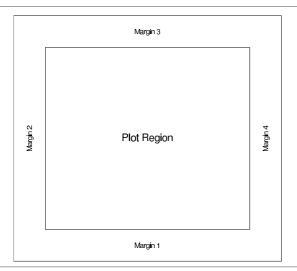
Computer Graphics

- Drawing graphics in a window on the screen of a computer is very similar to drawing by hand on a sheet of paper.
- We begin a drawing by getting out a clean piece of paper and then deciding what scale to use in the drawing.
- With those basic decisions made, we can then start putting pen to paper.
- The steps in R are very similar.



Setting the Axis Scales

Next we set the scales on along the sides of the plot. This determines how coordinates get mapped onto the page.

```
> plot.window(xlim = xlimits, ylim = ylimits)
```

The graphics system arranges for the specified region to appear on the page.

xlimits and ylimits are vectors which contain lower and upper limits which are to appear on the x and y axes.

For example,

...
$$xlim = c(-pi, pi), ylim = c(-1, 1), ...$$

might be suitable for plotting sine and cosine functions.

Aspect Ratio Control

There is also an optional argument to the function plot.window() which allows a user to specify a particular aspect ratio.

The use of asp=1 means that unit steps in the x and y directions produce equal distances in the x and y directions on the page.

This is important if circles are to appear as circles rather than ellipses.

Starting a New Plot

We begin a plot by first telling the graphics system that we are about to start a new plot.

```
> plot.new()
```

This indicates that we are about to start a new plot and must happen before any graphics takes place.

The call to plot.new chooses a default rectangular *plotting* region for the plot to appear in.

The plotting region is surrounded by four margins.

Controlling The Margins

The sizes of the margins can be changed by making a call to the function par before calling plot.new.

Set the margin sizes in inches.

```
> par(mai=c(2, 2, 1, 1))
```

Set the margin sizes in lines of text.

```
> par(mar=c(4, 4, 2, 2))
```

Set the plot width and height in inches.

> par(pin=c(5, 4))

Manipulating the Axis Limits

The statement

```
> plot.window(xlim = c(0, 1), ylim = c(10, 20))
```

produces axis limits which are expanded by 6% over those actually specified. This expansion can be inhibited by specifying xaxs="i" and/or yaxs="i".

For example, the call

produces a plot with 0 lying at the extreme left of the plot region and 1 lying at the extreme right.

Drawing Axes

The axis function can be used to draw axes at any of the four sides of a plot.

```
axis(side)
```

The value of the side arguments which axis is drawn.

side=1: below the graph (x axis),

side=2: to the left of the graph (y axis),

side=3: above the graph (x axis),

side=4: to the right of the graph (y axis).

A variety of optional arguments can be used to control the appearance of the axis.

Axis Customisation

The axis command can be customised. For example:

```
axis(1, at = 1:4, labels = c("A", "B", "C", "D"))
```

places the tick marks on the lower x axis at 1, 2, 3, and 4 and labels them with the strings "A", "B", "C" and "D".

Label rotation can be controlled with the value of the optional las argument.

las=0: labels are parallel to the axis,las=1: labels are horizontally oriented,las=2: labels are at right-angles to the axis,

las=3: labels are vertically oriented.

Plot Annotation

The function title can be used to place labels in the margins of a plot.

title(main=str, sub=str, xlab=str, ylab=str)

The arguments are as follows:

main: a main title to appear above the graph,sub: a subtitle to appear below the graph,

xlab: a label for the x axis, ylab: a label for the y axis.

Framing a Plot

It can be useful to draw a box around a plot. This can be done with the function box. The call

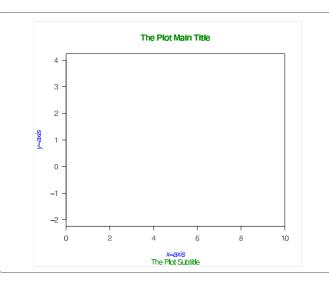
> box()

draws a box around the plot region (the region within the plot margins). The call

> box("figure")

draws a box around the figure region (the region containing the plot and its margins).

An optional col argument makes it possible to specify the colour for the box.



