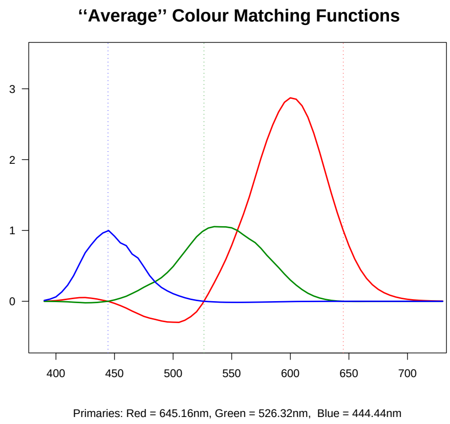
Statistics 120
Colourimetry



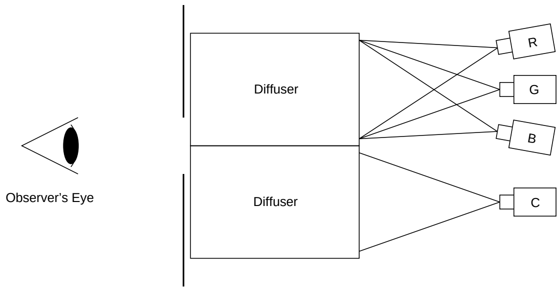
Colour Matching

- Much colour research was carried out in the 1920s and 1930s (mostly in the UK).
- There were two goals for this research
  - To understand colour vision.
  - To develop commercial technology for colour matching.

Negative Colour Amounts

- There are some colours *C* which can't be matched by any combination of primaries.
- A match can be made by adding a small amount of one of the primaries to *C*.
- When this is the case the amount of that primary in the match is taken to be negative.

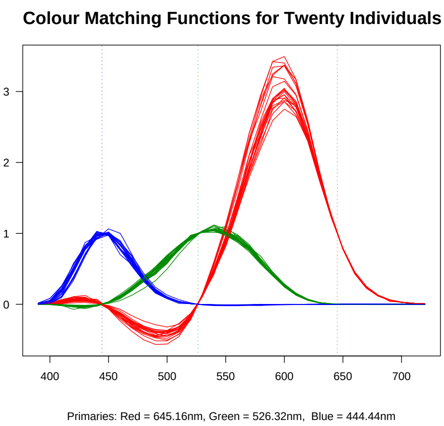
A Colour Matching Experiment



How much of the pure primary colours *R*, *G* and *B* does it take to produce a match to *C*?

Notes

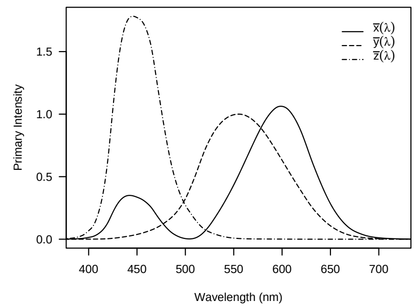
- Colour matching functions were found to vary very little between individuals.
- This meant that an average of the colour matching functions could be used as a description of a “standard observer.”
- The international colour standards organisation (the *Commission Internationale de l'Éclairage* or *CIE*) adopted the standard observer as the basis for colour description.



The CIE XYZ Primaries

- The CIE felt that the negative colour amounts which resulted from using RGB primaries were too difficult for practitioners to handle.
- Instead they adopted an alternative set of “imaginary” colour primaries *X*, *Y* and *Z* and used a mathematical conversion from RGB to XYZ to obtain the matching functions for XYZ.
- The XYZ matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ provide an absolute description of every possible colour.

The CIE Matching Functions



Note that $\bar{y}(\lambda)$ is proportional to the apparent brightness of the pure hue with wavelength λ.

Tristimulus Values

- A colour is defined by its spectral distribution $S(\lambda)$ (telling how much of each wavelength λ is present).
- Such a colour can be matched by the following amounts of X, Y and Z primaries.

$$
\begin{aligned}
X &= \int \bar{x}(\lambda)S(\lambda)d\lambda \\
Y &= \int \bar{y}(\lambda)S(\lambda)d\lambda \\
Z &= \int \bar{z}(\lambda)S(\lambda)d\lambda
\end{aligned}
$$

- These are called the tristimulus values for the colour.

