Heart Transplant Data

Arden Miller

Executive Summary

Data concerning heart transplant patients in the Standford heart transplant programme was analysed. It was found that younger patients had a higher probability of surviving at least 100 days than older patients. Having a good match between the tissue type of the donor and the recipient also increased the probability of survival. No evidence that the amount of time the patient spent in the programme affected the probability of survival was found. There also was no evidence that the probability of survival increased over the range of time (January 1968 to April 1974) covered by the data.

1 The Data

The data consists of observations for 61 patients from the Standford heart transplant programme who received heart transplants between January 1968 and April 1974. Measurements for four explanatory variables and a binary response (SUR) that indicates whether the patient survived longer than 100 days after the transplant or not are included for each patient. The four regressors represent variables that it was thought would have an impact on the probability of survival. This data is to be analysed to see if it provides evidence that this is in fact true.

The four explanatory variables are:

T5S: A measure of the degree to which the donor and recipient were mismatched with respect to tissue type. It was thought that this measure should be related to the phenomenon of rejection of the donor heart by the recipients immune system. A low score (< 1) indicates a good match and a high score represents a poor match.

AGE: The age of the patient at the time of the operation. It was thought the young patients should fare better.

WT: The number of days from the date the patient entered to the programme until the operation was performed.

CT: The number of days after 1 January that the operation was performed.

Table 1: Fitted probabilities of surviving 100 days

	Age		
T5S	40	50	60
0.50	0.83	0.68	0.47
1.50	0.69	0.48	0.28
2.50	0.49	0.29	0.15

Figure 1 contains pairwise scatterplots of the data. These plots suggest that T5S and AGE may both be having the expected impact on the probability of surviving more than 100 days: the probability appears to decrease as the value of T5S increases and as the age of the patient increases. There is no clear indication from these plots that either WT or CT has an impact on SUR. The plot between AGE and CT indicates that there was a trend to younger transplant recipients during this period.

2 Factors that Affect the Survival Time

The four regressors were chosen because it was believed that they should be related to the probability that a heart transplant patient survives at least 100 days. A statistical analysis of this data only found evidence that the T5S score and AGE affect survival time. No evidence that waiting time or calendar time are related to SUR was found.

A fitted logistic regression model that relates the probability of surviving 100 days (π) to the T5S score and the age of patient is:

$$\text{logit}(\pi) = \log\left(\frac{\pi}{1-\pi}\right) = 5.44 - 0.829(\text{T5S}) - 0.0855(\text{AGE})$$

This model indicates that the probability of surviving 100 days increases as both AGE and T5S decrease which is what was expected. Table 1 gives the predicted probabilities of surviving for patients with different combinations of AGE and T5S scores. Clearly, both factors are having a substantial impact on survival. A patient who is 40 and has a relatively low T5S score (0.5) has an estimated probability of surviving 100 days of 0.83 while a patient who is 60 and has a relatively high T5S score only has an estimated probability of surviving 100 days of 0.15.

It was found that there were two observations that were having an unusually large effect on the fitted model. These two observations corresponded to two patients that were relatively young (36 and 29) and had low T5S scores (0.00 and 0.61) but both failed to survive 100 days. It would be interesting to find out if there were any unusual circumstances related to these patients. Our fitted model estimates the probabilities of survival for these two patients as 0.91 and 0.92. If the model is refit omitting these two observations (which would be reasonable if they were found to be unusual circumstances associated with these 2 patients) then the fitted model would become

$$\text{logit}(x) = 9.73 - 1.41(\text{T5S}) - 0.156(\text{AGE}).$$

Notice that the estimated effects of both T5S and AGE become much more pronounced if these two points are deleted.