Additional Axis Customisation

Additional customisation can be produced with additional arguments to the axis function:

col: the colour of the axis and tickmarks,

col.axis: the colour of the axis labels,

font.axis: the font to be used for the axis labels.

Colours can be specified by name (e.g. "red", "green", etc) as well as in other ways (see later).

Fonts can be one of 1, 2, 3 or 4 for *normal*, *bold*, *italic* and *bold-italic*.

Customising Plot Annotation

The elements of the plot annotation can be customised with additional optional arguments.

font.main, col.main, cex.main
The font (1, 2, 3, or 4), colour and
magnification-factor for the main title.

font.sub, col.sub, cex.sub

The font, colour and magnification-factor for the subtitle.

font.lab, col.lab, cex.lab

The font, colour and magnification-factor for the axis labels.

Example: A "Bare" Plot

Some Drawing Primitives

- Points
- Connected Line Segments
- Straight Lines Across A Plot
- Disconnected Line Segments
- Arrows
- Rectangles
- Polygons
- Text
- Legends

Drawing Points

The basic call has the form:

points(x, y, pch=int, col=str)

where:

- pch specifies the plotting symbol. Values 1 to 25 are special graphical symbols, values from 33 to 126 are taken to ASCII codes. A quoted character will also work,
- col gives a colour specification. Examples are, "red", "lightblue", etc. (More on colour later.)

Plotting Symbols and Colour

- The colour of plotting symbols can be changed by using the col argument to points.
- Plotting symbols 21 through 25 can additionally have their interiors filled by using the bg argument to points.

Drawing Connected Line Segments

The basic call has the form:

lines(x, y, lty=str, lwd=num, col=str)

where:

 1ty specifies the line texture. It should be one of "blank", "solid", "dashed", "dotted", "dotdash", "longdash" or "twodash".

Alternatively the length of on/off pen stokes in the texture. "11" is a high density dotted line, "33" is a short dashed line and "1333" is a dot-dashed line.

• 1wd and col specify the line width and colour.

A Line Graph Example 88 88 1996 1998 2000 2002 2004 Time

Graphical Plotting Symbols

The following plotting symbols are available in R.

O 1	<u>^</u> 2	+ 3	× 4	
▽	⊠	* 8	⊕	⊕
6	7		9	10
\\	⊞	⊠	△	1 5
11	12	13	14	
•	1 7	♦	●	•
16		18	19	20
O	□		△	▽
21	22		24	25

Coloured Plotting Symbols

The effect of colour choice on plotting symbols.

<u> </u>	<u>^</u> 2	+	× 4	
▽ 6	⊠ 7	* 8	⇔ 9	⊕ 10
☆ 11	⊞ 12	⊠ 13	△ 14	15
• 16	1 7	◆ 18	1 9	20
<u> </u>	22	♦ 23	<u>^</u> 24	▽ 25

A Line Graph

Line Graph Variations

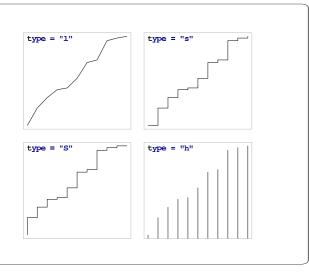
Additional forms can be produced by the lines function. This is controlled by the type argument.

type="1": line graph,
type="s": step function — horizontal first,
type="S": step function — vertical first,
side="h": high density (needle) plot.

Additional variations:

type="p": draw points,type="b": both points and lines,

type="o": over-plotting of points and lines,



Straight Line Example

Drawing Disconnected Line Segments

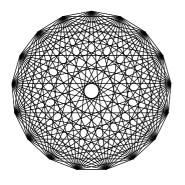
The basic call has the form:

```
segments(x0, y0, x1, y1)
```

where

- The x0, y0, x1, y1 arguments give the start and end coordinates of the segments.
- Line texture, colour and width arguments can also be given.