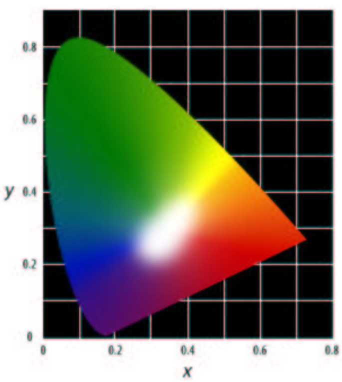
Chromaticities

- The tristimulus values include the effects of hue, saturation and brightness.
- It is common to remove the effect of brightness by converting to *chromaticities*:

$$
\begin{aligned}
x &= X/(X+Y+Z) \\
y &= Y/(X+Y+Z) \\
z &= Z/(X+Y+Z)
\end{aligned}
$$

- Since the chromaticities sum to 1, the z value can be dropped without loss. This makes it possible to plot colours in a two dimensional display called the chromaticity diagram.

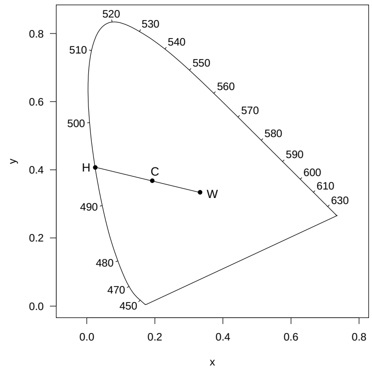
The Chromaticity Diagram



Colour Mixing

- The fundamental property of chromaticities is that the chromaticity for a mixture of two colours lies on the line joining the individual chromaticities.
- This property makes the chromaticity diagram ideal for describing colour mixing.

Colour Properties



Colour Properties

- Every colour can be obtained as a mixture of a pure hue and white.
- The wavelength of the pure hue is called the dominant wavelength of the colour.
- The relative distance of the colour from white to the hue is called the excitation purity of the colour.
- These are analogues to the hue and saturation parameters of the HSV model.

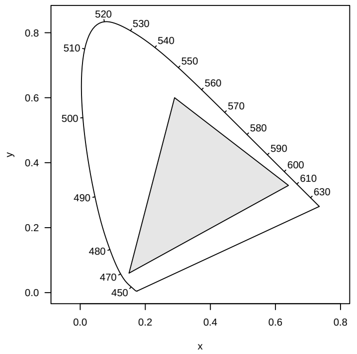
Complementary Colours

- Colours are said to be complementary if they have the same excitation purity and can be mixed to form white.
- Examples of complementary pairs are:
  - red/cyan
  - yellow/blue
  - green/magenta
- Complementary colours provide a vivid contrast. (Often too vivid for use in presentation graphics.)

Device Gamuts

- The chromaticity diagram provides a good way to describe the colour capabilities of devices like computer displays or printers.
- For RGB devices (like computer displays) all colours are a mixture of the Red, Green and Blue primaries.
- This means that all the colours which can be produced by the display must have chromaticities which lie in a triangle whose vertexes are the chromaticities of the primaries.

A Typical RGB Device Gamut



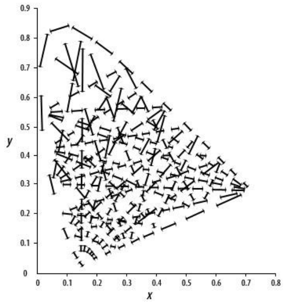
Colour Gamut Restrictions

- RGB displays have problems displaying truly saturated greens and purples.
- Colour printers use Cyan, Magenta and Yellow primaries rather than Red, Green and Blue (because inks provide subtractive rather than additive colour).
- The colour gamut restrictions of printers are far more restrictive that that of RGB displays.
- The set of colours which can be represented faithfully on both RGB displays and colour printers is very small.

Perceptual Uniformity

- The chromaticity diagram provides a complete description of all colours which humans can see.
- It does not, however, present the colours in a *perceptually uniform* way.
- Quite soon after the diagram was developed, researchers began investigating to see just how uniform the diagram was.
- They did this by looking at how large the least noticeable colour differences were.

Least Noticeable Chromaticity Differences



The lengths of the lines in this diagram are proportional to the least noticeable colour difference at the centre of the line.

