in the variation and trying to understand these in relation to the context; (4) reasoning with models – from the simple (such as graphs or tables) to the complex, as they enable finding patterns an summarising data in multiple ways; and (5) integrating the statistical and contextual – making the link between the two as an essential component of statistical thinking. Along with these fundamental types of thinking are more general types that could be considered part of problem solving (but not exclusively statistical problem solving): the investigative cycle (problem, plan, data, analysis, and conclusions) – the "procedures that a statistician works through and what the statistician thinks about in order to learn more from the context sphere" (Pfannkuch & Wild, 2004, p. 41); the interrogative cycle (generate, seek, interpret, criticise, and judge) – "… a generic thinking process that is in constant use by statisticians as they carry out a constant dialogue with the problem, the data, and themselves" (Pfannkuch & Wild, 2004, p. 41); and dispositions – such as scepticism and imagination (to name but two) (Wild & Pfannkuch, 1999).

A model (see Figure 1) for examining teacher knowledge in statistics was proposed by Burgess (2006) based on the mathematics teacher knowledge work of Hill, Schilling, and Ball (2004) and Ball, Thames, and Phelps (2005) and the statistical thinking model of Wild and Pfannkuch (1999).

			Statistical knowledge for teaching			
			Content knowledge		Pedagogical content knowledge	
			Common knowledge of content (ckc)	Specialised knowledge of content (skc)	Knowledge of content and students (kcs)	Knowledge of content and teaching (kct)
Statistical thinking in empirical enquiry	Types of thinking	Need for data				
		Transnumeration				
		Variation				
		Reasoning with models				
		Integration of statistical and contextual				
	Investigative cycle					
	Interrogative cycle					
	Dispositions					

Figure 1. Framework for examining teacher knowledge used in the classroom (Burgess, 2006)

By combining these two aspects, the resulting framework suggests that the day-to-day work of the teacher when teaching statistics through investigations is essentially statistical. Examining the teaching of statistics against the framework helps determine what teacher knowledge is needed in the classroom and whether aspects of knowledge on the framework are unnecessary for the classroom.

METHODS

A case study approach was adopted for the major study based on the post-positivist realism of Popper (1979, 1985). Popper argues that objective knowledge exists and that knowledge grows dynamically through the recognition of a problem and the development of a trial solution that is subject to the elimination of error, rather than growing by induction. Furthermore, Swann (2003) suggested that such knowledge is testable, falsifiable, and open to refutation.

This paper reports on two teachers, Linda and Rob (both pseudonyms). They are both in their second year of teaching. Linda teaches a Year 5 and 6 class (approximately 9-11 years old), while Rob teaches a Year 7 class (approximately 11-12 years old). They were chosen for the study because they were beginning teachers who were graduates of one teacher education programme, and the researcher wanted to make links between the research and the same teacher education programme in which he teaches.

Both teachers were given a teaching unit that had students investigating various multivariate data sets. The first data set included four category variables (or attributes) for 24 cases, while the subsequent data sets included four variables, with at least two of them being numeric. The data were provided to the students as a set of 24 data cards (see Figure 2 for examples of cards from three different data sets), which could be manipulated and sorted in order to discover interesting things in the data set. The first set has (clockwise from top) gender, do you do your best writing with you right or left hand, position in family (youngest, middle, or eldest), and can you whistle? The second set has gender, age (years), reaction time (measured by dropping a ruler and obtaining the distance it fell before it was caught between the thumb and first finger), and year level at school. The third set has gender, height (cm), age (years) and arm span (cm).

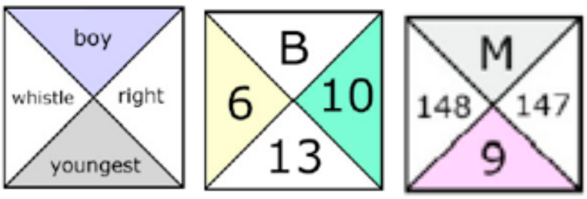

Figure 2. Examples of data cards for the three datasets, each with four variables

During an initial meeting with the researcher, the unit was discussed, as were the major statistical ideas involved, such as the use of the investigative cycle. Following this meeting the teachers developed their own sequence of lessons. The lessons (either four or five) were videoed, and the researcher edited each lesson video to focus on interesting and important episodes that had occurred in that lesson. The edited video was shown to the teacher (generally, although not always, on the same day and prior to the next lesson) and the subsequent discussion was audio-recorded. The video and audio recordings were analysed in relation to the cells of the framework - segments of video and audio were coded in relation to one of the four categories of teacher knowledge and to the components of statistical thinking. Some segments had multiple codes ‘attached’, as often more than one cell of the framework was in evidence in a particular classroom episode.

ANALYSIS

There were noticeable differences between the framework ‘profiles’ for Linda and Rob (see Figure 3). For each teacher, a shaded cell on the profile indicates one or more ‘incidents’ that arose within the teacher’s lessons or interviews for which that aspect of teacher knowledge was identified. In some cases, the incident revealed a situation in which an aspect of knowledge was not used. Such an incident was coded on the profile as a missed opportunity, shown by an

'M'. Some cells are both shaded and coded with an M, indicating there was at least one episode in which that aspect of teacher knowledge was present (shown by shading), but at some other time, there was a missed opportunity (M) with regard to that knowledge.