A Rosette with 17 Vertexes



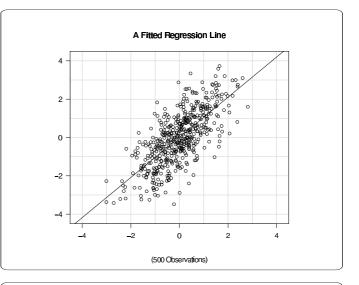
Drawing Straight Lines

The basic call has the forms:

```
abline(a=intercept, b=slope)
abline(h=numbers)
abline(v=numbers)
```

where

- The a / b form specifies a line in intercept / slope form.
- h specifies horizontal lines at the given y values.
- v specifies vertical lines at the given x values.
- Line texture, colour and width arguments can also be given.



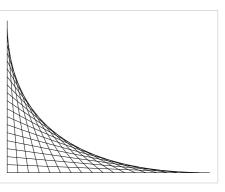
Rosettes

A rosette is a figure which is created by taking a series of equally spaced points around the circumference of a circle and joining each of these points to all the other points.

A Curve Envelope

Here is another example which shows how the eye can perceive a sequence of straight lines as a curve.

A Curve Envelope



Basic Arrows

Here is a simple diagram using arrows.

```
> plot.new()
> plot.window(xlim = c(0, 1), ylim = c(0, 1))
> arrows(.05, .075, .45, .9, code = 1)
> arrows(.55, .9, .95, .075, code = 2)
> arrows(.1, 0, .9, 0, code = 3)
> text(.5, 1, "A", cex = 1.5)
> text(0, 0, "B", cex = 1.5)
> text(1, 0, "C", cex = 1.5)
```

Using Arrows as Error Bars

Drawing Rectangles

The basic call has the form:

```
rect(x0, y0, x1, y1, col=str, border=str)
```

where

- x0, y0, x1, y1 give the coordinates of diagonally opposite corners of the rectangles.
- col and border specify the colour of the interior and border of the rectangles.
- line texture and width specifications can also be given.

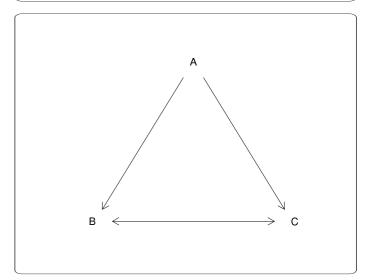
Drawing Arrows

The basic call has the form:

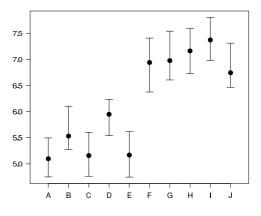
```
arrows(x0, y0, x1, y1, code=int, length=num, angle=num)
```

where

- The x0, y0, x1, y1 arguments give the start and end coordinates of the arrows.
- code=1 head at the start, code=2 head at the end and code=3 a head at both ends.
- length and angle length of the arrow head and angle to the shaft.

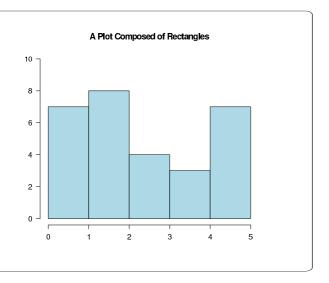


Using Arrows as Error Bars



Rectangle Example

The following code illustrates how a barplot or histogram could be constructed.



Drawing Polygons

The basic call has the form:

polygon(x, y, col=str, border=str)

where

- x and y give the coordinates of the polygon vertexes. NA values separate polygons.
- col and border specify the colour of the interior and border of the polygons.
- line texture and width specifications can also be given.

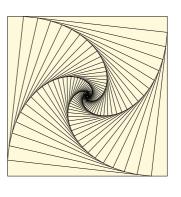
A Simple Polygon Example

Here is a simple example which shows how to produce a simple polygon in a plot.

A Simple Polygon

Spiral Squares

Spiral Squares



Drawing Text

The basic call has the form:

```
text(x, y, labels)
```

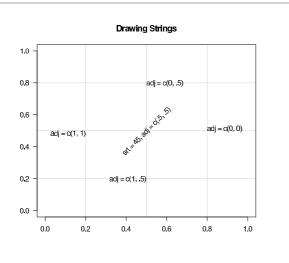
where

- x and y give the text coordinates.
- labels gives the actual text strings.

Optionally,

- font and col give the font and colour of the text,
- srt and adj give the rotation and justification of the strings.

