PRE-SERVICE ELEMENTARY SCHOOL TEACHERS' METAPHORS FOR THE CONCEPT OF STATISTICAL SAMPLE

RANDALL E. GROTH
Salisbury University
regroth@salisbury.edu

JENNIFER A. BERGNER
Salisbury University
jabergner@salisbury.edu

ABSTRACT

The study describes the nature of pre-service teachers' idiosyncratic metaphors for the concept of statistical sample. These metaphors were investigated because of their potential to provide insight about individuals' content knowledge and how that content knowledge is enacted during teaching. Personal metaphors were elicited from 54 pre-service teachers through writing prompts. The writing prompt responses revealed seven different categories of thinking. In some instances, pre-service teachers struggled to construct a metaphor for the concept of sample. In the majority of cases, they constructed a metaphor for sample and discussed its relationship to their knowledge of the concept. The categories of thinking highlight some of the aspects of the concept of sample that teacher educators need to attend to over the course of instruction, and they also point out directions for further research related to metaphorical thinking about statistical content and its interaction with teaching practice.

Keywords: *Statistics education research; Sample; Metaphor; Teacher education*

1. INTRODUCTION

The concept of sample is foundational to the practice of statistics. In many cases, statisticians are interested in drawing conclusions about a large population when it is impractical to gather data from each of its members. Well-designed samples lead to viable conclusions about the population of interest. Because of the importance of samples in statistical reasoning, various curriculum documents recommend that students begin to develop understanding of the concept during elementary school (e.g., National Council of Teachers of Mathematics, 2000; American Association for the Advancement of Science, 1993).

Teachers' content knowledge impacts their abilities to carry out curriculum recommendations (Ball, Lubienski, & Mewborn, 2001; Shulman, 1987). Because of this, the development of content knowledge is an important part of many university-level mathematics courses for pre-service elementary teachers (Conference Board of the Mathematical Sciences, 2001; National Research Council, 2001). Teacher education

---

Statistics Education Research Journal, 4(2), 27-42, http://www.stat.auckland.ac.nz/serj

efforts can be highly effective when curricula are designed to be responsive to teachers' thinking (Mewborn, 2003). Curricula designed without the benefit of a research base are likely to be ineffective (Battista & Clements, 2000). The present study sought to inform the construction of curricula aimed at building pre-service teachers' statistical content knowledge by examining their metaphorical thinking in regard to the concept of sample.

2. METAPHOR AS A WINDOW ON TEACHERS' THINKING

Metaphor can be defined as, "A figure of speech in which a word or phrase literally denoting one kind of object or idea is used in place of another to suggest a likeness or analogy between them" (Mish, 1991). Under this definition, "metaphor is a window" is itself an example of a metaphor. Any metaphor one constructs will have both ground and tension (Presmeg, 1998). Similarities between the objects involved in a metaphor constitute the ground, while dissimilarities constitute the tension. For instance, the ground of the metaphor "metaphor is a window" includes the aspect that both objects allow one to observe phenomena that might otherwise remain unseen. The tension of the metaphor includes the aspect that a window is a physical object, while a metaphor is not. The reader can likely identify various other similarities and dissimilarities between "windows" and "metaphors."

2.1. METAPHOR IN MATHEMATICAL THINKING

Metaphor plays a prominent role in mathematical thinking. Presmeg (1992) illustrated how personal metaphors can influence problem-solving behavior. She gave an example of a high school student named Alison who thought of the x-axis as a "water level" in working with trigonometric ratios. Her problem-solving behavior was described in the following manner:

> Alison…used the metaphor of a ship sailing on a water level (x-axis) to remind her to use 180 degrees and 360 degrees in obtaining the acute angle she requires for the trigonometric ratios rather than 90 degrees or 270 degrees which would be contrary to her metaphor and would give an incorrect acute angle for the trigonometric ratio she is using (Presmeg, 1992, p. 600).

Alison's self-invented, idiosyncratic metaphor played a significant role in her mathematical activity. It should be noted that this is not simply an isolated case of metaphor playing a role in mathematical problem-solving. Other common mathematical metaphors include thinking of a function as a machine (Cuevas & Yeatts, 2001) and thinking of the arithmetic mean as a balance point for a set of numerical data (Mokros & Russell, 1995).

Reliance upon metaphors to guide mathematical thinking is not a phenomenon restricted to students. Sfard (1994) discussed how the thinking of mathematicians is guided by metaphors. She argued that commonly accepted and used mathematical words such as "increasing and decreasing," "closed and open," and "saturated and stable" all "clearly have their roots in the world of material objects" (Sfard, 1994, p. 47). Further, the mathematicians she interviewed stressed the use of analogy and metaphor in their work. For example, some spoke of using analogies mapping elements from one domain of mathematics to another to guide conjecture formation. Lucid analogies played a role in providing evidence that a result from one mathematical situation was likely to carry over to another similar situation.

Given the prominent role of metaphor in mathematical thinking, Lakoff and Nunez (2000) have gone so far as to argue that the very substance of mathematical thinking is metaphorical in nature. While a comprehensive critique of their argument is beyond the scope of this paper, it is important to note that Lakoff and Nunez (2000) have been cited for their lack of attention to the role that idiosyncratic metaphors play in individuals' mathematical thinking and learning (Presmeg, 2002). Such metaphors can provide instructors with valuable insights about students' construction of knowledge (Presmeg & Bergner, 2002). Therefore, the investigation of individuals' idiosyncratic metaphors for the concept of sample was the focus for the present study.

