Introductory Statistics Tutorial
Chapter 7 – Sampling Distributions of Estimates

1. A random sample of size n is drawn from a population with mean, μ, and standard deviation, σ. Let $\overline{X}$ be the sample mean.

(a) What is the:

(i) mean of $\overline{X}$?

(ii) standard deviation of $\overline{X}$?

(b) If we are sampling from a Normal distribution then $\overline{X}$ is **exactly / approximately** (circle **one**) Normally distributed.

(c) **(i)** If we are sampling from a non-Normal distribution then for large samples (ie, n is large) $\overline{X}$ is **exactly / approximately** (circle **one**) Normally distributed.

(ii) The result in **(i)** is called the

2. A random sample of size n is drawn from a population in which a proportion p has a characteristic of interest. Let $\hat{P}$ be the sample proportion.

(a) What is the:

(i) mean of $\hat{P}$?

(ii) standard deviation of $\hat{P}$?

(b) For large samples $\hat{P}$ is
exactly / approximately (circle **one**) Normally distributed.

3. **(a)** A __________________________________ is a numerical characteristic of a population.

(b) An _____________________________________ is a known quantity calculated from data in order to estimate an unknown parameter.

4. Suppose that $X_1, X_2, \ldots, X_{16}$ is a random sample from a Normal distribution with mean of 50 and a standard deviation of 10. Then the distribution of the sample mean $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_{16}}{16}$ has mean, $\mu_{\overline{X}}$, and standard deviation, $\sigma_{\overline{X}}$, given by:

(1) $\mu_{\overline{X}} = 50, \quad \sigma_{\overline{X}} = 6.25$

(2) $\mu_{\overline{X}} = 50, \quad \sigma_{\overline{X}} = 0.625$

(3) $\mu_{\overline{X}} = 800, \quad \sigma_{\overline{X}} = 10$

(4) $\mu_{\overline{X}} = 50, \quad \sigma_{\overline{X}} = 2.5$

(5) cannot be determined because $n = 16$ is too small for the central limit effect to take effect.

5. The fuel consumption, in litres per 100 kilometres, of all cars of a particular model has mean of 7.15 and a standard deviation of 1.2. A random sample of these cars is taken.

Calculate the mean and standard deviation of the sample mean if:

(a) one observation is taken.

(b) four observations are taken.

(c) sixteen observations are taken.

6. About 65% of all university students belong to the student loan scheme. Consider a random sample of 50 students. Let $\hat{P}$ be the proportion of these 50 students who belong to the student loan scheme.

(a) In words, describe p.

(b) State the distribution of $\hat{P}$.

(c) What is the probability that the sample proportion is more than 70%? Use the output below to help you.

Cumulative Distribution Function

Normal with mean = 0.650000 and standard deviation = 0.0674537

x	P(X <= x)
0.4000	0.0001
0.4500	0.0015
0.5000	0.0131
0.5500	0.0691
0.6000	0.2293
0.6500	0.5000
0.7000	0.7707
0.7500	0.9309

7. The owner of a large fleet of courier vans is trying to estimate her costs for next year's operations. Fuel purchases are a major cost. A random sample of 8 vans yields the following fuel consumption data (in km/L):

10.3 9.7 10.8 12.0 13.4 7.5 8.2 9.1

Summary statistics: $n = 8$, $\bar{x} = 10.125$, $s = 1.9477$

Assume that the distribution of fuel consumption of the vans is approximately Normal.

Construct a two-standard-error interval for the mean fuel consumption of all of her vans.

8. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size n from a distribution with mean μ and standard deviation σ. Let $\bar{X}$ represent the sample mean.

Which **one** of the following statements is **false**?

(1) Since $\text{E}(\bar{X}) = \mu$, $\bar{x}$ is an unbiased estimate of μ.

(2) If X is Normally distributed then $\dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ is a random variable with a Student's t-distribution with parameter degrees of freedom, df, where $df = n - 1$.

(3) If X is Normally distributed then $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is a random variable with a standard Normal distribution.

(4) The Central Limit Theorem says that for large samples from a non-Normal distribution, the distribution of $\dfrac{\bar{X} - \mu}{\text{sd}(\bar{X})}$ is approximately the standard Normal distribution.

(5) The Student's t-distribution with parameter degrees of freedom, df, shows increased variability as df increases.