A Text Example



Legend

Drawing Curves

- There are no general curve drawing primitives available in R (yet).
- To draw a curve you must approximate it by a sequence of straight line segments.
- The question is how many line segments are required to obtain a visually "smooth" approximation to the curve.

Lack of Smoothness

- Using 31 points to approximate the curve, there is a noticeable lack of smoothness in regions where the curve has high curvature.
- This is because our eye-brain system is good a detecting large changes of direction but sees changes in direction of less than 5° as "smooth."
- Checking the changes of angle in the approximation shows that there are some very large changes of angle.
- Increasing the number of approximating points to 331 means that there are no changes of direction which exceed 5°.

Drawing a Legend

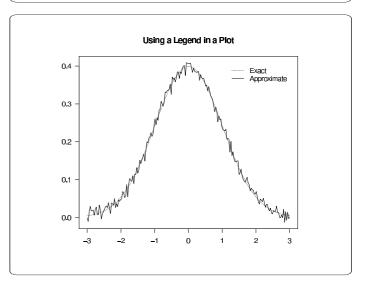
A simple example has the form:

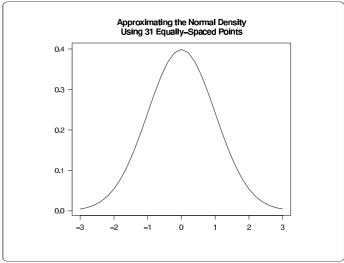
legend(xloc, yloc, legend=text
 lty=linetypes, lwd=linewidths,
 pch=glyphname, col=colours,
 xjust=justification, yjust=justification)

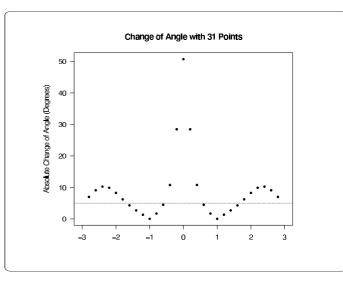
where

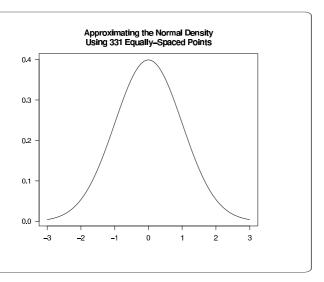
xloc and yloc give the coordinates where the legend is to be placed and xjust and yjust give the justification of the legend box with respect to the location. The other values describe the legend contents

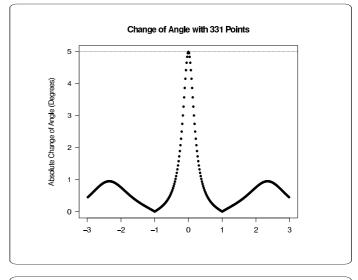
The legend function is very flexible. Consult its manual entry for details.





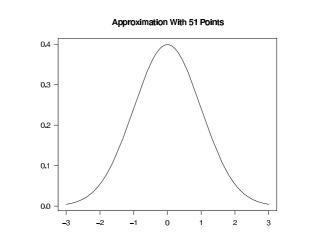




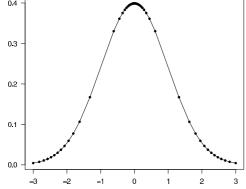


Nonuniform Point Placement

- It is wasteful to use equally spaced points to approximate a curve. Regions with high curvature require closely packed points while regions of low curvature may need only a few points.
- This means that techniques which take account of curvature can lead to approximations with many fewer points.
- One technique is to place points so that the segment to segment direction change is less than a fixed threshold (e.g., 5°).



Approximation With 51 Points



Circles

for $t \in [0, 2\pi)$.

The circle with centre (x_c,y_c) and radius R is defined by the equation

$$(x-x_c)^2 + (y-y_c)^2 = R^2$$
.

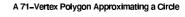
It can also be defined parametrically with the equations

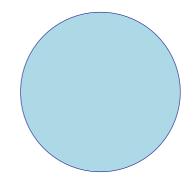
$$x(t) = R\cos t$$
$$y(t) = R\sin t$$

There is no simple R function for drawing a circle. Circles must be approximated with a regular polygon.