2.2. METAPHOR IN THINKING ABOUT STATISTICAL SAMPLES

In studies concerned with the nature of students' thinking in regard to statistical samples, elements of metaphorical thinking have been revealed even when the researchers were not explicitly looking for them. For example, when Jacobs (1997) posed the question, "What is a sample?" to middle school students, one response was that a sample was "a piece of food or carpet [that] gives you an idea of what the real thing is" (p. 28). This particular student appeared to make sense of the idea of sample metaphorically, by relating it to concrete objects. There were other similar instances, in which samples were thought of in terms of grocery store food samples, samples of products sent in the mail, and student writing samples (Jacobs, 1997). Jacobs (1999) subsequently recommended that teachers need to be aware of and build on children's informal thinking about samples during instruction.

In another study, when Watson and Moritz (2000a) posed the question, "If you were given a 'sample,' what would you have?" to a group of students, the students also related the concept to concrete objects. One student, for example, stated, "I'd have, if it was for clothes, a small piece of material" (Watson & Moritz, 2000a, p. 119). A unique aspect of the Watson and Moritz (2000a) study is that they used the SOLO Taxonomy (Biggs, 1992; Biggs & Collis, 1982) to place a hierarchical structure on the categories of children's definitions. Four distinct levels were identified. At the lowest level, students' responses did not capture any relevant aspects of the concept of sample. At the second level, responses included the idea that a sample is a "bit," or "part," but made no reference to a whole. Responses at the third level clearly characterized a sample as a part of a whole. The highest level observed included responses that characterized a sample as a representative part of a whole. Watson (2004) found that students generally moved to higher levels of response about the concept of sample as they progressed through school, suggesting that the hierarchy may map a developmental sequence.

The Jacobs (1997, 1999) and Watson and Moritz (2000a) studies suggest that metaphorical thinking plays a role in the development of understanding of the statistical idea of sample. Personal metaphors for "samples" reveal aspects of an individual's thinking about them. Therefore, in the present study, we elicited idiosyncratic metaphors from pre-service teachers in order to better understand their content knowledge in regard to the concept of sample, and also compared the content knowledge displayed against the hierarchy described by Watson and Moritz (2000a).

2.3. IMPACT OF METAPHOR ON TEACHING ACTIONS

While we expected pre-service teachers' idiosyncratic metaphors to provide a window on the structure of their content knowledge, we also expected the metaphors to provide some insight about the role their content knowledge might play in teaching. Educators in the fields of mathematics, science, and statistics education have recommended the use of metaphor and other types of analogy as teaching devices (delMas, 2004; English, 1999; Glynn, 1991; Martin, 2003). When teachers share well-formed concrete analogies with students it can enhance their learning (Mayer, 1987). Gentner and Holyoak (1997) provided some insight about why analogies can be helpful to learning. They stated,

> One basic mechanism [for learning] is analogy – the process of understanding a novel situation in terms of one that is already familiar. The familiar situation – often termed the base or source analog – provides a kind of model for making inferences about a particular situation – the target analog (p. 32).

By examining the metaphors for sample constructed by pre-service teachers, we expected to gain some insight about their ability to construct and use metaphors in their own classrooms. Therefore, while learning about teachers' content knowledge, we also expected to gain insight about their pedagogical content knowledge (Shulman, 1987), which enables teachers to make subject matter understandable to students.

It should also be noted that even if teachers do not consciously construct and share metaphors and other analogies during the course of teaching, students' thinking can be influenced by the spontaneous metaphors implicit in teachers' presentations of subject matter. Presmeg (1992) argued that although students ultimately must construct their own individual meanings for mathematics concepts, ideas encountered within classrooms have an impact on the sort of meanings that are constructed. She provided an example of a student who had a higher level of mathematics achievement than other students in his class because his thinking was influenced by his teacher's operating metaphor of "pure logic is beauty." The metaphor was bound together with the mathematical notions of generalization, abstraction, and patterns. Hence, even if teachers are not aware that they are sharing metaphors in teaching, the metaphors they unknowingly convey and operate under can have an impact on the meanings their students construct for concepts, because students are likely to construct meanings at least partially from their interactions with teachers in classroom communities.

2.4. SUMMARY

Metaphor is a useful tool for examining individuals' mathematical and statistical knowledge. Individuals' idiosyncratic metaphors for concepts provide a window on their understanding of them. In addition, the personal metaphors held by teachers are related to the manner in which they are able to convey information to students, and also to their abilities to help develop students' understanding. Hence, the purpose of the present study was to elicit and analyze idiosyncratic metaphors generated by pre-service elementary teachers in regard to the concept of statistical sample. We expected this analysis to provide insights about factors teacher educators should consider in developing instructional programs to enhance pre-service elementary teachers' statistical content knowledge. Just as teachers should build on students' informal conceptions of sample during instruction (Jacobs, 1999), teacher educators need to become aware of and build on teachers' conceptions of sample, because successful models of teacher education use teachers' thinking as a starting point for instructional design (Mewborn, 2003).

