TOWARDS A SUITABLY DESIGNED INSTRUCTION ON STATISTICAL REASONING: UNDERSTANDING SAMPLING DISTRIBUTION WITH TECHNOLOGY

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Earlier studies on sampling distribution, its founding concepts, misconceptions about sampling distributions, and the use of simulation highlighted that (1) learning of statistics requires an understanding of multifaceted issues and relations among them; (2) learning may be examined in terms of task, technique, theory, and learner's profile, each of which is influenced by instructional context; and (3) learning environments should be designed to stimulate flexible travelling along the network of these issues. Considering these emerged findings we attempt to outline a possible instructional design to teach sampling distribution with technology. Suggestions for training teachers in statistics education are included.

INTRODUCTION

Recent international studies suggest that mathematics teaching and learning in schools face serious difficulties. For example, a quarter of all 2003 Grade 8 students did not attain basic mathematical knowledge on Trends in International Mathematics and Science Study (TIMSS) (Mullis, Martin, Gonzalez & Chrostowski, 2004), and half of 15-year old Programme for Inetrnational Student Assessment (PISA) students from the Organisation for Economic Co-operation and Development (OECD) countries could only deal with simple problems related to everyday experience (OECD, 2004). One of the main reasons for students' inadequate mathematical knowledge is, according to Kokol-Voljc (1998), the fact that students usually receive instruction focusing only on one mathematical perspective (competence, knowledge type, or representation). Little room is thus left for the acquisition and coordination of different mathematical perspectives required for genuine learning (Kadijevich, 2007a).

The teaching and learning of statistics also faces similar difficulties. Research has uncovered a broad range of students' misconceptions concerning statistical thinking (e.g., Castro, Vanhoof, Van den Noortgate & Onghena, 2007). Some reasons for these misconceptions are similar to the issues discussed above for mathematics education. According to Garfield and Ben-Zvi (2004), it can be said that because of a one-sided instruction focusing on individual statistical concepts and skills, the majority of students do not understand overreaching ideas in the subject and do not conceive the relations among them. One of the key concepts in statistics is the sampling distribution, which is crucial for the understanding of the ideas of confidence level and significance. In order to comprehend the concept of sampling distribution, students must understand and be able to integrate various concepts such as sampling, distribution, and variability (see Chance, delMas & Garfield, 2004).

By focusing on the concept of sampling distribution, this study searched for initial answers to three questions raised in the Joint ICMI/IASE Study Discussion Document: (1) What challenges do teachers face and what support do they need when teaching statistics?; (2) What might be the theoretical basis for teacher learning in statistics?; and (3) How can teachers be empowered to create an instructional design that allows students to acquire key ideas of statistics?

SAMPLING DISTRIBUTION AND ITS FOUNDING CONCEPTS

Research studies on sampling distribution may be clustered into two groups: (1) sampling distribution and its founding concepts; and (2) misconceptions about sampling distributions and the use of simulation to address them. The concept of sampling distribution rests on the notion of distribution, which is the pattern of variation in data (Wild, 2005) linked with the notion of variability. Below, we summarize current research on variability, distribution, and sampling distribution. For additional research see, e.g., Shaughnessy (2007).

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Variability

Distribution assumes some level of variability in the data. When variation is detected in the data then we try to explain it by using appropriate models (Gould, 2003). When comparing two groups, two kinds of variation are possible: variation within group and variation between groups (Makar & Confrey, 2004). Empirical and theoretical distributions relate to variation from individual to individual, whereas the sampling distributions relates to variation from study to study (Wild, 2005). To explain variability, students tend to collapse data into a few categories and focus on the frequencies within such data slices (Makar & Confrey, 2004). However, deep understanding of variability according to Garfield and Ben-Zvi (2005) includes many components such as developing intuitive ideas of variability, describing and representing variability, using variability to make comparisons, recognizing variability in special kinds of distributions, identifying patters of variability in fitting models, using variability to predict random samples or outcomes, and considering variability as part of statistical thinking. Due to their complexity and interdependence, these components "must be constantly revisited along the statistics curriculum from different points of views, context and levels of abstraction, to create a complex web of interconnections among them" (see p. 95).

Distribution

The concept of sampling distribution requires an understanding of distribution. Distribution can be approached from three perspectives: as a collection of single points, as a collection of slices comprising it (triads, modal clumps or distributions chunks), or as an aggregate described by its mean, shape, standard deviation and outliers (Makar & Confey, 2004). Along with a data centered view (distribution as a collection of data results), distribution should be perceived as a modeling entity displaying a variation from an ideal model (e.g., the normal distribution) caused by noise or random errors included in that model (Prodromou & Pratt, 2005). Also, distribution can be empirical (a variability pattern of concrete data) and theoretical (a variability pattern of idealized data). A successful coordination of all these views and a stronger bi-directional linking of the concepts of variation and distribution are key to the understanding and successful use of distributions in class (see Pfannkuch & Reading, 2005). Because distribution is characterized by its centre, spread, density, skewness and outliers, a hierarchy of reasoning about distribution may be based upon a student's sophistication in dealing with these features. His/her reasoning may not refer to the features, focus on one feature, focus on more than one feature, or integrate (make links between) various features. Furthermore, when a student uses distribution for statistical inference, he/she may recognize the concept of distribution but not use it for this inference, make one inferential statement evidencing a correct understanding of the concept, make more than one such statement evidencing this understanding, or make several inferential statements and integrate them evidencing the understanding (Reading & Reid, 2005).