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				M
	Reasoning with models			M	
	Integration of statistical and contextual				M
Investigative cycle					M
Interrogative cycle					
Dispositions					

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Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				M
	Reasoning with models		M	M	M
	Integration of statistical and contextual			M	M
Investigative cycle				M	M
Interrogative cycle		M	M	M	
Dispositions					

Key: [grey box] = direct evidence of that knowledge used; [hatched box] = indirect evidence of that knowledge; M = missed opportunity related to that knowledge.

Figure 3. Profiles for Linda (left) and Rob (right)

Linda's profile reveals that most aspects of teacher knowledge in relation to statistical thinking were needed and used within her teaching. There were only four missed opportunities, situations in which knowledge could have been used but was not. As one example, the missed opportunity in relation to knowledge of content and students, reasoning with models, involved Linda listening to a student's explanation of what she had found in the data. The student's explanation was not very clear, but Linda did not check the validity of the statement against a two-way table that had previously been used. If Linda had done so, she would have realised that the student's finding was not valid for the data. This was a relatively significant missed opportunity whereby she could have encouraged students to use such models to back up or verify claims being made about the data. In comparison, another missed opportunity for knowledge of content and teaching, integration of statistical and contextual, was relatively insignificant. The students were being encouraged to make predictions prior to collecting data about whether they were right or left handed. However, Linda did not explicitly encourage the students to think about the real world relative incidence of right and left-handedness. She only asked them to make a guess as to how many of each there would be. Because of the direction of the subsequent discussion, this missed opportunity did not appear to have a detrimental impact on the discussion. The blank cells in the profile indicate there was no 'call' on the use of knowledge in relation to those cells during any of her lessons.

Rob's profile is significantly different from Linda's profile. There were missed opportunities in relation to all knowledge categories linked to transnumeration; most knowledge categories linked to the interrogative cycle; all statistical thinking components related to knowledge of content and students; and most components related to knowledge of content and teaching. Rob's missed opportunities in the classroom manifested themselves in various ways. One example was Rob's failure to seek further clarification from a student, when it was not clear what the student was meaning or intending by his or her statement with regard to using a mean to help answer a question. The students were considering what questions could be answered by the data. One student suggested that the cards be sorted by year level, age, and gender. Rob then asked what they were trying to find out by doing this. Another student said that they could "add them together and do averages". At this point, Rob did not challenge the students or explore with them what data would be averaged, or what the averages would be used for. This indicated that Rob was not using specialised knowledge of content, transnumeration; he did not ask the students to clarify how the average would help answer the question. A second example was when Rob did not recognise that a student's suggestion for how the cards could be sorted would not be useful for making sense of the data and answering the question at hand. This related to Rob's lack of specialised knowledge of content, reasoning with models. He did not connect the student's suggestion of how they would sort the data to the focus question and was not able to guide the students to a more useful way of sorting the data. In another situation, Rob's students suggested comparing the heights of the boys with the heights of the girls by summing the heights for each group, even though the groups were of unequal size. Rob recognised the inappropriateness of this, but he did not know a way of helping the students recognise this for themselves, and he did not follow this up any further. This indicated a problem with Rob's knowledge of content and teaching, transnumeration and reasoning with models.

In comparison with Rob's missed opportunities, Linda showed a more comprehensive base of teacher knowledge across most cells of the framework. This resulted in situations in which she was able to guide her students to make more sense of the data sets, as shown by the statements that they formulated and were able to justify with the data. For example, Linda was aware of the difficulty that students would have by trying to sort the data by too many variables. This indicated that she had knowledge of content and students, transnumeration. As a consequence of this knowledge, she had considered how to overcome the students' potential difficulty by guiding the students to sort the cards more 'slowly', by dealing with one variable initially, and only then extending the sorting to a second variable. This situation corresponded to Linda having appropriate knowledge of content and teaching, transnumeration. Another situation for Linda arose when a student was attempting to make a statement about the data set. Linda struggled to make sense of what the student was saying, so she developed a different representation of the data as the student talked. This representation helped both Linda and the other students make sense of the student's statement. This exemplified Linda's use of specialised knowledge of content, reasoning with models and knowledge of content and teaching, reasoning with models.

CONCLUSIONS

The framework for teacher knowledge in relation to statistical thinking was useful for determining the teacher knowledge that was needed during the teaching of the investigations. Also, the framework was used to profile the two teachers' knowledge. The profiles revealed some of the reasons why Linda's students, in spite of being younger than Rob's students, were able to progress further with their investigations and make more appropriate statements that could be supported by the data.

The study was able to describe the components of teacher knowledge that were called on during the teaching of statistics investigations and how a lack of appropriate teacher knowledge created missed opportunities in relation to the teaching and learning of statistics.

This paper has briefly described the differences between two inexperienced teachers in relation to the statistics knowledge needed for teaching very similar lessons. The descriptions of teacher knowledge and the profiles based on the framework suggest that the various components are highly connected (as shown by the often multiple coding of episodes). The implications for both preservice (or initial) teacher education and inservice teacher education (or professional development) are that teacher education cannot focus on any one type of knowledge but must engage teachers in all aspects simultaneously. As most teachers have not experienced the learning of statistics through investigations, they should be immersed in their own investigations, thereby focusing on common knowledge of content. In order to develop knowledge of content and students as well as specialised knowledge of content, teachers need to engage with students and observe (through watching and listening) how the students deal with aspects of investigations. For preservice teachers, this may need to be through the use of video examples from the classroom. From developing these aspects of knowledge, knowledge of content and teaching can be considered. It is apparent that teachers need to develop all four categories of knowledge linked with all the components of statistical thinking, in order to make the most of the teaching and learning opportunities that arise in the classroom.

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