3. METHODOLOGY

In the present study, we collected and analyzed qualitative data in order to explore the nature of pre-service teachers' idiosyncratic metaphors for sample. We used a qualitative design because the study sought to investigate and describe thinking patterns among a given group of pre-service teachers rather than to make statistical generalizations to a larger population (Bogdan & Biklen, 1992; Strauss & Corbin, 1990). The specific components of that design are outlined in this section.

3.1. PARTICIPANTS

Idiosyncratic metaphors were elicited from 54 pre-service elementary teachers. The pre-service teachers were distributed among three different sections of a university-level mathematics teaching methods course at a university in the Eastern United States. They comprised approximately 75% of the population of students enrolled in all of the sections of the course. Four of the participants were male and 50 were female. This distribution of gender is not at all atypical in the U.S., as currently just 9% of all elementary school teachers in the country are male (National Education Association, 2005).

At the time of the study, statistical content and teaching methods had not yet been topics of study in the participants' teaching methods course. Each participant had, however, taken and passed an introductory statistics course at the college level before taking the methods course that provided the setting for the present study. Some had taken the introductory statistics course at the same university where the methods course took place, while others had transferred the courses from other institutions. Regardless of where it had been taken, it was required to satisfy the following course description: "descriptive and inferential analysis of raw data, emphasizing appropriate assumptions, computer use, and interpretation; consideration of parametric and non-parametric methods and comparison of their powers." While this course description leaves considerable room for interpretation by individual instructors, the essence of the required course mirrored what Cobb and Moore (1997) characterized as the traditional introductory statistics course, where heavy emphasis is placed on selection and use of formal probability models. In such a course, the idea of sample is implicit in concepts such as random samples and sampling distributions, but it may or may not be explicitly defined.

3.2. DATA GATHERING PROCEDURE

During a session of the participants' teaching methods course, they were introduced to the idea of using metaphors in teaching mathematical concepts. Following Glynn (1991), the first author, who was the instructor of the course, recommended to the participants that when using metaphors to teach concepts, the teacher should discuss both the ground of the metaphor and its tension. If such discussions do not take place, students may be more likely to make invalid conclusions about which aspects of a concept a metaphor captures and which it does not. To illustrate the application of the Glynn (1991) model for using metaphor in teaching, the methods course instructor asked the participants to consider the ground and the tension inherent in the metaphor "a function is a machine." Participants identified several aspects of the ground, such as the fact that there is an output for any given input in a function and also in some types of machines. They also identified several aspects of the tension, such as the fact that a machine is a physical object, while a function is not. This was the first point during the methods course when the idea of metaphor was discussed.

Immediately following the initial discussion of the function machine metaphor, participants were given a writing prompt (Figure 1) in which they were asked to write a metaphor for the concept of statistical sample, and to identify the ground and tension inherent in the metaphor they had written. Writing prompts were chosen as a data collection method because of their empirically demonstrated ability to reveal aspects of students' thinking related to mathematical concepts (Aspinwall & Aspinwall, 2003; Miller, 1992). It should be noted that one of the limitations of using writing prompts to elicit thinking is that students sometimes do not express everything they know about any given piece of content in writing (Aspinwall & Aspinwall, 2003). In order to minimize the impact of this limitation, we informed participants that it was important for their writing to be as reflective of their knowledge as possible, because the writing prompts would be used to design instruction for the methods course. Participants were also told in advance that with their permission, the material in the writing prompts would be used for this study. While there may still have been aspects of the participants' thinking that were not revealed in their writing, we chose to use writing prompts because of their demonstrated ability to enhance teachers' knowledge of students' thinking and in turn influence teaching (Miller, 1992). The use of writing prompts also facilitated the collection of data from a fairly large number of study participants.

Metaphor Construction Exercise
Background and Instructions

Definition

Metaphor: A figure of speech in which a word or phrase literally denoting one kind of object or idea is used in place of another to suggest a likeness or analogy between them (Mish, 1991).

Example of a metaphor

The world is a stage.

Discussion

Consider the metaphor "the world is a stage." There are many reasons why the world can be thought of as a stage (people are actors, life is a drama, etc.). However, there are some characteristics of the world that the idea of "stage" does not capture (its size, chemical composition, etc.). This is true of metaphors in general. Any given metaphor will capture some of the characteristics of any given idea while missing others.

Task

In this exercise, I am asking you to write a metaphor for a statistical idea. After writing the metaphor, discuss how and why the metaphor works (which characteristics of the statistical idea are captured). Also discuss how and why it does not work (which characteristics of the statistical idea are missed or perhaps misrepresented). Please do the exercises in writing, individually.

Statistical idea: Sample

(1) Write a metaphor for the statistical idea.
(2) Explain how and why your metaphor works.
(3) Explain how and why your metaphor does not work.

