# Introductory Statistics Tutorial Answers
# Chapter 10 – Data on a Continuous Variable

**Section A: Paired Comparisons or Two Independent Samples**

**1.** **(a)** Paired data.
**(b)** Two independent samples.
**(c)** Paired data.

**2.** **(a)** A *t*-test on the differences. A pair of observations is made on the same subject so this is paired comparison data. A *t*-test on the differences is more appropriate.

**(b)** $H_0$: $\mu_{\text{Diff}} = 0$ vs $H_1$: $\mu_{\text{Diff}} \neq 0$.
*P-value* = 0.033
We have some evidence against there being no difference between the mean amounts of current and previous spending. It appears that, on average, access to the cable network increases spending in viewers. With 95% confidence, we estimate that viewers spent, on average, between \$3.10 and \$62.50 more when they had access to the cable network.

**(c)** Since a *t*-test on the differences is used we look at the dot plot, Normal probability plot and *W*-test on the differences. The dot plot shows moderate positive skewness but the *t*-test is robust to such departures from Normality. The *W*-test for Normality has a *P-value* greater than 10% so it is believable that the data has an underlying Normal distribution. The results of the *t*-test should be valid in this situation.

**(d)** **Hypotheses:** $H_0 : \tilde{\mu}_{Diff} = 0$ vs $H_1 : \tilde{\mu}_{Diff} \neq 0$
**Signs of the differences:** + - + + - 0 + - + + - + - + + +
10 positive signs, 5 negative signs, 1 zero.
Since this is a two-tailed test, evidence against the null hypothesis will be provided by either a large number of observations above the hypothesised median of zero (and therefore a small number of observations below zero) or a small number of observations above the hypothesised median of zero (and therefore a large number of observations below zero).
***P-value*** = 2 x pr($Y \geq 10$) by considering positive signs, or
2 x pr($Y \leq 5$) by considering negative signs, where $Y \sim$ Binomial ($n = 15$, $p = 0.5$)
= 2 x 0.1509
= 0.3018
**Interpretation:** We have no evidence against the null hypothesis. We have no evidence to suggest that the median spending changes when the viewers have access to the cable network.

**(e)** The *t*-test is more appropriate. When the assumptions of the *t*-test are met reasonably well choose a *t*-test in preference to a nonparametric test. This is because a *t*-test will more readily detect departures from the null hypothesis when these departures do exist (That is, the *t*-test is more powerful than the Sign test).
Note that the *t*-test gave a *P-value* of 3.3% while the Sign test gave a *P-value* of 30.2%.

**3.** They are less sensitive to outliers and do not assume any particular underlying distribution for the observations.



**4.** **(a)** A one-tailed test is appropriate because, prior to the collection of the data, there was a strong reason (from the customer's complaints) for believing that the average was lower than the claimed 1.8 hours. (See page 380 of *Chance Encounters*.)
**(b)** The dot plot shows the presence of a possible outlier. The *t*-test is sensitive to the presence of outliers, especially with a small sample size of 11. Because the Normality assumption is not satisfied a nonparametric test should be used.
**(c)** $H_0 : \tilde{\mu} = 1.8$ vs $H_1 : \tilde{\mu} < 1.8$.
**(d)** *P-value* = 0.1719
**(e)** We have no evidence against the null hypothesis. We have no evidence that the median operating time for hedge trimmers was less than 1.8 hours.
**(f)** A high number of observations below the hypothesised median (and therefore a small number of observations above the hypothesised median) provide evidence against the null hypothesis.
*P-value* = pr($Y \leq 3$) by considering positive signs, or
pr($Y \geq 7$) by considering negative signs, where $Y \sim$ Binomial ($n = 10$, $p = 0.5$).

**Section B: More Than Two Independent Samples**

**1.** **(a)** When we want to investigate differences between the underlying means of more than two groups.
**(b)** $H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (The population means are all equal.)
$H_1$: At least one of the population means is different from the other three.

**(c)** Assumption: The samples are independent of each other.
Check: Ensure independence in the design of the experiment or study – read the story.
Assumption: The underlying distribution of each group is Normally distributed.
Check: By plotting the data. The choice of plot will depend on the sample sizes.
Assumption: The population standard deviations of each group are equal.
Check: By plotting the data and/or looking at the sample standard deviations (We require that the ratio of the largest sample standard deviation to the smallest sample standard deviation is less than 2.).

**(d)** $f_0 = \dfrac{s_B^2}{s_W^2}$

**(e)** $df_1 = k - 1$
$df_2 = n_{\text{tot}} - k$

**(f)** $s_B^2$ is called the between mean sum of squares and it measures the variability between the sample means.
$s_W^2$ is called the within mean sum of squares and it measures the average variability within the samples (that is, the internal variability within the samples themselves).

2. **(a)** The times for the Neither group are centred higher and more spread out than those for the Drug and Placebo groups. There does not appear to be a great difference between the centre and spread of times for the Drug and Placebo groups. There are signs of moderate positive skewness in all three of the samples.

