Newsletter No. 68 **October 2007**

It was interesting to see how much creativity August's open-ended problem engendered. 'Does this work in the classroom?' you may ask. Well, in my experience it does. Perhaps not with all students but certainly with those who are inclined to respond less interestedly to more direct questions. They are often the same young people, incidentally, who in turn ask, 'Why are we doing all this?'

Some time ago I was trialling a series of in-depth problems in schools. One of them went like this: *At night three lighthouses are visible from a ship. Their positions are known on a map and the angles between the rays of light coming from them have been measured on board the ship. Plot the position of the ship on the map provided.*

Some teachers felt that it was unreasonable to expect students to be able to solve this open-ended problem and provided specific angles between the lighthouses. Others felt that if no specific data was given (i.e. distances, scales or angles), students would gain a greater understanding of the general problem, which relates to intersecting circles, since they would have to 'search and find' solvable cases. It is certainly true that when the problem was offered in the more open-ended form solutions handed in by students showed more mathematical insight than those where specific data was supplied.

If children don't know the problem is difficult they might find it easy.

Geoff Giles

This is the 68th edition of our newsletter and 68 is a Happy number. That is, if you sum the squares of its digits and keep repeating the process with subsequent results, eventually the number one is generated. About 12% of whole numbers are Happy. Unhappy numbers occur when the process yields an infinite loop, not the number one. 68 is also a Perrin number (see Afterthoughts below) and the atomic number of erbium.

Boris Pasternak, the Russian author, was 68 when awarded the Nobel Prize for Literature in 1958. Ogden Nash, Dmitri Shostakovich and Sir Robert Walpole died aged 68.

INDEX

Booke Review

The Canterbury Puzzles by H.E. Dudeney

Henry Dudeney (1847-1930) has been called England's greatest inventor of mathematical puzzles. We reviewed his book *Amusements in Mathematics* last month. His talent for creating fascinating puzzles and his knowledge of unusual side-paths of the subject enabled him to create some of the most ingenious mathematical and logical problems of all time.

The Canterbury Puzzles was Dudeney's first book. Although it is likely that he got the idea of presenting its puzzles not as individual problems but as incidents in connecting stories from Lewis Carroll's *A Tangled Tale* (see next month's Booke Review), it was a very unusual idea at the time. The first 31 problems, for example, are amusingly posed by the pilgrims of Chaucer's *Canterbury Tales* while others are set as part of the *Adventures of the Puzzle Club, The Strange Escape of the King's Jester* and the *Merry Monks of Riddlewell*. All the puzzles are ingenious and most are entirely original. Some require an exercise in logic, others good spatial awareness. The book includes comprehensive solutions and added analyses of mathematical aspects where appropriate. The book will keep you busy for hours.

Multiplication by Sleight of Hand by Jim Hogan

(Jim is a Secondary Mathematics Adviser and Regional Coordinator Secondary Numeracy in the Waikato area. Some of his work can be found on nzmaths. If you are near his lap top any time ask him to show you the incredible mathematical things he has there.)

This trick is quite a cool way to get students thinking about products of numbers from 5 x 5 to 10 x 10 and also to practice some place value ideas as well. Knowing the trick is 'doing numbers' but explaining how it works is 'doing mathematics'.

Hold your hands up and close each hand like a fist. That represents 5. Lift a thumb for 6, add an index finger to get 7, and so on to 8, 9 and finally a full hand representing 10.

If we want to multiply 6 x 8 we make a 6 on one hand and an 8 on the other.

Hold the free fingers together. Add these to get the 4 'tens' (the thumb on one hand and a thumb and two fingers on the other). Multiply the folded fingers left over, 4 x 2, to get 8 'ones'.

Add the 4 tens and the 8 ones to get the answer 48.