Figure 1. Metaphor Writing Prompt Given to Study Participants

3.3. DATA ANALYSIS

Both authors read the writing prompt responses generated by the participants. The first author used a clustering procedure (Miles & Huberman, 1994) to group similarly-structured responses into categories. The research pertaining to students' understanding of sample discussed earlier (Jacobs, 1997; Watson, 2004; Watson & Moritz, 2000a) highlighted relevant cognitive issues to which to attend in judging whether or not a group of responses was similarly-structured. For example, some participants' responses conceptualized a sample as a representative part of a whole, and since this sort of response was similar to a level of thinking documented by Watson and Moritz (2000a), the responses were clustered together. The second author then independently analyzed the responses and attempted to place them into the categories formed by the first author. At the conclusion of the independent analyses, the authors agreed on the placement of 30 out of the 54 responses.

After analyzing writing prompt responses independently, the authors met in order to determine the source of the 24 disagreements in coding. Ten of the 24 disagreements were caused by responses that were ultimately judged to be pedagogically awkward in their characterizations of the concept of sample. These ten instances became a new category, and its characteristics along with sample responses are given in the results section. The rest of the disagreements were resolved as the authors discussed their rationale for coding. Ultimately, consensus was reached about the category in which each response was to be placed.

4. RESULTS

Seven different categories of responses to the metaphor writing prompts were formed during the data analysis process. The names of the categories formed and the numbers of responses fitting each one are shown in Table 1. The characteristics of each category along with some sample responses are discussed in this section.

Table 1. Categories of Response to Metaphor Writing Prompt

Category descriptor	Frequency
No Metaphor Provided	3
Sample as a Collection of Objects	4
Sample as Part of a Whole	23
Sample as a Representative Part of a Whole	10
Metaphor for Place of Sample within Mathematics and Statistics	2
Metaphors Describing Actions to be Taken Upon Samples	2
Pedagogically Awkward Characterizations	10

4.1. CATEGORY 1: NO METAPHOR PROVIDED

Three participants gave no metaphor in response to the writing prompt. One of the three participants wrote, "A sample is not satisfactory" in response to the request to write a metaphor for the statistical idea of sample. Another of the three wrote, "A sample is" and did not finish her thought. The last of the three did not write anything in response to the prompt. The fact that three participants did not write any metaphor appears to have been caused by the cognitive complexity of thinking metaphorically rather than unwillingness to cooperate with the writing prompt exercise. None of the students handed in a paper without first spending time thinking through the process of writing an appropriate metaphor, but several remarked that they found it difficult to generate one.

This first category of response strongly resembles the lowest category in the hierarchy described by Watson and Moritz (2000a). In that category, students' definitions did not capture any of the relevant aspects of the concept of sample. Likewise, in this first category, responses were off-target. It should be noted, however, that the reason for the off-target responses in this category in the present study may well be due to cognitive difficulties in producing a metaphor rather than with lack of statistical content knowledge.

4.2. CATEGORY 2: SAMPLE AS A COLLECTION OF OBJECTS

Four of the writing prompt responses characterized a sample as a collection of objects with no mention of a larger population from which the objects were drawn. One participant provided the metaphor, "A sample is a forest," explaining that, "a forest is a group of trees." Another wrote the metaphor, "A sample is a car lot," because "a sample is like a group." The third of the students in the first category wrote "A sample is pizzas in a pizza parlor." She justified the metaphor by saying, "This shop has different sizes and types of pizza, just like a sample is dynamic. There are different aspects that come together to make the sample a whole." The final response in the first category started with the metaphor, "A sample is a bag of bagels," because "it includes more than one item from a wide range of possibilities." While each of the metaphors in the second category related the idea that samples often consist of diverse members or objects, they did not characterize samples as parts of larger populations.

The second category of response in the present study connects to the second level in the Watson and Moritz (2000a) hierarchy. In both instances, a sample is characterized as a part, but at the same time, the whole from which the part is drawn is not described.

4.3. CATEGORY 3: SAMPLE AS PART OF A WHOLE

The most common type of response characterized a sample as part of a whole. Twenty-three responses fit this category. Some examples include:

> Metaphor: A sample is one toy off of a toy shelf. Ground: My metaphor works because one toy is a sample from the toy shelf. Tension: My metaphor does not work because a sample does not always have to be a toy off of a toy shelf.
>
> Metaphor: A sample is the sugar in a cookie. Ground: To make a cookie, sugar is one of the ingredients. Likewise, a sample is one part of a whole set. Tension: You need more than just sugar to make a cookie and more than just one number to complete a set of data.
>
> Metaphor: A sample is a piece of the pie. Ground: A sample is one piece of data from an entire experiment. A piece of pie is only one part of the whole. Tension: A sample is only supposed to be one person's data, not the whole.

In the discussions of ground and tension for responses in this category, there was no mention of a sample being a representative part of a whole. While some of the metaphors provided, such as "a sample is a piece of a pie," had the potential to be used to illustrate the idea of representativeness (e.g., a piece of pie could be thought of as representative of the taste of the entire pie), that idea was not touched upon in accompanying discussion of ground and tension. The idea that a sample, as part of a whole, can serve to represent the whole, was absent from responses in the third category.

The third category of response in the present study is structurally similar to the third level in the Watson and Moritz (2000a) hierarchy. In the third level of their hierarchy, students conceptualized a sample as part of a whole, and explicitly referred to the population from which the sample was drawn. This pattern resonates with pre-service teacher responses in the third category, since in each instance the whole population was an integral part of the metaphors that were constructed (e.g., in the case of the toy shelf metaphor, the toy was thought of as a sample, and the toy shelf was referred to as the population from which it was drawn).

