

Statistics 120

Mosaic Plots

Who Listens To Classical Music?

The following table of values shows a sample of 2300 music listeners classified by age, education and whether they listen to classical music.

| | Education | | | |
|--------------|-----------------|-----------|------------|-----------|
| | <i>High</i> | | <i>Low</i> | |
| Age | Classical Music | | | |
| | <i>Yes</i> | <i>No</i> | <i>Yes</i> | <i>No</i> |
| <i>Old</i> | 210 | 190 | 170 | 730 |
| <i>Young</i> | 194 | 406 | 110 | 290 |

This is a $2 \times 2 \times 2$ contingency table.

Old Versus Young

The effect of age and education on musical taste can be investigated by breaking the observations down into more homogenous groups. The most obvious split is by age. There are 1300 older people and 1000 younger people.

| <i>Old</i> | <i>Young</i> |
|------------|--------------|
| 56.5% | 43.5% |

This is almost certainly a result of the way in which the sample was taken.

Education Level

Within the old and young groups we can now find the proportions falling into each of the high and low education categories.

| <i>Old</i> | | <i>Young</i> | |
|-----------------|----------------|-----------------|----------------|
| <i>High Ed.</i> | <i>Low Ed.</i> | <i>High Ed.</i> | <i>Low Ed.</i> |
| 30.8% | 69.2% | 60.0% | 40.0% |

The *young* group is clearly more highly educated than the *old* group.

Music Listening

Finally, we can compute the proportion of people who listen to classical music in each of the age/education groups.

| <i>Old</i> | | <i>Young</i> | |
|-----------------|----------------|-----------------|----------------|
| <i>High Ed.</i> | <i>Low Ed.</i> | <i>High Ed.</i> | <i>Low Ed.</i> |
| 52.5% | 18.9% | 32.3% | 27.5% |

The music-listening habits of younger people seem to be fairly independent of education level. This is not true for older people.

Summary

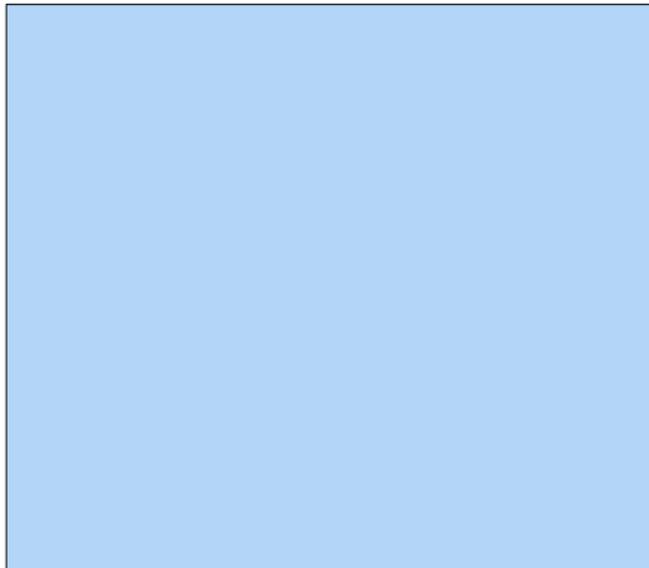
The result of our “analysis” is a series of tables. From these tables we can see:

1. There are slightly more old people than young people in the sampled group.
2. The younger people are more highly educated than the older ones.
3. The likelihood of listening to classical music depends on both age and education level.

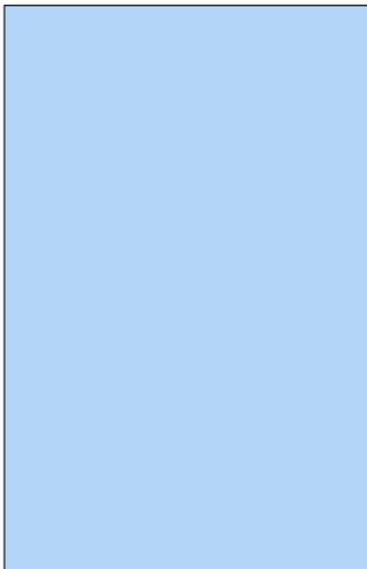
Mosaic Plots

- Mosaic plots give a graphical representation of these successive decompositions.
- Counts are represented by rectangles.
- At each stage of plot creation, the rectangles are split parallel to one of the two axes.