Using at least 71 vertexes for the polygon ensures that the change of direction between edges is less than or equal to 5° .

Drawing Circles





Ellipses

An ellipse is a generalisation of circle defined by the equation:

$$\left(\frac{x-x_c}{a}\right)^2 + \left(\frac{y-y_c}{b}\right)^2 = 1.$$

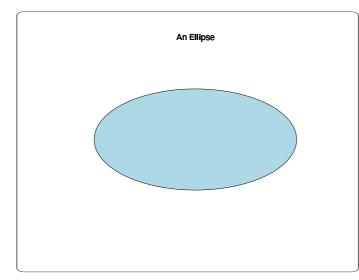
An ellipse can be defined in parametric form by:

$$x(t) = a\cos t + x_c,$$

$$y(t) = b\sin t + y_c,$$

with $t \in [0, 2\pi)$.

The distortion of the ellipse happens in such a way that it can be approximated by the same number of straight line segments as a circle.



Rotation Formulae

If the point (x,y) is rotated though an angle θ around the origin to a new position (x',y') then

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta,$$

or in matrix terms

$$\left(\begin{array}{c} x'\\ y'\end{array}\right) = \left(\begin{array}{cc} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{array}\right) \left(\begin{array}{c} x\\ y\end{array}\right).$$

Drawing Ellipses

Rotation

We want to rotate (x,y) through an angle θ about the origin to (x',y').

In polar coordinates:

$$x = R\cos\phi,$$

$$y = R\sin\phi.$$

and:

$$\begin{aligned} x' &= R \cos(\phi + \theta) & y' &= R \sin(\phi + \theta) \\ &= R (\cos \phi \cos \theta - \sin \phi \sin \theta) & = R (\cos \phi \sin \theta + \sin \phi \cos \theta) \\ &= x \cos \theta - y \sin \theta & = x \sin \theta + y \cos \theta \end{aligned}$$

Rotated Ellipses

Often it is useful to consider rotated ellipses rather than ellipses aligned with the coordinate axes. This can be done by simply applying a rotation.

If the ellipse is rotated by an angle θ , its equation is

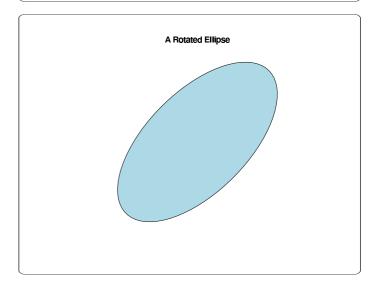
$$x(t) = a\cos t \cos \theta - b\sin t \sin \theta + x_c$$

$$y(t) = a\cos t \sin \theta + b\cos t \sin \theta + y_c$$

for $t \in [0, 2\pi)$.

Again, the same number of straight line segments can be used to approximate the ellipse.

Drawing Rotated Ellipses



Ellipses in Statistics

Suppose that (X_1, X_2) has a bivariate normal distribution, with X_i having mean μ_i and variance σ_i^2 , and the correlation between X_1 and X_2 being ρ .

If we define

 $d = \arccos \rho$

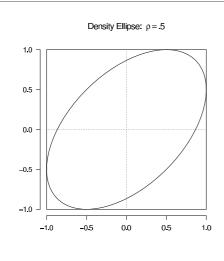
the equations

$$x = \mu_1 + k \sigma_1 \cos(t + d),$$

$$y = \mu_2 + k \sigma_2 \cos(t),$$

describe the contours of the density of (X_1, X_2) .

Choosing the appropriate value for k makes it possible to draw prediction ellipses for the bivariate normal distribution.



Spirals

A spiral is a path which circles a point at a radial distance which is changing monotonically.

$$x(t) = R(t)\cos t$$
$$y(t) = R(t)\sin t$$

for t > 0.

In particular, an exponential spiral is obtained when

$$R(t) = \alpha^t$$

where $\alpha < 1$.

Such a path resembles a snail or nautilus shell.

An Exponential Spiral

Statistical Ellipses

Here
$$\mu_1 = \mu_2 = 0$$
, $\sigma_1 = \sigma_2 = 1$ and $k = 1$.