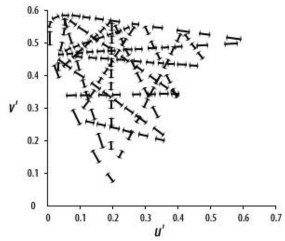
The CIE 1976 u', v' Diagram

- In 1976 the CIE introduced a transformed chromaticity diagram which was much more uniform than the original chromaticity diagram.
- The transformations which achieved by the following transformations:

$$u' = 4X/(X+15Y+3Z) = 4x/(-2x+12y+3)$$
$$v' = 9Y/(X+15Y+3Z) = 9y/(-2x+12y+3)$$

- The transformations, although not perfect make the diagram much more perceptually uniform.

Least Noticeable Chromaticity for CIE u', v'



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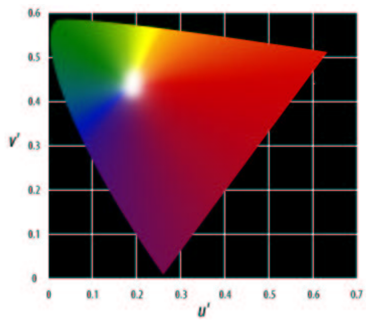
The CIE 1976 u', v' Diagram



Properties of the CIE u', v' Diagram

- The u',v' diagram retains the same properties as the original chromaticity diagram.
- Most importantly, the chromaticity for a mixture of two colours lies on the line joining the individual chromaticities.
- Colour gamuts etc. retain the same interpretation.

The CIELUV Space

- The u', v' diagram gives chromaticity information. It does not give full colour.
- To produce full colour information, information about apparent brightness must be added.
- This was done by using a perceptually uniform transformation of the apparent brightness Y.
- This is the CIE 1976 lightness L^*, defined by

$$L^* = \begin{cases} 116(Y/Y_n)^{1/3} - 16 & \text{for } Y/Y_n > 0.008856, \\ 903.3(Y/Y_n) & \text{otherwise,} \end{cases}$$

Where Y_n is the apparent brightness for white.

CIELUV Coordinates

- The full set of CIELUV coordinates is:

$$\begin{aligned} L^* &= \begin{cases} 116(Y/Y_n)^{1/3} - 16 & \text{for } Y/Y_n > 0.008856, \\ 903.3(Y/Y_n) & \text{otherwise,} \end{cases} \\ u^* &= 13L^*(u' - u'_n) \\ v^* &= 13L^*(v' - v'_n) \end{aligned}$$

- The space is also (more formally) know as the CIE 1976 $L^*u^*v^*$ space.

Hue, Chroma and Luminance

- The u^* and v^* axes are not intuitive ones when making colour choices.
- It is useful to transform the u^* and v^* scale to hue angle chroma, defined by:

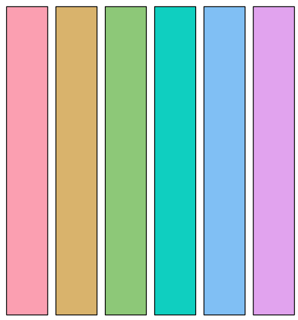
$$\begin{aligned} h_{uv} &= \arctan v^*/u^* \\ C^*_{uv} &= (u^{*2} + v^{*2})^{1/2} \end{aligned}$$

- The three variables h_{uv}, C^*_{uv} and L^* provide a description of CIELUV which can be interpreted in the same way as the axes of HSV space.

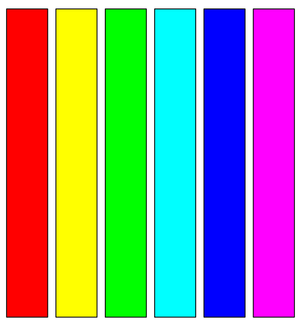
Using CIELUV

- As an example of using the CIELUV space, consider the problem of choosing a set of colours which could be used for colouring the slices of a pie chart or the bars of a bar chart.
- We would like to have the colours have "equal impact" so that they do not influence our perception of what is happening in a graph.
- We would also like to have them be as easy to distinguish from one another as possible.
- One way in which to choose such equal impact colours is to choose them to have equally spaced hue angles with equal chroma and luminance.

Equal Impact Colours



HSV Based Colours



Equal Impact Colours In Practice