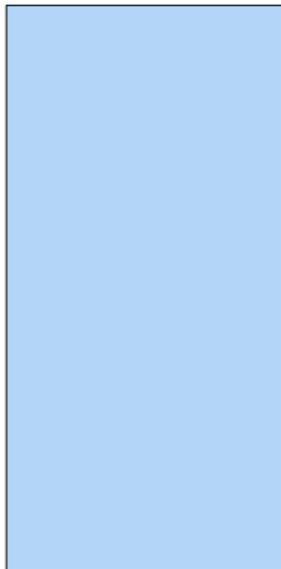
Everyone

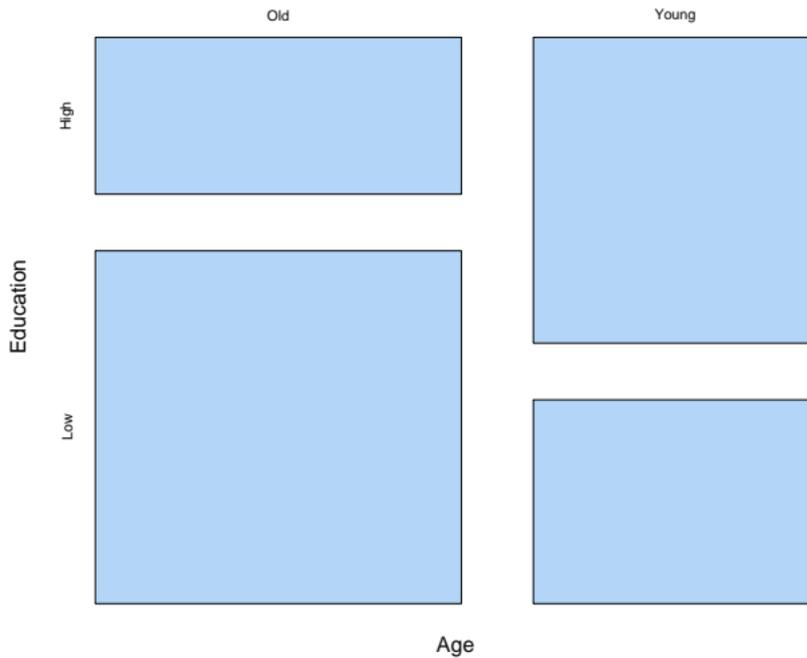


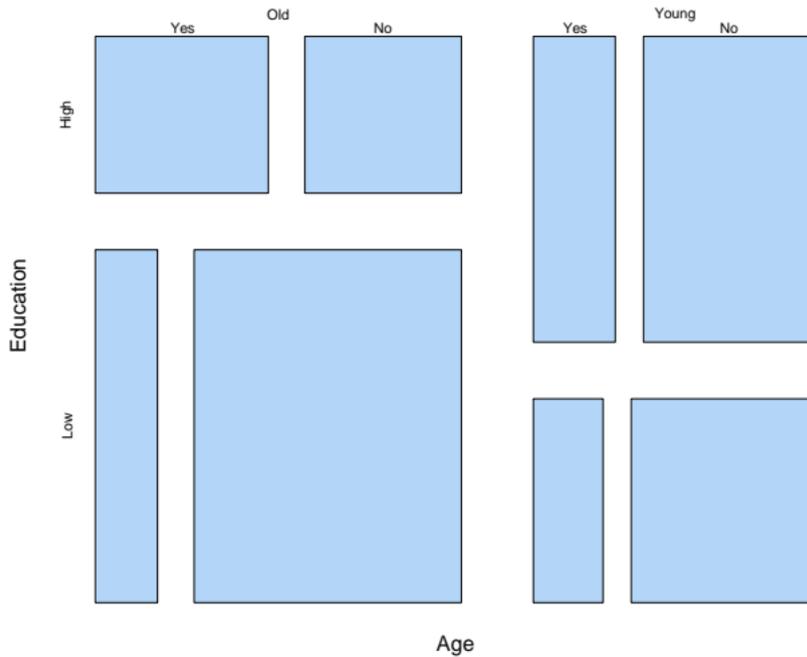
Old



Young







The Perceptual Basis for Mosaic Plots

- It is tempting to dismiss mosaic plots because they represent counts as rectangular areas, and so provide a distorted encoding.
- In fact, the important encoding is length.
- At each stage the comparison of interest is of the lengths of the sides of pieces of the most recently split rectangle.