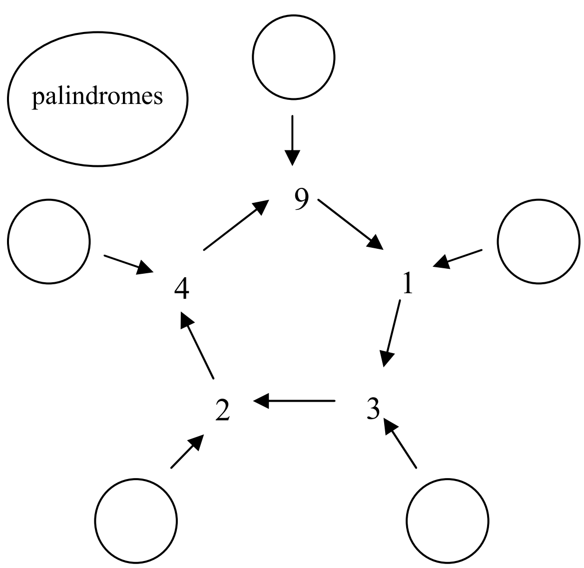
Investigate with these

(a) 7 x 7 (b) 9 x 7 (c) 8 x 6 (d) 6 x 6 (e) 7 x 6 (f) 10 x 10

Notice the last three problems have some place value ideas that need to be considered. 10 x 10 has 'no' ones.

Vive La Différence! II

Of course the trouble with 3-digit numbers is that they are larger than 2-digit numbers. Last month we looked at what happened if we took any 2-digit number, reversed it and took the small of these numbers from the larger, then did the same thing to the answer; and to that answer; and to that answer; … This only stopped if something interesting happened and fortunately it happened pretty quickly. I ended up with a diagram like the one below.



The obvious thing to do now is try it all over again for 3-digit numbers.

Take an example. It's the only way I can work with anything (unless I've done it before). If I start at 634 I get

$$\begin{array}{c} 634 \\ \underline{436} \\ 198 \end{array}$$

I'll think of the numbers as pairs, so I really started with the pair (634, 436). Now I'll spare you the pain of all the calculations but, continuing then with (891, 198) I get the following pairs in order: (693, 396), (792, 297), (594, 495), (990, 99) and back to (891, 198) again.

What can we learn from that one example? Well it looks as if we are going to find a cycle of numbers here in exactly the same way that we did for the 2-digit numbers. Here our cycle covers the five pairs (891, 198), (693, 396), (792, 297), (594, 495), (990, 99). And these are ***all*** the numbers with a 9 in the middle.

But isn't it clear that any 3-digit number when reversed and then subtracted in the canonical way, gives a 9 in the middle? Oh well, not the palindromes, of course. And so does every number, other than a cycle number, immediately latch on to one of these cycle numbers and then just cycle round?