> n = 72

> rho = 0.5

> d = acos(rho)

> t = seq(0, 2 * pi, length = n)[1:(n-1)]

> plot.new()

> plot.window(xlim = c(-1, 1), ylim = c(-1, 1), asp = 1)

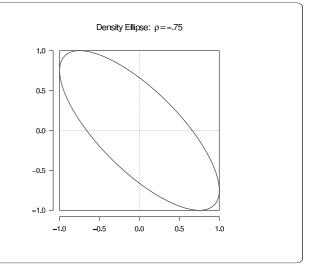
> rect(-1, -1, 1, 1)

> polygon(cos(t + d), y = cos(t))

> segments(-1, 0, 1, 0, lty = "13")

> segments(0, -1, 0, 1, lty = "13")

> axis(1); axis(2, las = 1)



Drawing a Spiral

These commands draw a spiral, centred on (0,0). The spiral does 5 revolutions:

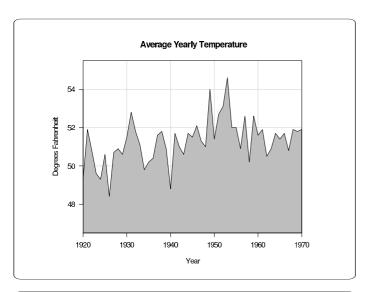
Filling Areas In Line Graphs

Annual year temperatures in New Haven (1920-1970).

```
> y
[1] 49.3 51.9 50.8 49.6 49.3 50.6 48.4
[8] 50.7 50.9 50.6 51.5 52.8 51.8 51.1
[15] 49.8 50.2 50.4 51.6 51.8 50.9 48.8
[22] 51.7 51.0 50.6 51.7 51.5 52.1 51.3
[29] 51.0 54.0 51.4 52.7 53.1 54.6 52.0
[36] 52.0 50.9 52.6 50.2 52.6 51.6 51.9
[43] 50.5 50.9 51.7 51.4 51.7 50.8 51.9
[50] 51.8 51.9
```

The corresponding years.

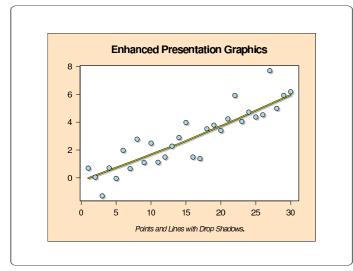
```
> x = 1920:1970
```

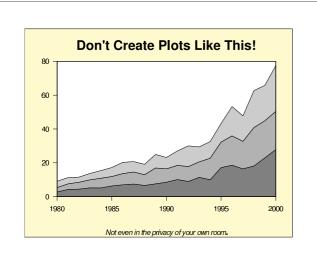


Finishing up

Add the axes, bounding box and annotation.

```
axis(1)
axis(2, las = 1)
box()
title(main = "Average Yearly Temperature")
title(ylab = "Degrees Fahrenheit")
title(xlab = "Year")
```





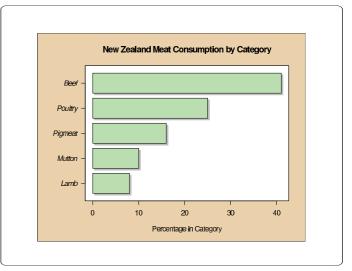
Plot Construction

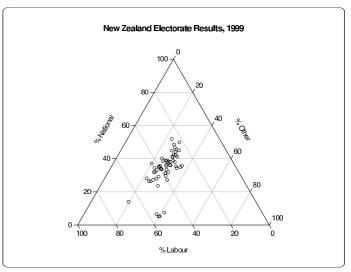
Setting up the plot and drawing the background grid.

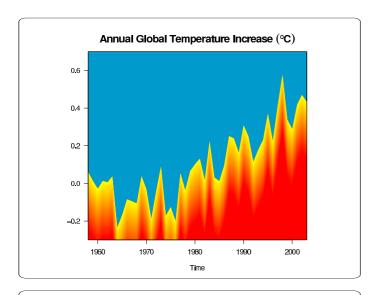
Some Graphics Examples

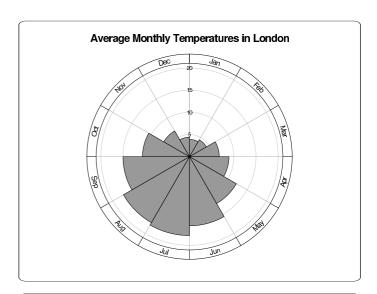
Here is a short set of examples to show the kind of graphics that is possible to create using the R graphics primitives.