Sampling distribution

Sampling distribution, i.e., distribution of a sample statistic, is probably the most important but also one of the most difficult concepts in statistics learning. Students may understand why a sampling distribution gets narrower as we increase sample size yet still not be able to answer why the mean of a 5-element sample would be greater than a specified value above the population mean more often than the mean of a 20-element sample. Because of that, as underlined by Chance, delMas and Garfield (2004; p. 314; italic added):

> it is vital for teachers to spend substantial time in their course on concepts related to sampling distributions. This includes not only the ideas of sampling, distributions of statistics, and applications of the Central Limit Theorem but also foundational concepts such as distribution and variability. Focus on these early foundational concepts needs to be *integrated* throughout the course so students will be able to apply them and understand their use in the context of sampling distributions.

According to these researchers, students' reasoning about sampling distributions and their statistical reasoning in general may develop along several interrelated dimensions including procedural and conceptual knowledge fluency, conceptual and procedural knowledge integration, and inferential statements coherence (consistence in reasoning, confidence in reasoning, and equilibrium in reasoning). Again, an integration of different dimensions and perspectives is a key to success in learning statistics.

MISCONCEPTIONS ABOUT SAMPLING DISTRIBUTIONS AND USE OF SIMULATIONS TO ADDRESS THEM

Misconceptions about distributions

Students may hold various misconceptions about sampling distributions. Having a poorly developed idea of variability, students may believe in the law of small numbers: because small samples are very similar to the population, they should have distributions like the population. Students may extrapolate the central limit theorem wrongly when they believe that the distribution of any statistic can be, for large samples, approximated by a normal distribution, which may not be viewed as a theoretical distribution but rather as a model described by empirical data. By erroneously equating different kinds of distributions, students may believe that the sampling distribution of a statistic of a variable should be similar to the population or sample distribution of that variable. Also, not understanding the variability in distributions of sample means, students may claim that the confidence interval of the mean is not affected by sample size, that it increases with sample size, or that, for the same data, its 95% confidence interval is narrower than its 90% confidence interval (see Castro et al., 2007).

Use of simulations to address these misconceptions

Carefully-designed simulations can in general help bring statistical concepts and procedures to life for the students. With regard to sampling distributions, they would convince students that the law of small numbers is erroneous or that a sampling distribution may look different than that of the sample or population. However, although students may understand that the sample mean is getting closer to the population mean as sample size increases, they may believe that to get good information about the population mean we must sample repeatedly (Hodgson & Burke, 2000). Furthermore, the outcome of simulations with small samples may reinforce old misconceptions (Batanero et al., 2005). Finally, although interactive dynamic visualizations help students realize important patterns (e.g., sampling distribution gets narrower as we increase sample size), an understanding of the relationships causing these patterns is usually missing (Chance, delMas & Garfield, 2004).

To improve the impact of statistical software on students' reasoning about sampling distribution, Chance, delMas and Garfield (2004) suggest asking students to predict how the features of a sampling distribution (such as distribution shape and sample size) are related and then to verify their predictions with technology. These researchers also suggest assessing students' reasoning about this topic repeatedly and in a rigorously increased way with diverse procedural and conceptual tasks (see such tasks at https://app.gen.umn.edu/artist/). To avoid pitfalls of simulations in general, Hodgson and Burke (2000) recommend instruction that makes use of pre-organizers (essential aspects of simulation that direct students to salient features of the learning activity) and debriefing (looking back at the learning activity where students share their understandings and give reasons for them). Debriefing should include a "what's behind the simulation" part that aims to clarify statistical theory behind the simulation, and this part of debriefing should be supported by appropriate learning materials such as Erickson (2003) that combine carefully structured simulations with theory behind them.

THREE QUESTIONS AND INITIAL ANSWERS TO THEM

This part summarizes initial answers to our three research questions, giving a number of suggestions for training teachers in statistics education. The implementation of some of these suggestions may be examined in further research.

What challenges do teachers face and what support do they need when teaching statistics?

Learning of statistics requires an understanding of various multifaceted issues (statistical ideas, object perspectives, or statistical reasoning dimensions) and relations among them, many of which are prone to various misconceptions. Teachers thus need be supported not only in developing individual concepts and skills but also in developing this understanding within a complex web of interrelated objects (see Garfield & Ben-Zvi, 2004; 2005). To achieve this, appropriate teacher education programs should be developed and utilized.