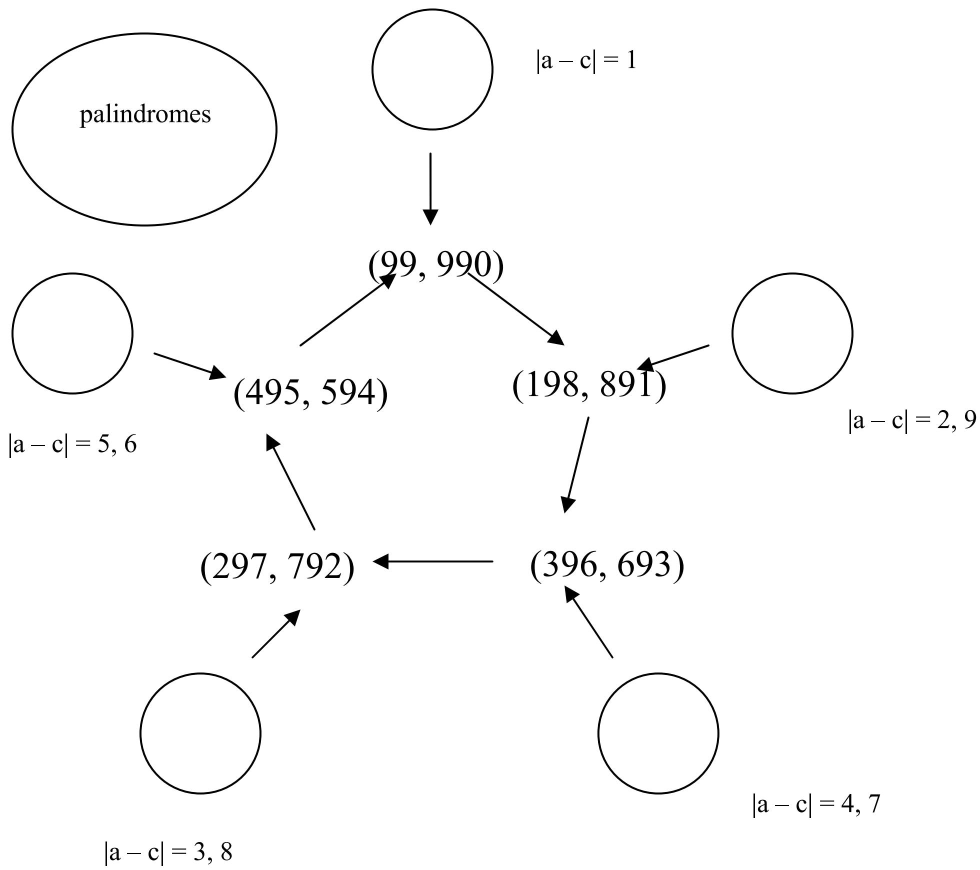
Well if that is the case we get a similar diagram to the one for 2-digit numbers. The only question that occurs to me then is, "Which numbers attach themselves to the different pairs of the cycle?" Can we say exactly who is going to land on (891, 198)?

This requires a bit of algebra I think. Take any 3-digit number abc with $a \geq c$. Then

$$abc - cba = 99(a - c).$$

Now if I want $99(a - c)$ to be 198, I need $a - c$ to be 2. Clearly there are a lot of ways to do this, though $a = 4$ and $c = 2$ will do. (And it doesn't matter at all what b is.) On the other hand if I want $99(a - c)$ to be 891, then $a - c$ would need to be 9. Hmm. Not so many ways to do this.

I should be able to decide what numbers go where. Can you check to see if I have got the 3-digit case right? (see diagram on the next page)



You can see that I've got a never ending series going here. Next month I'll do 4-digit numbers, then 5-digit ones and so on for quite some time.

Solution to September's problem

Last month we invited you to write a Limerick for us. We had no difficulty judging the results. So here is one to show that you could all have done better.

There once were some young men, you'd fear 'em,
When doing geometry just don't go near 'em.
The sides they confuse,
With the hypotenuse,
In their use of Pythagr's's Theorem.

Come on, you can do better than that in the break. Let's have something in for next month.

This Month's Problem

The ABC Computer Company offers its 350 employees a bonus of $1000 to each man and $835 to each woman. All the women accept but a certain proportion of the men refuse (they object to the women's bonus being less than theirs - no chauvinism with them!).

If, knowing what this proportion is, the total amount paid out in bonuses can be calculated, what is the total amount paid out to the women?

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

More on Those Sequences

The good old 3, 1, 4, 1, 5, … sequence won't go away. Here are two more from Richard Catterall of Hutt International Boys' School. He says:

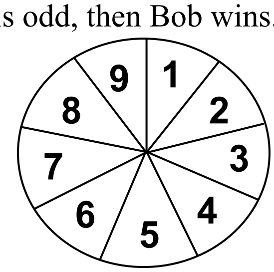
Two sequences have just occurred to me;

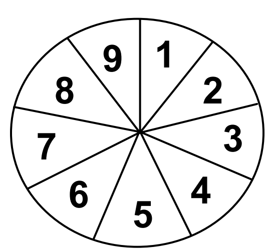
3 1 4 1 5 9 2 3 5 8 1 4 5 7 9 0 1 3 4 5 6 1 …
This one will not go on for ever; but has a whole family of possibilities because the rule is to write a smaller positive integer, then a larger, a smaller, then two consecutive larger, a smaller, three consecutive larger, and so on. If I moved on to double digits or more, and wandered into negatives there would be an explosion of possibilities.