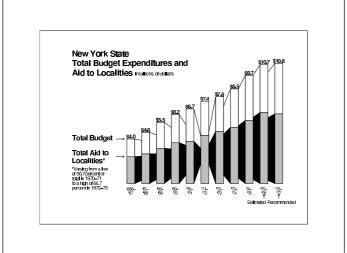
These are not necessarily all "good" graphs. They just show what is possible with a little effort.

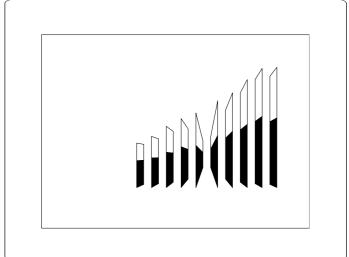


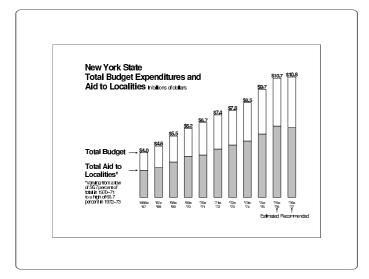


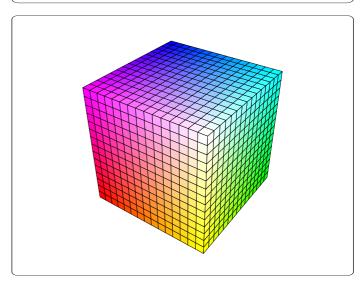


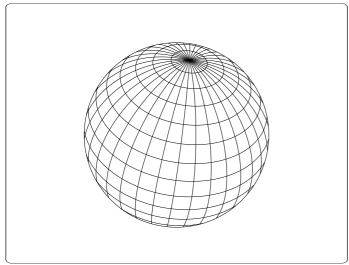


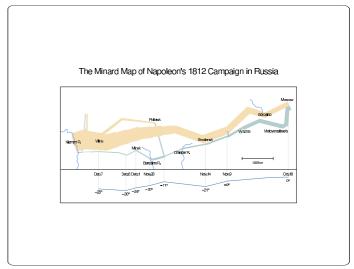












Packaging Graphics Functionality

We have seen lots of ways of drawing graphs. Now let's look at how this capability can be packaged as general purpose tools (i.e. R functions).

There are two types of tool to consider.

- Tools which set up and draw a complete plot,
- Tools which add to existing plots.

The tools are slightly simplified versions of real tools which are part of R, or can be found in extension libraries.

Scatterplot Code

Here are the key steps required to produce a scatterplot.

ullet Determine the x and y ranges.

```
xlim = range(x)
ylim = range(y)
```

• Set up the plot window.

```
plot.new()
plot.window(xlim = xlim, ylim = ylim)
```

• Plot the points.

```
points(x, y)
```

Using The Scatterplot Function

We can use this function just like any other R function to produce scatter plots.

```
> xv = 1:100
> yv = rnorm(100)
> scat(xv, yv)
> title(main = "My Very Own Scatterplot")
```

Customisation

The scat function is very restricted in what it can do. Let's add a little flexibility.

- Optional plotting symbol specification
- Optional colour specification
- Optional range specification
- Optional logarithmic axes
- Optional annotation

A Scatterplot Function

There are a number of tasks which must be solved:

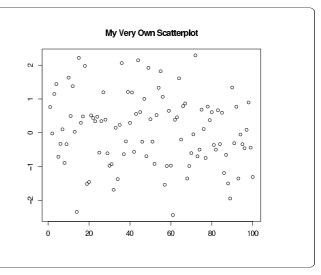
- ullet Determining the x and y ranges.
- Setting up the plot window.
- Plotting the points.
- Adding the plot axes and frame.

Each of these tasks is relatively simple.