Creating Mosaic Plots

- In order to produce a mosaic plot it is necessary to have:
 - A contingency table containing the data.
 - A preferred ordering of the variables, with the “response” variable last.

Entering the Data

- To enter the data, we must settle on the order in which the values.
- The order of values in an R array is with the first subscript varying most quickly, the second subscript varying next most quickly, etc.
- In the case of the music data we can take the first subscript to correspond to *Age*, the second to *Education* and the third to *Listening*.
- The steps are then (i) entering the data, (ii) shaping it as an array and (iii) labelling the extents.

Order for Data Entry

| | Education | | | |
|--------------|-----------------|-----------|------------|-----------|
| | <i>High</i> | | <i>Low</i> | |
| | Classical Music | | | |
| Age | <i>Yes</i> | <i>No</i> | <i>Yes</i> | <i>No</i> |
| <i>Old</i> | 210 | 190 | 170 | 730 |
| <i>Young</i> | 194 | 406 | 110 | 290 |

| Data Order | | | |
|------------|---|---|---|
| 1 | 5 | 3 | 7 |
| 2 | 6 | 4 | 8 |

Data Entry

```
> music = c(210, 194, 170, 110,  
            190, 406, 730, 290)  
  
> dim(music) = c(2, 2, 2)  
  
> dimnames(music) =  
  list(Age = c("Old", "Young"),  
        Education = c("High", "Low"),  
        Listen = c("Yes", "No"))
```

Data Inspection

```
> music
, , Listen = Yes
```

```
      Education
Age    High Low
Old    210 170
Young  194 110
```

```
, , Listen = No
```

```
      Education
Age    High Low
Old    190 730
Young  406 290
```

Producing A Mosaic Plot

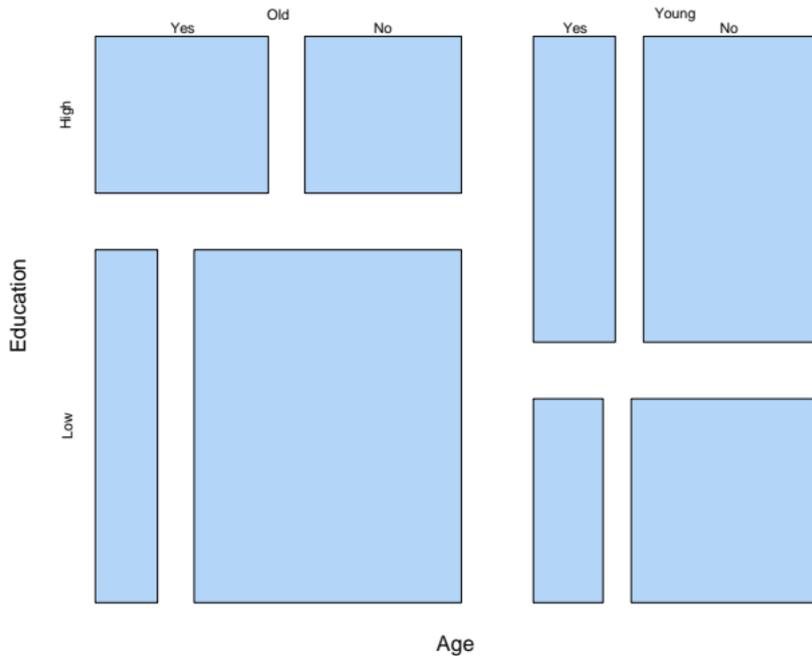
The R function which produces mosaic plots is called `mosaicplot`. The simplest way to produce a mosaic plot is:

```
> mosaicplot(music)
```

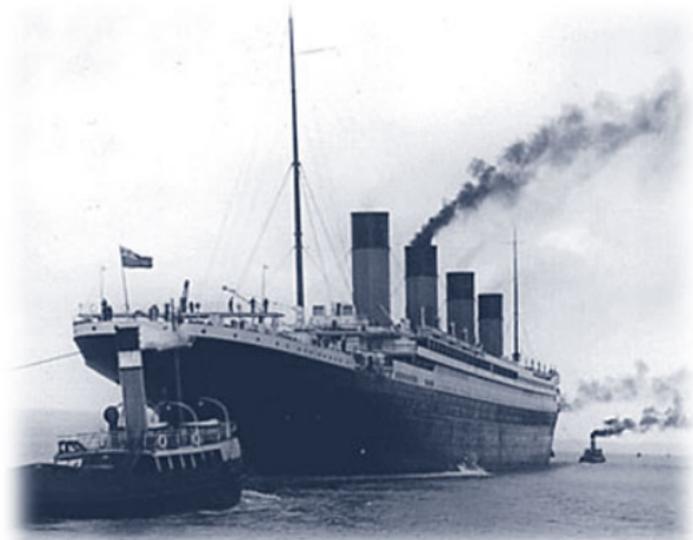
It is also easy to colour the plot and to add a title.

```
> mosaicplot(music, col = hcl(240),  
             main = "Classical Music Listening")
```