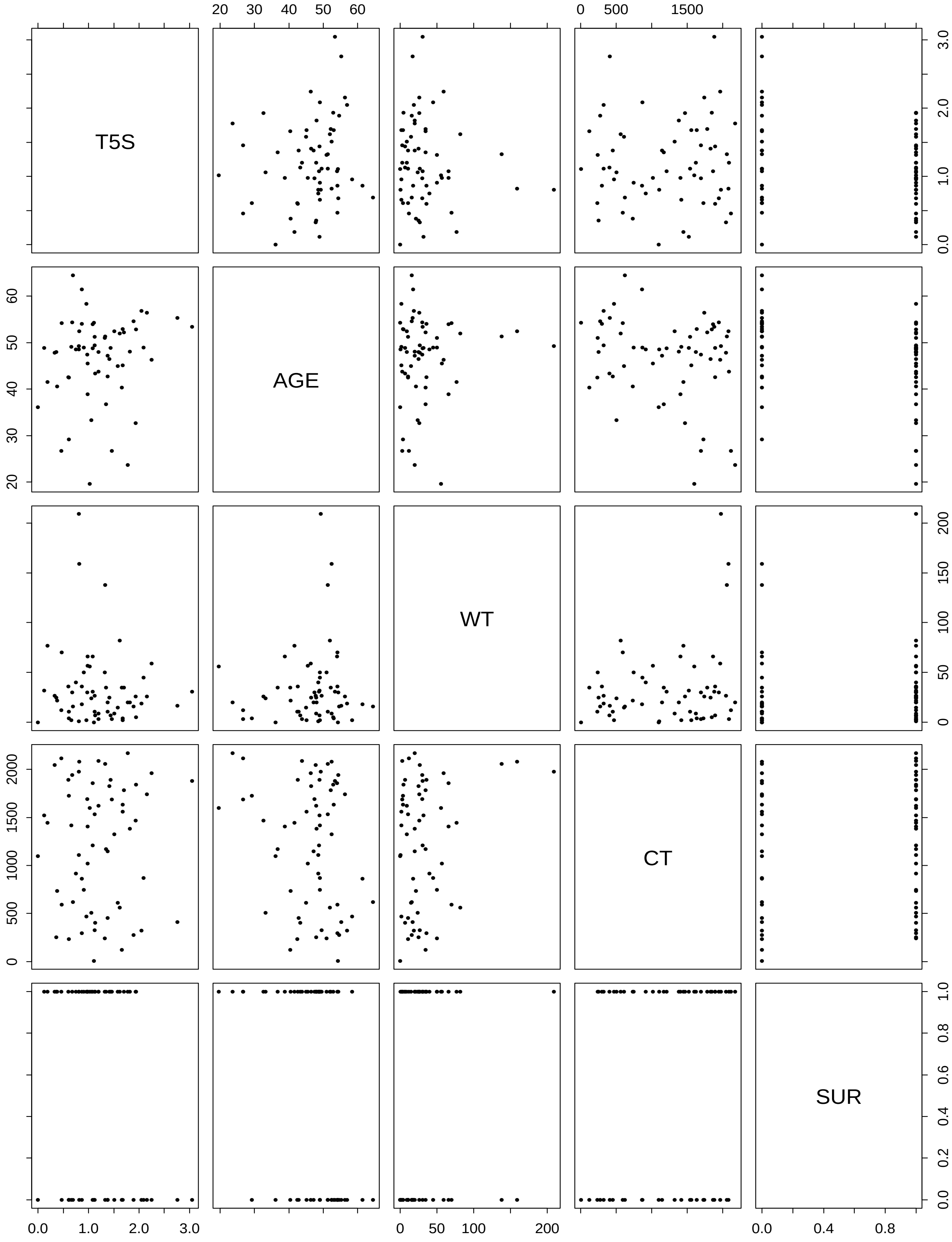

Figure 1: Pairwise scatterplots of data

It is interesting that no evidence of a time trend was found since we would certainly expect that the doctors would gain valuable experience from the first operations. It could be that the learning process occurred very quickly. In this case it is unlikely that we would have enough information about the probability of survival early in the programme to be able to detect the improvement over time.

No evidence was found that waiting time affected the probability survival. We need to be careful how we interpret this since we only have data for people in the programme who actually received a heart transplant. We don't know how many patients died while waiting for transplant. As a result if we are asked about the trade-off between accepting a poor mismatch score or increasing waiting time we can only speculate. It would seem that as long as the patient survives long enough for a better match to be found, it is worth waiting since we have found clear evidence that a better T5S score increases the probability of surviving 100 days. However, we have no information about the length of time it takes to find a good match or how long a patient waiting for a transplant can be expected to survive.