4.4. CATEGORY 4: SAMPLE AS A REPRESENTATIVE PART OF A WHOLE

Ten writing prompt responses characterized a sample as a representative part of a whole. This group of responses generally contained the richest descriptions and most vivid ideas. Some typical responses include:

> Metaphor: A sample is a handful of cereal out of the cereal box. Ground: My metaphor works because it is an equal representation of the box of cereal. It is taking a part of the whole and examining it instead of the whole. Tension: My metaphor doesn't work because every population is not a box of cereal.

> Metaphor: A sample is a handful of M&M's out of the candy dish. Ground: The metaphor shows that the sample is a smaller group than what is being studied. A handful of M&M's represents the whole dish of M&M's like a small group can represent a whole group. You don't need to take all the M&M's because a handful is the same. Tension: It doesn't work because all samples are not candy or bright colored.

> Metaphor: A sample is one WalMart store. Ground: A sample is a way of looking at a large amount of data by examining a small amount that can represent it. When you've been in one WalMart store – you've been in them all. Therefore – one WalMart can be a representative of all of them. Tension: A sample isn't really a store.

The responses in this category included the idea of sample as a part of a whole that was present in the previous category. However, they also incorporated a unique dimension in characterizing a sample as representative of the population from which it is drawn.

Since category 4 in the present study involved the conceptualization of sample as a representative part of a whole rather than simply part of a whole, the category strongly resonates with the fourth level in the Watson and Moritz (2000a) hierarchy. It is a significant conceptual leap to think of a sample as representative of the population from which it was drawn. This idea was missing from the previous category (e.g., the "toy on the toyshelf" metaphor and its accompanying discussion said nothing about how or if the toy selected might tell something about the larger population of toys on the shelf).

4.5. CATEGORY 5: METAPHOR FOR PLACE OF SAMPLE WITHIN THE FIELD OF STATISTICS

Two writing prompt responses were very different from the others in that they presented metaphors for the place held by the concept of sample within statistical data analysis. One of these students wrote, “A sample is a pathway on a long journey.” Her discussion of ground for the metaphor stated, “Taking a sample is only part of the process of data analysis, so it is like a journey because at some pathway you can choose to turn left or right, and, just like a sample, this determines the end result.” The second student in this category wrote that “Statistics is one small fish in a big ocean.” The explanation of the ground for the metaphor read, “It works because statistics is only part of the math world. There is also algebra, trig, calculus, geometry, and so on. A sample is an even smaller part of math because it is a part of statistics.” These two metaphors mark a sharp break from those in the previously discussed categories, since they seek to situate the concept of sample within the disciplines of statistics and mathematics rather than to illustrate intrinsic characteristics of the concept itself.

These two writing prompt responses appear to fall outside the levels described in the Watson and Moritz (2000a) hierarchy. None of the students in that study were reported as describing the place of sample within the field of statistics. Part of the reason for this may be that the students in the Watson and Moritz (2000a) study were younger, and hence had less opportunity to think about how various branches of mathematics fit together. Another part of the reason for this new category of response may be that the task was slightly different. Whereas Watson and Moritz (2000a) requested a definition of “sample,” in the present study a metaphor was requested. Asking students to write a metaphor may trigger different cognitive processes than asking for a definition.

4.6. CATEGORY 6: METAPHORS DESCRIBING ACTIONS TO BE TAKEN UPON SAMPLES

In two instances, participants characterized sample as a collection of data from which statistics are to be calculated. In one such instance, a student stated that “A sample is a phone bill.” She justified the metaphor by stating,

> A sample is a bunch of data that you get and perform computations with to get more data. Once you have the data, you might find the mode, median, mean, range, etc. of the data to draw further conclusions. Similarly, when you receive a phone bill you are receiving a whole bunch of data…Then, from this info. one might calculate mode, median, mean, etc., to figure out who they made the most calls to, what was the length of their longest call, what time most of their calls were made, etc.

The other response in this category began by characterizing a sample as, “a small part of the big picture.” The discussion of the tension for the metaphor included the thought that the metaphor “doesn’t describe any parts of the math process involved,” again implying that mathematical calculations are keys to defining the concept of sample. This sort of characterization of sample was similar to that conveyed by textbook problems which present a sample of numerical data and ask students to perform mathematical operations upon it.

This sixth category appears to be another that is not related to the Watson and Moritz (2000a) hierarchy. As with the fifth category, it is possible that both the structure of the task and the ages/experiences of the students involved played roles in this occurrence.

Experiences of students may play an especially pronounced role, since it is conceivable that years of doing nothing but performing computations on numerical samples would foster a resilient belief that samples exist for the purpose of being acted upon with arithmetic algorithms. This sort of "number crunching" characterization is not an uncommon portrayal of the field of statistics in traditional school curricula. Scheaffer (2002) noted that, "Statistics is often presented as a collection of techniques and tools rather than as a process for quantitative reasoning and problem solving" (p. 6). The responses in category six may possibly reflect a strong indoctrination into that particular view of statistics.