**(b)** There are three independent random samples.
Though there are signs of moderate positive skewness in all three of the samples, with the equal sample sizes and the moderate size of the three samples this should not cause any concern with the validity of the *F*-test.
The assumption of equality of the standard deviations of each underlying distribution is suspect because the sample standard deviation of the Neither group is larger than the sample standard deviations of the Drug and Placebo groups. However this difference is not large enough to cause concern with the assumption because the *F*-test is quite robust with respect to this assumption. Note that the ratio of the largest sample standard deviation to the smallest standard deviation is less than 2 ( $\frac{23.14}{15.22} = 1.52$ ).

**(c)** $H_0$: $\mu_{\text{Drug}} = \mu_{\text{Neither}} = \mu_{\text{Placebo}}$, where $\mu_{\text{Drug}}$, $\mu_{\text{Neither}}$ and $\mu_{\text{Placebo}}$ are the mean number of minutes for patients to fall asleep in the Drug, Neither and Placebo groups respectively. That is, the mean number of minutes to fall asleep is the same for each of the three groups.

$H_1$: At least one of the three underlying means is different from the other two.

**(d)** ANOVA table:

| | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Treatment | **2** | 3330 | 1665 | **4.83** | 0.011 |
| Error | **72** | 24868 | 345 | | |
| Total | **74** | 28198 | | | |

**(e)** The *P-value* of 0.011 means that we have strong evidence against the null hypothesis. We have strong evidence that at least one of the groups has a different underlying mean number of minutes for people to fall asleep.

**(f)** With 95% confidence we estimate that the mean time for people taking the placebo to fall asleep is somewhere between 8.9 minutes shorter and 16.2 minutes longer than the mean time for people taking the drug to fall asleep.

**(g)** **(i)** Yes. The 'Neither' level of treatment.
**Note:** Zero is not in the confidence interval for $\mu_{\text{Drug}} - \mu_{\text{Neither}}$ so we have evidence of a real difference in these underlying means. Zero is in the confidence interval for $\mu_{\text{Neither}} - \mu_{\text{Placebo}}$, but only just. The *P-value* for a test for no difference between $\mu_{\text{Neither}}$ and $\mu_{\text{Placebo}}$ would be only just greater than 5%. So we have some evidence of a real difference between these underlying means.

**(ii)** No. We have no evidence of a real difference between the underlying means of the Drug and Placebo groups.



3. **(a)** Plot the data.
**(b)** It is the quickest way to see what the data say.
It often reveals interesting features that were not expected.
It helps prevent inappropriate analyses and unfounded conclusions.
Plots also have a central role in checking the assumptions made by formal methods.

4. All observations in the sample are independent and are from the same distribution.

5. **(a)** (i) and (iv)
**(b)** (iv)
**(c)** (i) [in that the underlying distribution of the differences is assumed to be Normal] and (iii)
**(d)** (i), (ii) and (iv)
**(e)** (i), (ii), (iv) and (v).

**Section C: Identifying Appropriate Type of Analysis**

1. **Scenario 1**
**(i)** Exam – quantitative, Attend – qualitative.
**(ii)** The scenario is exploring a relationship (between Exam and Attend).
**(iii)** Side-by-side dot plot or box plot on the same scale.
**(iv)** **D:** Two-sample *t*-test on a difference between two means

**Scenario 2**
**(i)** Pass – qualitative.
**(ii)** The scenario is not exploring a relationship.
**(iii)** One-way table of counts (frequency table) or bar graph comparing the counts or proportion who pass and fail.
**(iv)** **B:** One-sample *t*-test on a proportion

**Scenario 3**
**(i)** Assign – quantitative, Test – quantitative.
**(ii)** The scenario is exploring a relationship (between Assign and Test).
**(iii)** Scatter plot of Test against Assign (**<u>or</u>** dot plot, box plot, stem-and-leaf plot or histogram of the differences).
**(iv)** **C:** One-sample *t*-test on a mean of differences / Paired-data *t*-test

**Scenario 4**
**(i)** Exam – quantitative, Degree – qualitative.
**(ii)** The scenario is exploring a relationship (between Exam and Degree).
**(iii)** Side-by-side dot plot or box plot on the same scale.
**(iv)** **F:** *F*-test for one-way analysis of variance

**Scenario 5**
**(i)** Pass – qualitative, Attend – qualitative.
**(ii)** The scenario is exploring a relationship (between Pass and Attend).
**(iii)** Two-way table of counts (frequency table) or bar graphs of the counts or proportion who pass and fail for <u>each</u> of the groups, attending and non-attending.
**(iv)** **E:** Two-sample *t*-test on a difference between two proportions

2. **(a)** Two-sample *t*-test on a difference between two proportions
**(b)** *F*-test for one-way analysis of variance
**(c)** Two-sample *t*-test on a difference between two means
**(d)** One-sample *t*-test on a mean of difference / Paired-data *t*-test