Statistical Appendix

The fitted regression model that contains all 4 regressors gives the following output:

```
Coefficients:
                    Value   Std. Error    t value
(Intercept)  5.0054470257 2.1150503858  2.3665852
        T5S -0.8412732074 0.5068424119 -1.6598319
        AGE -0.0826516738 0.0399513567 -2.0688077
         WT  0.0009613783 0.0077887058  0.1234324
         CT  0.0002374699 0.0004633104  0.5125504

    Null Deviance: 82.56946 on 60 degrees of freedom
Residual Deviance: 71.8974 on 56 degrees of freedom
```

The overall test of H_o: all coefficients are 0 gives a P-value of 0.03. Therefore there is evidence that this model has some predictive value. The t-values for WT and CT are quite small indicating that we should be able to drop these variables from the model. Constructed variable plots were produced for all 4 explanatory variables in the above model. The plots for T5S and WT gave weak evidence that transformations may be useful and the plots for AGE and CT gave no evidence. Some transformations of T5S and WT were tried but these only had a minor impact on how well the model fitted. In all cases, WT and CT were not significant.

The variables WT and CT were dropped from the model. The S-plus output for the model that just contains T5S and AGE is:

```
Coefficients:
                  Value Std. Error   t value
(Intercept)  5.44129167 1.96937664  2.762951
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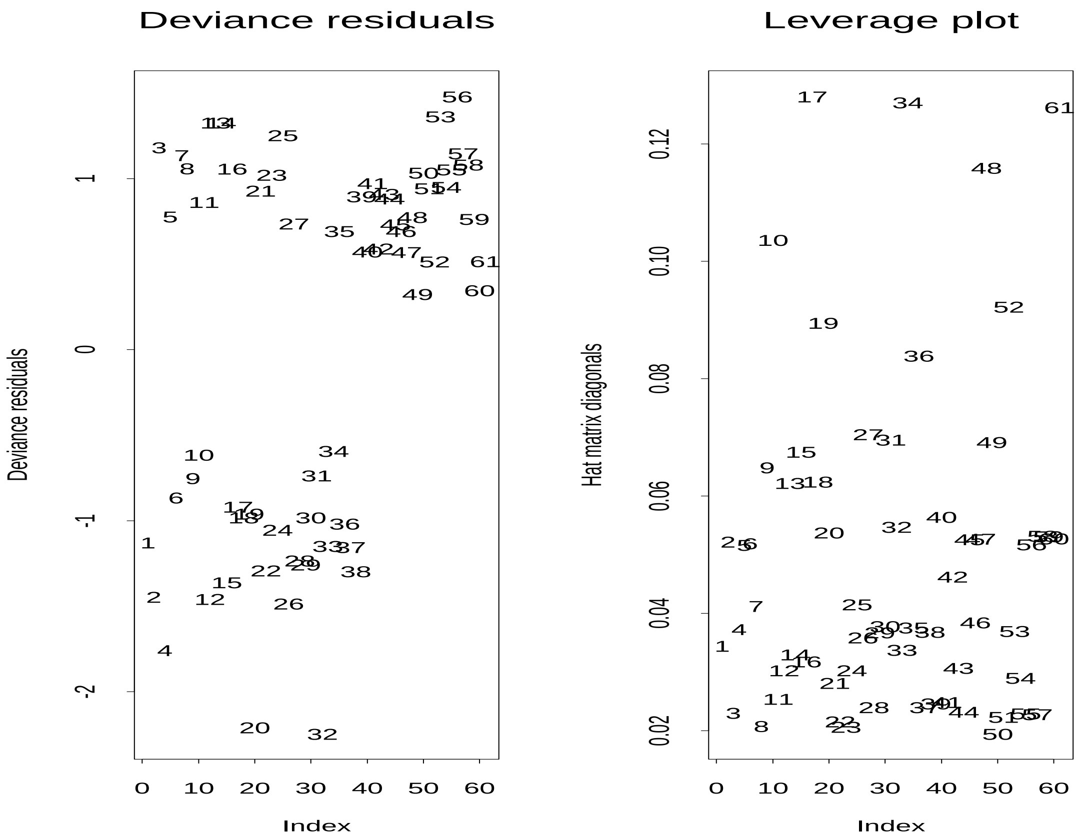

Figure 2: Plots of Deviance Residuals and Leverages