4.7. CATEGORY 7: PEDAGOGICALLY AWKWARD CHARACTERIZATIONS

In ten cases, it was judged that writing prompt responses were pedagogically awkward in their characterization of sample. For example, one participant in this category used the metaphor of "one in a million" for the concept of sample, which suggests that samples are generally not representative of the populations from which they are drawn, since the phrase generally is taken to mean that one has encountered something or someone unique. However, the same participant who constructed the "one in a million" metaphor stated that the tension of the metaphor was "the one sample does not accurately represent the population." Hence, the metaphor that was constructed appeared to obscure the participant's actual conception of sample to a large extent. Therefore, such responses were considered pedagogically awkward because of their inhibited potential to communicate meaning.

Other responses placed in the seventh category offered metaphors that were not clearly written or explained. For instance, one participant wrote, "a sample is like an example" without explaining why that would be an apt metaphor. Another wrote, "a slice is a piece of a pie," leaving the authors to speculate that she may have meant to write that a sample is a slice of pie or a piece of pie. However, those meanings could not be directly inferred from the writing. Since the authors agreed that these types of responses were highly ambiguous, they were considered likely to be awkward in conveying meaning in a teaching situation.

Since teachers' content knowledge regarding the concept of sample was not clearly displayed in responses fitting the seventh category, it is not possible to say where these participants may fall in relation to the Watson and Moritz (2000a) hierarchy. While it is difficult to gain any insight about the statistical content knowledge of teachers whose responses fit this seventh category, some information is gained about their pedagogical content knowledge (Shulman, 1987). It appears that the ten teachers whose responses fit the seventh category needed further learning experiences before they could incorporate metaphor as a teaching tool for helping students understand the important statistical aspects of the concept of sample.

5. DISCUSSION

The present study, exploratory in nature, provides directions for further research and some pedagogical implications. To conclude, both areas will be discussed.

The present study holds implications for instruction designed to help teachers use metaphors and other analogies explicitly as tools for articulating statistical ideas in everyday language. In discussing analogy as a teaching tool in the field of science education, Glynn (1991) suggested that teachers identify the ground and the tension in the analogy for students after introducing it. Participants' responses highlight that it is also important for teachers to attend closely to the initial construction of the metaphor. If a metaphor with a great deal of tension is used, the power of the metaphor may well work against the content that the teacher desires to convey, even if the tension inherent in the metaphor is explicitly discussed with students. For instance, using the metaphor of "one in a million" for sample creates a strong initial impression that samples identify a unique aspect of a group rather than serving the purpose of representing it. Although the teacher may not mean to convey this meaning, the power of the initial metaphor has the potential to obscure the meaning the teacher wishes to convey. Teacher educators might encourage pre-service teachers to share their idiosyncratic metaphors with one another and with young students to obtain a sense of what types of images the metaphors tend to invoke in others' minds.

The categories of participants' responses also suggest areas for teacher educators to focus upon in developing pre-service teachers' statistical content knowledge. Some participants exhibited very limited notions of the concept of sample by characterizing it simply as a group of objects or a collection of numerical data to perform mathematical procedures upon. Teacher educators must be conscious of designing instruction to expand and revise the personal metaphors of pre-service teachers holding such limiting conceptions. If limited personal metaphors are not replaced with richer ones, those metaphors may well constrain the types of tasks in which they have their future students engage, since one's personal metaphors guide teaching actions (Chapman, 1997; Sfard, 1998).

While it is beyond the scope of the present study to decisively state which teaching methods best foster understanding of the concept of sample, directions for further research in this area are illuminated by participants' responses to the metaphor task. Of particular interest is the fact that less than 20% of the pre-service teachers mentioned representativeness in discussing samples. Further research might focus on investigating the extent to which various teaching methods are able to improve on this figure. Various pedagogical methods for introducing the concept of sample are suggested in existing published texts. Many of the participants in this study had taken a course using a text which first formally defined a sample as a subset of units from a population, and then later gave a separate formal definition for "representative" sample (McClave & Sincich, 2003). Other authors take the approach of beginning with a formal definition for sample which encompasses the idea that information gathered from a sample is used to draw conclusions about the population from which it is drawn (e.g., Moore, 1997). Of course, these two approaches do not exhaust all possible patterns of teaching the concept, as individual instructors are likely to have their own pedagogical patterns. Even those who use the aforementioned texts in their courses may deviate from the text's presentation, as the interactions between intended and implemented curricula are often complex (Schmidt, McKnight, & Raizen, 1997). It would be beneficial to have future studies that focus on the effectiveness of various teaching approaches in building understanding of the representative functions of statistical samples and the role that formal definition plays in developing students' understanding.

Another possible direction for further research would be to attempt to determine how widespread the documented categories of metaphorical thinking are among pre-service teachers. Although no statistical generalizations were sought in the present study, we do expect some overlap between the characteristics of the group studied and other groups of pre-service elementary teachers, therefore making the study valuable for informing pre-service teacher instruction in other settings (Eisner, 1998; Glesne, 1999). Researchers desiring statistical generalizability could take the categories described in the present study as a working framework for larger-scale studies.