Classical Music Listening



Example: Survival on the Titanic



On Sunday, April 14th, 1912 at 11:40pm, the RMS Titanic struck an iceberg in the North Atlantic. Within two hours the ship had sunk. At best reckoning 705 survived the sinking, 1,523 did not.

The Data

- There is very good documentation on who survived and who did not survive the sinking of the Titanic.
- R has a data set called “Titanic” which gives data on the passengers on the Titanic, cross-classified by:
 - Class: 1st, 2nd, 3rd, Crew.
 - Sex: Male, Female.
 - Age: Child, Adult.
 - Survived: No, Yes.

| Adults | <i>Survivors</i> | | <i>Non-Survivors</i> | |
|------------------|------------------|---------------|----------------------|---------------|
| | <i>Male</i> | <i>Female</i> | <i>Male</i> | <i>Female</i> |
| <i>1st Class</i> | 57 | 140 | 118 | 4 |
| <i>2nd Class</i> | 14 | 80 | 154 | 13 |
| <i>3rd Class</i> | 75 | 76 | 387 | 89 |
| <i>Crew</i> | 192 | 20 | 670 | 3 |

| Children | <i>Survivors</i> | | <i>Non-Survivors</i> | |
|------------------|------------------|---------------|----------------------|---------------|
| | <i>Male</i> | <i>Female</i> | <i>Male</i> | <i>Female</i> |
| <i>1st Class</i> | 5 | 1 | 0 | 0 |
| <i>2nd Class</i> | 11 | 13 | 0 | 0 |
| <i>3rd Class</i> | 13 | 14 | 35 | 17 |
| <i>Crew</i> | 0 | 0 | 0 | 0 |

Producing a Mosaic Plot

First load the “Titanic” data from the R data library.

```
> data(Titanic)
```

Next produce the mosaic.

```
> mosaicplot(Titanic,  
             main = "Survival on the Titanic",  
             col = hcl(c(240, 120)),  
             off = c(5, 5, 5, 5))
```

Note the use of `col=` to produce alternating coloured rectangles — green for survivors and blue for non-survivors. Also note that the `off=` argument is used to squeeze out a little of the space between the blocks.

Example: Sexual Discrimination at Berkeley

- In the 1980s, a court case brought against the University of California at Berkeley by women seeking admission to graduate programs there.
- The women claimed that the proportion of women admitted to Berkeley was much lower than that for men, and that this was the result of discrimination.

| <i>Gender</i> | <i>Admitted</i> | <i>Rejected</i> | <i>%Admitted</i> |
|---------------|-----------------|-----------------|------------------|
| Male | 1198 | 1493 | 44.5 |
| Female | 557 | 1278 | 30.4 |

- It is clear that a higher proportion of males is being admitted.