```
        T5S -0.82887716 0.49805771 -1.664219
        AGE -0.08551241 0.03890246 -2.198124

    Null Deviance: 82.56946 on 60 degrees of freedom
Residual Deviance: 72.23767 on 58 degrees of freedom
```

A plot of the deviance residuals and a plot of hat matrix diagonals indicated that none of the observations were outliers or had an usually large leverage (see Figure 2). Both the Cook's distance plot and the plot of Deviance Changes (see Figure 3) indicate that observations 20 and 32 are having an unusually large impact on the fitted model. These observations were dropped and the model re-fitted.

```
Coefficients:
                Value Std. Error   t value
(Intercept)  9.762933 2.97208691  3.284875
        T5S -1.409698 0.62955652 -2.239193
        AGE -0.155477 0.05591242 -2.780724

(Dispersion Parameter for Binomial family taken to be 1 )

    Null Deviance: 78.90332 on 58 degrees of freedom

Residual Deviance: 59.31695 on 56 degrees of freedom
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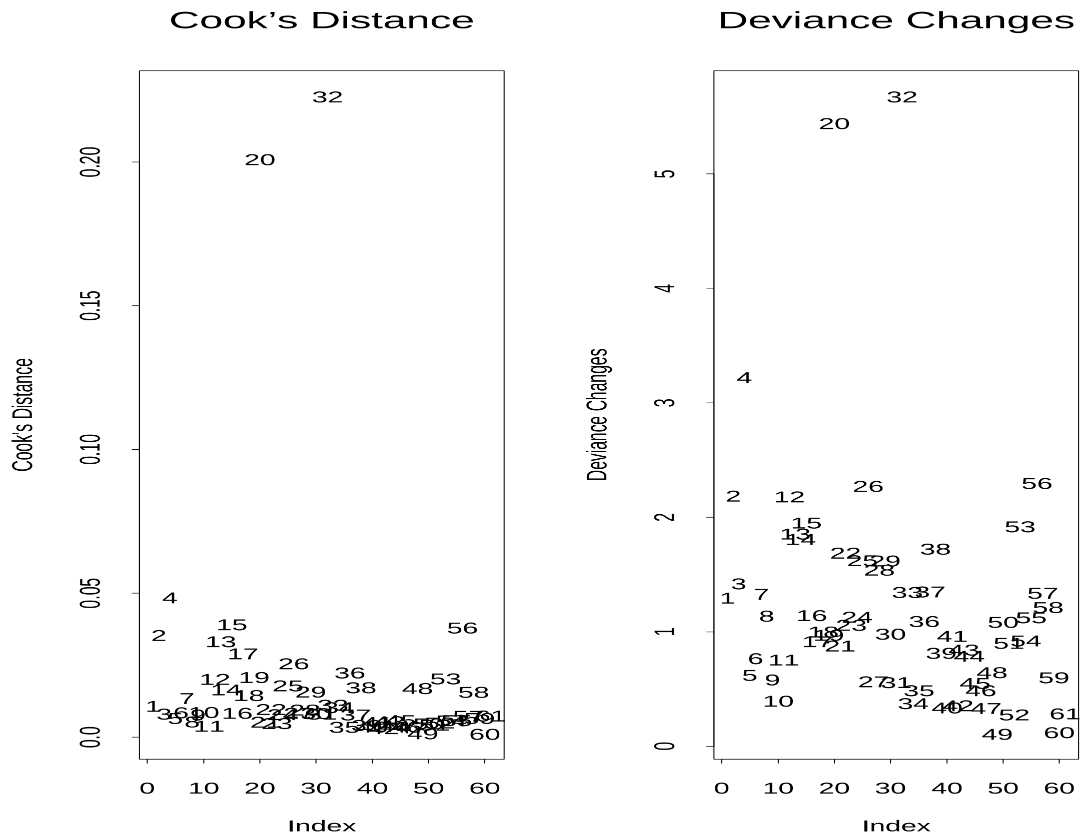