While the methodology used to ascertain teachers' thinking patterns in the present study yielded some useful data, it is not without its limitations. Eliciting written idiosyncratic metaphors yielded a wide range of categories of response, but even richer data could be gathered in future studies by coupling written metaphors with interviews in which participants are asked to elaborate further upon their personal metaphors. Such interviews would allow researchers to further unpack reasons for impoverished and ill-formed metaphors. Also, while personal metaphors provide a window on an individual's thinking about a mathematics concept, it is important for future studies to also investigate the relationship between elicited and spontaneous metaphors. It may be in some cases the spontaneous metaphors that occur in the course of instruction do not match the metaphors that are constructed upon the request of a researcher. It is important to understand teachers' spontaneous metaphors that occur during the course of instruction, given that they can influence students' thinking (Presmeg, 1992). As we begin to understand the types of spontaneous metaphors used in instruction and how they relate to elicited metaphors, we can become more effective in helping teachers re-think and re-construct the metaphors that guide their instruction of content.

Finally, it should be noted that the present study deals largely with describing teachers' knowledge of the concept of sample, but not with its use in problem solving situations. Therefore, we primarily learned about teachers' thinking within what Watson (1997) called "tier 1" of statistical literacy, which involves understanding of statistical terminology. Tier 2 involves "An understanding of statistical language and concepts when they are embedded in the context of wider social discussion" (Watson, 2000a, p. 54) and tier 3 involves, "A questioning attitude that can apply more sophisticated concepts to contradict claims made without proper statistical foundation" (Watson, 2000a, p. 54). Studies that delve into tiers 2 and 3 with an eye toward the concept of sample have, however, taken place. For example, Metz (1999) explored children's conceptions of sampling within the context of designing their own studies, and found that many of them were not convinced of the power of sampling even after designing and executing a research project. Also, Watson (2000b) has investigated pre-service teachers' strategies for solving the famous "hospital problem" (Kahneman & Tversky, 1972) and found that some strategies were based on intuition, some on mathematical principles, and others on both. Finally, the research of Watson and Moritz (2000a, 2000b) provides more examples of tasks that can be used to begin to understand the nature of thinking in regard to sample within tiers 2 and 3. Partially replicating these studies (Metz, 1999; Watson, 2000b; Watson & Moritz, 2000a, 2000b) while maintaining an eye toward the spontaneous metaphors teachers construct in such problem solving situations could yield rich insights about their understanding of sample within tiers 2 and 3 of statistical literacy.

6. CONCLUSION

Personal metaphors impact the manner in which individuals function on a day-to-day basis. This small-scale study brought to light a variety of personal metaphors that pre-service teachers hold for the concept of statistical sample. In some cases, the metaphors were rich and captured a number of important attributes of the concept. In other cases, the metaphors were impoverished and in need of further development. The impoverished patterns of thinking suggest that pre-service teachers need to experience problem-solving activities, such as those that require drawing samples for the purpose of making an inference to a larger population (e.g., Morita, 1999), that allow them to begin to conceive of a sample as more than just a collection of objects or a set of numerical data from which statistics are to be calculated. Teacher educators also need to construct learning trajectories (Simon, 1995) that aid the construction of richer personal metaphors. The present study provides a starting point in the construction of such trajectories, since it provides an overview of the patterns of thinking one might expect to encounter among pre-service teachers at the beginnings of interventions that include the goal of enhancing knowledge of the concept of statistical sample. This ultimately can improve statistics education for teachers by allowing pre-service curricula to have their foundation built on knowledge of teachers' statistical thinking.

REFERENCES

American Association for the Advancement of Science (1993). *Benchmarks for scientific literacy*. New York: Oxford University Press.

Aspinwall, L., & Aspinwall, J. S. (2003). Investigating mathematical thinking using open writing prompts. *Mathematics Teaching in the Middle School, 8*, 350-353.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed.) (pp. 433-456). New York: Macmillan.

Battista, M. T., & Clements, D. H. (2000). Mathematics curriculum development as a scientific endeavor. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 737-760). Mahwah, NJ: Erlbaum.

Biggs, J. B. (1992). Modes of learning, forms of knowing, and ways of schooling. In A. Demetriou, M. Efklides, & A. Efklides (Eds.), *Neo-Piagetian theories of cognitive development: Implications and applications for education* (pp. 31-51). London: Routledge.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning: The SOLO taxonomy*. New York: Academic.

Bogdan, R. C., & Biklen, S. K. (1992). *Qualitative research for education: An introduction to theory and methods*. Boston: Allyn and Bacon.

Chapman, O. (1997). Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics, 32*, 201-228.

Cobb, G., & Moore, D. (1997). Mathematics, statistics, and teaching. *American Mathematical Monthly, 104*, 801-823.

Conference Board of the Mathematical Sciences (2001). *The mathematical education of teachers*. Providence, Rhode Island: American Mathematical Society.

Cuevas, G. J., & Yeatts, K. L. (2001). *Navigating through algebra in grades 3-5*. Reston, VA: National Council of Teachers of Mathematics.

delMas, R. (2004). Mathematical and statistical reasoning. In D. Ben-Zvi & J. Garfield, (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 79-95). Dordrecht, The Netherlands: Kluwer.

Eisner, E. (1998). *The enlightened eye: Qualitative inquiry and the enhancement of educational practice*. Upper Saddle River, NJ: Merrill.