Teachers usually do not have (enough) knowledge and skills to choose appropriate technological tools, use them, and understand how they affect student learning (Chance & Garfield, 2002). Teachers thus need support not only to become aware of suitable statistical software and to become competent users of that software but also to develop their knowledge of affordances and possible pitfalls of this software by using suitable materials and workshops (Lavicza & Koch, forthcoming). In short, there is a great need for more learning materials supporting the existing software and more relevant educational research (e.g., Clements, 2006).

Teachers may not be aware of students' characteristics that limit learning of statistics: viewing statistics in a narrow way as a collection of techniques to be used properly (Petocz & Reid, 2005); preferring textual representation of statistics (Schuyten & Dekeyser, 2007); viewing technology not as a tool that expands learner's thinking but rather as a master or servant (Galbraith, 2002). Teachers need be supported not only in realizing but also in helping their students to overcome one-sided views and preferences.

What might be the theoretical basis for teacher learning in statistics?

The learning of statistics may be examined in terms of the following four interrelated components: task, technique, theory, and learner's profile, each influenced by instructional context (needs, values, learning support offered, etc.). According to the task-technique-theory (T-T-T) framework (Artigue, 2002), task involves objects to be learned, technique, which usually has both pragmatic and epistemic values, stands for technique for solving tasks (paper-and-pencil and, if available, its software variant), whereas theory deals with statistical theory learned, typical learner's misconceptions as well as mathematical knowledge built into technology applied. Consider, for example, tasks "Given a distribution of the annual income in your country, compare (with respect to distribution center, spread and shape) a distribution of sample means for 100 samples of size 5 with that for 100 samples of size 50" and "What sample size should one use to predict the outcome of the forthcoming presidential election accurately?". By considering the T-T-T framework, learner's profile and the instructional context, analyze techniques (paper-and-pencil and computer-based) and theoretical issues to emerge (have emerged) from solving these tasks for different learners' profiles and the given institutional context (extrapolated from Kieran & Drijvers, 2006). As the T-T-T framework seems suitable for various kinds of software (see Kieran & Drijvers, 2006), teachers may thus not only be supported in understanding and using this framework to cope with the complexity of learning statistics (both their own and their students) but also scaffolded through the transition from using an impersonal software tool to a personal digital instrument.

How can teachers be empowered to create an instructional design that allows students to acquire key ideas of statistics?

If we assume that the knowledge of statistics is constructed through the coordination of its key ideas, different perspectives of statistical objects, different dimensions of statistical reasoning and so on, the learning of statistics may be based on a four-component design framework comprising learning network of statistical objects, learning paths connecting them, bridging tools that activate these paths, and learning issues of what should be done and realized by using particular bridging tools (extrapolated from Abrahamson & Wilensky, 2007). In other words, learning environments should be designed to stimulate flexible travelling along the network of statistical objects by means of bridging tools that can reconcile related objects (e.g., different perspectives of construct) as well as groups of related objects taken as wholes (e.g., different constructs), pointing out in some ways the limitations of one-sided views and preferences. A simple application of this complex design requirement can be recognized in Prodromou and Pratt (2005) where two views of distribution (collection of data results versus variation from an ideal model) are reconciled by technology. A detailed application of the requirement for these two views of sampling distribution can be found in Erickson (2003) where technology separates the outcomes for different sample size that help learners not only coordinate the two views but also relate the shape of this distribution and sample size. Of course, learning activities within this design should make use of pre-organizers and debriefing in the way mentioned above. To help teachers develop and use such a tailored instruction, appropriate teacher education programs are to be designed and implemented by using various suggestions given above. These programs may support teachers' learning by means of connection levers involving inquiry, multiple iterations, validation, resources, and sustained support and feedback (Makar, 2007).

CLOSING REMARKS

Although this contribution gives just preliminary ideas about the examined questions, it helps us realize some standards for well-designed instruction on statistical reasoning including (1) learn and teach within a network of objects that can be connected flexibly; (2) reconcile related objects by means of technology; and (3) avoid pitfalls of technology by using pre-organizers, debriefing, and a continuous assessment described in the subsection on using simulations, which are relevant to training teachers of statistics (for other requirements, see Shaughnessy, 2007). As we should agree upon some standards for statistical literacy and reasoning and use them in student and teacher training (Garfield & Ben-Zvi, 2004), researchers in statistics education may focus on developing these standards and search for critical issues of their implementation. Of course, good standards cannot guarantee appropriate teaching, successful learning, or desired research. Despite that, they would help us spread the agreed philosophy of statistics education and its research, recruit its followers among skilled and open-minded educators and researchers, manage their professional development, and assess the effects of their outcomes, enabling proper improvements (adapted from Kadijevich, 2007b).

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