A Scatterplot Function

By "wrapping" the steps in a function definition we can produce a simple scatter plot function.

```
> scat =
    function(x, y) {
        xlim = range(x)
        ylim = range(y)
        plot.new()
        plot.window(xlim = xlim, ylim = ylim)
        points(x, y)
        axis(1)
        axis(2)
        box()
}
```



The Customised Scatterplot Function

An Ellipse Drawing Function

Now we show a function which can be draw a single ellipse with centre (ax,yc), axis lengths a and b and rotated by theta degrees.

It is possible to pass the function parameters which change the colour of the ellipse and its border, and to change the line type used for the border.

A real ellipse drawing function would be more complex (but harder to fit onto a single slide).

Querying and Specifying Graphics State

The par function provides a way of maintaining graphics state in the form of a variety of *graphics parameters*.

The call

```
> par(mar = c(4, 4, 2, 2))
```

sets the plot margins to consist of 4, 4, 2 and 2 lines of text.

The call

```
> par("mar")
```

returns the current setting of the mar parameter.

There are a *large* number of graphics parameters which can be set and retrieved with par.

User Coordinate System Enquiries

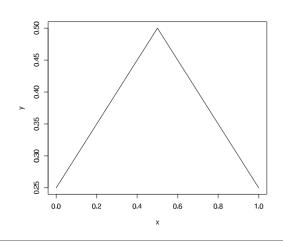
The upper and lower x and y limits for the plot region may not be exactly those specified by a user (because of a possible 6% expansion). The exact limits can be obtained as follows:

After this call, usr will contain a vector of four numbers. The first two are the left and right x scale limits and the second two are the bottom and top y scale limits.

A call to par can also be used to change the limits.

```
> par(usr = c(0, 1, 10, 20))
```

The specified limits must be sensible.



An Ellipse Drawing Function

Device, Figure and Plot Size Enquiries

The graphics system uses inches as its basic measure of length. Note that 1 inch = 2.54 cm.

```
par("din"): the device dimensions in inches,
par("fin"): the current figure dimensions in inches,
par("pin"): the current plot region dimensions in inches,
par("fig"): NDC coordinates for the figure region,
par("plt"): NDC coordinates for the plot region,
```

NDC = normalised device coordinates.

Computing Direction Change in Degrees

Here is a sketch of how the change of angle computations were done in the "smooth curve" examples. This works by transforming from data units to inches.

```
> x = c(0, 0.5, 1.0)
> y = c(0.25, 0.5, 0.25)
> plot(x, y, type = "l")
> dx = diff(x)
> dy = diff(y)
> pin = par("pin")
> usr = par("usr")
> ax = pin[1]/diff(usr[1:2])
> ay = pin[2]/diff(usr[3:4])
> diff(180 * atan2(ay * dy, ax * dx) / pi)
[1] -115.2753
```

Multifigure Layouts

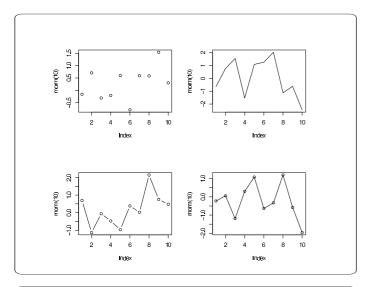
par can be used to set up arrays of figures on the page. These arrays are then filled row-by-row or column-by-column.

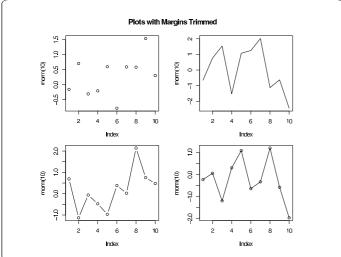
The following example declares a two-by-two array to be filled by rows and then produces the plots.

```
> par(mfrow=c(2, 2))
> plot(rnorm(10), type = "p")
> plot(rnorm(10), type = "l")
> plot(rnorm(10), type = "b")
> plot(rnorm(10), type = "o")
```

A two-by-two array to be filled by columns would be declared with

```
> par(mfcol = c(2, 2))
```





Eliminating Waste Margin Space

It can be useful to eliminate redundant space from multi-way arrays by trimming the margins a little.

Here we have trimmed space from the top of each plot, and placed a 4 line *outer margin* at the top of of the layout.