English, L. D. (1999). Reasoning by analogy: A fundamental process in children's mathematical learning. In L.V. Stiff & F.R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12* (pp. 22-36). Reston, VA: National Council of Teachers of Mathematics.

Gentner, D., & Holyoak, K. J. (1997). Reasoning and learning by analogy. *American Psychologist, 52*, 32-34.

Glesne, C. (1999). *Becoming qualitative researchers: An introduction* (2nd ed.). New York: Longman.

Glynn, S. M. (1991). Explaining science concepts: A teaching-with-analogies model. In S. M. Glynn, R. H. Yeany, & B. K. Britton (Eds.), *The psychology of learning science* (pp. 219-240). Hillsdale, NJ: Erlbaum.

Jacobs, V. R. (1997). *Children's understanding of sampling in surveys*. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago.

Jacobs, V. R. (1999). How do students think about statistical sampling before instruction? *Mathematics Teaching in the Middle School, 5*, 240-263.

Kahneman, D., & Tversky, A. (1972). Subjective probability: A judgement of representativeness. *Cognitive Psychology, 3*, 430-454.

Lakoff, G., & Nunez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* New York: Basic Books.

Martin, M. A. (2003). It's like…you know: The use of analogies and heuristics in teaching introductory statistical methods. *Journal of Statistics Education*, *2*. Retrieved February 14, 2005 from http://www.amstat.org/publications/jse/v11n2/martin.html.

Mayer, R. E. (1987). *Educational psychology*. Reading, MA: Addison Wesley.

McClave, J. T., & Sincich, T. (2003). *A first course in statistics* (8th ed.). Upper Saddle River, NJ: Pearson.

Metz, K. E. (1999). Why sampling works or why it can't: Ideas of young children engaged in research of their own design. In F. Hitt & M. Santos (Eds.), *Proceedings of the 21st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 492-498). Cuernavaca, Mexico: PME.

Mewborn, D. S. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 45-52). Reston, VA: National Council of Teachers of Mathematics.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis*. Thousand Oaks, CA: Sage.

Miller, D. L. (1992). Teacher benefits from using impromptu writing prompts in algebra classes. *Journal for Research in Mathematics Education, 23*, 329-340.

Mish, F. C. (Ed.) (1991). *Webster's ninth new collegiate dictionary*. Springfield, MA: Merriam-Webster.

Mokros, J., & Russell, S.J. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education, 26*, 20-39.

Moore, D. S. (1997). *Statistics: Concepts and controversies* (4th ed.). New York: W.H. Freeman.

Morita, J. G. (1999). Capture and recapture your students' interest in statistics. *Mathematics Teaching in the Middle School, 4*, 412-418.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Education Association (2005). Wanted: More male teachers. Accessed online June 24, 2005: http://www.nea.org/teachershortage/03malefactsheet.html

National Research Council (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Presmeg, N. C. (1992). Prototypes, metaphors, metonymies and imaginative rationality in high school mathematics. *Educational Studies in Mathematics, 23,* 595-610.

Presmeg, N. C. (1998). Metaphoric and metonymic signification in mathematics. *Journal of Mathematical Behavior, 17*, 25-32.

Presmeg, N. C. (2002). Mathematical idea analysis: A science of embodied mathematics – A review of where mathematics comes from: How the embodied mind brings mathematics into being. *Journal for Research in Mathematics Education, 33*, 59-63.

Presmeg, N. C., & Bergner, J. A. (2002). Cognitive elements of students' use of metaphors in a college geometry class. In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant, & K. Nooney (Eds.), *Proceedings of the twenty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education: Volume 2* (pp. 755-766). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Scheaffer, R. L. (2002). Data analysis in the K-12 curriculum: Teaching the teachers. *New England Mathematics Journal, 34*(2), 6-25.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (Eds.) (1997). *A splintered vision: An investigation of U.S. science and mathematics education.* Dordrecht, The Netherlands: Kluwer.

Sfard, A. (1994). Reification as the birth of metaphor. *For the Learning of Mathematics, 14* (1), 44-55.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), 4-13.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114-145.

Strauss, A. L., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, CA: Sage.

Watson, J. M. (1997). Assessing statistical literacy using the media. In I. Gal & J. B. Garfield (Eds.), *The assessment challenge in statistics education* (pp. 107-121). Amsterdam: ISO Press and the International Statistical Institute.

Watson, J. M. (2000a). Statistics in context. *Mathematics Teacher, 93*, 54-58.

Watson, J. M. (2000b). Preservice mathematics teachers' understanding of sampling: Intuition or mathematics. *Mathematics Teacher Education and Development, 2*, 121-135.

Watson, J. M. (2004). Reasoning about samples. In D. Ben-Zvi & J. Garfield, (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 277-294). Dordrecht, The Netherlands: Kluwer.

Watson, J. M. & Moritz, J. B. (2000a). Development of understanding of sampling for statistical literacy. *Journal of Mathematical Behavior, 19*, 109-136.

Watson, J.M., & Moritz, J. B. (2000b). Developing concepts of sampling. *Journal for Research in Mathematics Education, 31*, 44-70.

RANDALL E. GROTH
119 Emily Dr.
Salisbury, MD 21804
USA