3 1 4 1 5 9 6 1 7 1 8 9 9 1 1 0 1 1 1 3 …
Here I have a family of primes, 31, 41, 59; 61, 71, 89; …using the first with each consecutive starting digit(s), unless that is not prime, in which case I use the last. There are several possibilities for when we get to a starter without a prime; perhaps you would like to explore that.

Solution to October's Junior Problem

Last month we had this problem.
Ann and Bob's Game: Ann and Bob take it in turns to spin both of the spinners shown below. The numbers that come up are added. If the result is even, then Ann wins; if the result is odd, then Bob wins.





Is this a fair game? If it is, why is it fair? If it isn't, why isn't it and who is better off as a result?

If the size of the problem was too much, look at something smaller. If we had only 1 number on each spinner, then the only sum that would come up would be 2. That's even so Anne will always win. This is certainly not a fair game.

So suppose that each spinner had 1 and 2 on and that each number had equal space. Then you can get a few more possibilities. I list them in the table below.

	1	2
1	2	3
2	3	4

On average we'd expect 2 to come up once in every four spins; 3 to come up twice in every four spins; and 4 to come up once in every four spins. So odds and evens come up equally often and the game is fair.

But if we have 1, 2, 3 on both spinners it's not fair again. I'll do the table below and then you can easily see that there are slightly more even occurrences.

	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

But if you think about it the 3-spinner case has to be unfair. There are 3 x 3 different outcomes, and 9 is odd. So you can't have an equal number of occurrences of odd an even. And the same thing is true for the 9-spinner case that we started with. There are 81 outcomes (not all of which have the different sums) which is odd. So the game can't be fair. If you draw up the table, then you'll see that Anne has a slight advantage.

You might like to think about how you could make Anne and Bob's game fair. One way would be to divide the spinners in half and put the even numbers on one side and the odd numbers on the other. It doesn't matter how much room you give to each individual number you'll get odd and even sums equally often. This is because you are essentially putting 1 on one half of the spinners and 2 on the other.

What happens with three spinners or four or 399?

This Month's Junior Problem.

This section contains a monthly problem competition for students up to Year 8 with a $20 book voucher available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so that we can send you the voucher if you are the winner.

All of the different letters in the following correct multiplication represent different digits. Which letter is which digit?

$$\begin{array}{r} \text{TWO} \\ \underline{\times\ \ \text{TWO}} \\ \text{THRE E} \end{array}$$

Afterthoughts

(1) **The Perrin sequence** of numbers is an analogue of the better-known Fibonacci sequence. The latter begins 1, 1 with subsequent terms being the sum of the previous two. The Fibonacci sequence continues; 2, 3, 5, 8, 13, …. The Perrin sequence begins 3, 0, 2 with subsequent terms being the sum of the two terms immediately before the last one. It continues; 3, 2, 5, 5, 7, 10, ….. You might like to investigate the contexts in which these and similar sequences arose.

(2) Here is a mathematical explanation of how it all works

Xin Xin's Solution

Let X be a number from the set {5, 6, 7, 8, 9, 10}. Let Y be another number and from the same set. (In the figure, X is 6 and Y is 8.) Then X – 5 and Y – 5 represent the free figures so 10(X – 5) + 10 (Y – 5) is the number of tens you get from the free fingers.

There are 10 – X and 10 – Y folded fingers. Multiply these to give (10 – X)(10 – Y).

So the count we have made is

$$\begin{aligned} &10(X-5) + 10(Y-5) + (10-X)(10-Y) \\ &\qquad = 10X - 50 + 10Y - 50 + 100 - 10X - 10Y + XY \\ &\qquad = XY \end{aligned}$$

which is the product of X and Y as required.

But then, of course we have to ask, why does this only work for X and Y between 5 and 10 inclusive? Can we force it to work in these cases too?