Figure 3: Plots of Cook's Distance and Deviance Changes

The magnitudes of both coefficients have increased significantly and the fit of the model is much better (the residual deviance is much smaller relative to the Null deviance for this model). Observations 20 and 32 should be investigated. I also tried fitting the model with all 4 regressors but WT and CT are still not significant.

475.330 Assignment 3; Marking Guide

This assignment asks the students to analyse data collected on heart transplant patients at Stanford. The object was to identify the factors that were important for longer survival and to discuss the practical implications of these results (they were given a few specific questions to answer).

Report on the Heart Transplant Data

(13 Marks)

This part of the assignment should summarise their findings on the factors that affect the probability that a heart transplant patient survives at least 100 days. The results should be presented in a manner that is understandable by someone who is not familiar with statistical jargon. Their report should cover the 4 questions asked in the assignment.

- 5 marks for a generally well laid out, coherent report. The reader should not have to search for the important parts among a lot of details. Give: (1) 5 marks for a clear, precise report, (2) 3 marks if the report would be difficult for a non-statistician to understand, and (3) 1 mark if it would be difficult for anyone to understand.
- 8 marks for a sensible discussion about the factors affecting the survival of heart transplant data. Look for a sensible model and a correct interpretation of that model (3 marks for this part). They should demonstrate what their model says about the survival rate of patients (i.e. give examples of how the probability of survival changes for different values of the important regressors).
  - They should find that T5S and AGE are important in predicting SUR. Some of them may have transformed T5S (any power of T5S from 1 to 4 is ok) but this should not affect the conclusions. Both T5S and AGE affect SUR as expected (increasing T5S or AGE decreases the probability of SUR). They should illustrate the affect that T5S and AGE have on the probability of SUR.
  - A few students may have included the (T5S:AGE) interaction in the model – this is fine (although they weren't expected to look for interactions) and should not affect their conclusions very much.
  - It is possible that some students may have found that the T5S:AGE:WT interaction is quite large and used it in their model. This makes the interpretation quite a bit harder – if you get any assignments like this bring them to me and I will mark them.
  - Cases 20 and 32 will show up as being influential. The students should comment on these points and the affect they have on the fitted model (they should not simply drop them).
  - There is no clear evidence of a time trend although there is a hint that the probability of survival was lower for the first few cases. It may be the learning process occurred quite quickly making it hard to detect with this type of data.

- The student's were asked to comment on a possible trade-off between T5S score and WT. Since only T5S shows up as affecting SUR, it would appear that the operation should always be postponed until a favourable value of T5S is obtained. Of course this is too simplistic since it does not take into account that a patient may die while waiting. What the data indicates is that given the patient actually has a heart transplant the length of waiting time does not appear to affect the probability of survival. So as long as the patient is not in danger of dieing, it makes sense to wait for a good match. Some students may point out that WT may be flawed as a regressor in that it may be confounded with other factors – critically ill patients may tend to have lower waiting times since it is not feasible to delay the operation. Give extra credit if they identify this.

Statistical Appendix (7 Marks)

Thus appendix should outline the reasons that they came to the conclusions they presented in the first part of the analysis. They are not required to give a detailed account of everything they did and they most certainly should not just have a whole bunch of computer output. Rather they should present a coherent account of the parts of their analysis that led them to their model for savings. For this assignment they need to discuss influence diagnostics and residual plots for their final model (including the output and plots is not absolutely necessary but it is helpful). Give: (1) 7 marks for a well documented account (2) 5-6 marks if they leave some important aspects unexplained or if they include lots of unimportant details (3) 3-4 marks if some of the conclusions they made are clearly not supported by their analysis (4) 2 or less marks if they just present output without reasons for their conclusions or if their reasons are not sensible.