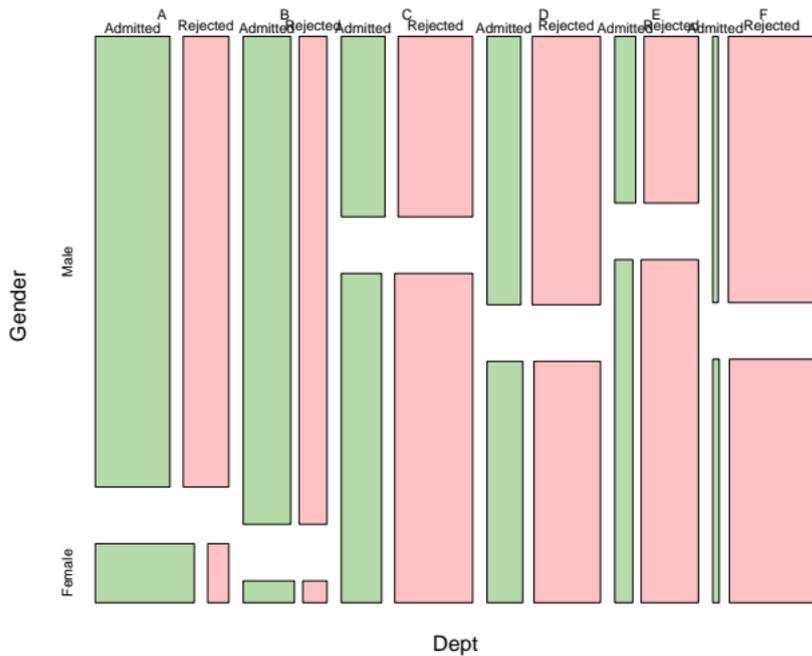
The University Case

The Dean of Letters and Science at Berkeley was a famous statistician (called Peter Bickel) and he was able to argue that the difference in admissions rates was not caused by sexual discrimination in the Berkeley admissions policy, but was caused by the fact that males and females generally sought admission to different departments.

The Dean broke the admissions data down by department and showed that within each program there was no admission discrimination against women. Indeed, there seemed to be some admissions bias in favour of women.

| | | <i>Male</i> | <i>Female</i> |
|---------------------|-----------------|-------------|---------------|
| <i>Department A</i> | <i>Admitted</i> | 512 | 89 |
| | <i>Rejected</i> | 313 | 19 |
| <i>Department B</i> | <i>Admitted</i> | 353 | 17 |
| | <i>Rejected</i> | 207 | 8 |
| <i>Department C</i> | <i>Admitted</i> | 120 | 202 |
| | <i>Rejected</i> | 205 | 391 |
| <i>Department D</i> | <i>Admitted</i> | 138 | 131 |
| | <i>Rejected</i> | 279 | 244 |
| <i>Department E</i> | <i>Admitted</i> | 53 | 94 |
| | <i>Rejected</i> | 138 | 299 |
| <i>Department F</i> | <i>Admitted</i> | 22 | 24 |
| | <i>Rejected</i> | 351 | 317 |

Student admissions at UC Berkeley



Producing The Plot

To produce the mosaic plot of the UCB data we must reverse order of the extents of the data array. We can either do this using `aperm`, or by using the `sort=` argument to `mosaicplot`.

```
> data(UCBAdmissions)
```

```
> mosaicplot(UCBAdmissions, sort = 3:1,  
             col = hcl(c(120, 10)),  
             main = "Student admissions at UC Berkeley")
```

(The order of the variables in the R data set is: Admit, Gender, Dept – the reverse of what `mosaicplot` expects.)

```
> UCBA admissions
```

```
, , Dept = A
```

| Admit | Gender | |
|----------|--------|--------|
| | Male | Female |
| Admitted | 512 | 89 |
| Rejected | 313 | 19 |

```
, , Dept = B
```

| Admit | Gender | |
|----------|--------|--------|
| | Male | Female |
| Admitted | 353 | 17 |
| Rejected | 207 | 8 |

```
, , Dept = C
```

| Admit | Gender | |
|----------|--------|--------|
| | Male | Female |
| Admitted | 120 | 202 |
| Rejected | 205 | 391 |

```
, , Dept = D
```

| Admit | Gender | |
|----------|--------|--------|
| | Male | Female |
| Admitted | 138 | 131 |
| Rejected | 279 | 244 |

etc.

Collapsing Contingency Tables

- The Berkeley Admissions contingency table has three dimensions.
 - Admit: Admitted, Rejected.
 - Gender: Male, Female.
 - Dept: A, B, C, D, E, F.
- To obtain the overall effect, ignoring Dept, we have to compute the table which would result by summing over departments.
- This is called *collapsing* the original table.

Collapsing Tables in R

The process of collapsing contingency tables is easy in R. The `apply` function can be used to collapse over the third subscript of the `UCBAdmissions` table.

```
> apply(UCBAdmissions, 1:2, sum)
```

```
      Gender
Admit  Male Female
Admitted 1198   557
Rejected 1493  1278
```

The effect of the `apply` statement is to sum the table over all the subscripts not given in the second argument.

Collapsing Tables in R

The order of the subscripts specified in an apply statement determines the order of the subscripts in the result. Reversing the order of subscripts in the previous apply statement produces the transpose of the original result.

```
> apply(UCBAdmissions, 2:1, sum)
```

```
      Admit
Gender  Admitted Rejected
Male      1198     1493
Female     557     1278
```