

## Introductory Statistics Tutorial

### Chapter 9 – Significance Testing: Using Data to Test Hypotheses

#### Section A: Quiz

1. What is the difference between the null hypothesis and the alternative hypothesis?
  
2. List the three alternative hypotheses that are possible when testing the null hypothesis  $H_0 : \theta = \theta_0$ .  

(a)
(b)
(c)
3. On what basis do we decide whether to do a one-tailed or a two-tailed test?

4. The formula for the  $t$ -test statistic is  $t_0 = \frac{\hat{\theta} - \theta_0}{\text{se}(\hat{\theta})}$ .

Write this formula in words.

5. Fill in the gaps in this description of the  $P$ -value from a  $t$ -test:

The  $P$ -value is the probability that, if the \_\_\_\_\_ was true, sampling variation would produce an estimate that is at least as far away from the hypothesised value as our data estimate.

6. Fill in the gap.

The  $P$ -value measures the \_\_\_\_\_ of evidence against the null hypothesis.

7. In the table below, fill in the descriptions used for the given  $P$ -values:

$P$ -value	Evidence against $H_0$
$> 0.12$	
$\approx 0.10$	
$\approx 0.05$	
$\approx 0.01$	
$\leq 0.001$	

8. What does the  $P$ -value tell us about the size of the effect?
  
9. What tool do we use to tell us about the size of an effect?
  
10. If a test is significant at the 5% level, what does this tell you about the  $P$ -value?

**Section B: Doing Tests by Hand**

1. Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. The results follow.

Variable		Prisoners with tuberculosis	Total number of prisoners
Gender	Male	556	984
	Female	36	90
Race	White	496	886
	Gypsy	74	152
	Other	22	36
Intravenous Drug Users	Yes	361	629
	No	231	445
HIV Positive	Yes	186	294
	No	406	780
Re-imprisonment	Yes	272	456
	No	320	618

Is there any evidence to suggest that the race of the prisoner (White or Gypsy) makes any difference to whether they contracted tuberculosis? Carry out a significance test to answer this question and then calculate an appropriate 95% confidence interval.

Let  $p_W$  be the proportion of White prisoners infected with TB and  $p_G$  be the proportion of Gypsy prisoners infected with TB.

- (a) Identify the parameter  $\theta$ .
  
  
- (b) State the hypotheses.
  
  
- (c) Write down the estimate and its value.

- (d) Calculate the value of the  $t$ -test statistic.

- (e) Find the  $P$ -value.

- (f) Interpret the  $P$ -value.

- (g) Answer the original question.

- (h) Calculate a 95% confidence interval for the parameter.

- (i) Interpret the 95% confidence interval.

### Section C: Interpreting Output and Interpretation Issues

1. A businessperson is interested in buying a coin-operated laundry and has a choice of two different businesses. The businessperson is interested in comparing the average daily revenue of the two laundries, so she collects some data. A simple random sample for 50 days from the records for the past five years of the first laundry and a simple random sample for 30 days from the records for the past three years of the second laundry reveal the following summary statistics:

	Sample size	Sample mean	Sample standard deviation
Laundry 1	50	\$635.40	\$71.90
Laundry 2	30	\$601.60	\$77.70

#### MINITAB output

#### Two Sample T-Test and Confidence Interval

Two sample T for Laundry1 vs Laundry2

	N	Mean	StDev	SE Mean
Laundry1	50	635.4	71.9	10
Laundry2	30	601.6	77.7	14

95% CI for mu Laundry1 - mu Laundry2: ( -1, 69)

T-Test mu Laundry1 = mu Laundry2 (vs not =): T = 1.94 P = 0.057 DF = 57

- State the parameter used in this analysis.
- State the hypotheses used in this  $t$ -test.
- Write down the estimate and its value.
- Write down the value of the test statistic.
- Interpret the test.

- Interpret the confidence interval.

- If this analysis were done by hand the value of  $df$  would have been 29. Why does the output show that  $df = 57$ ?

2. Which **one** of the following statements regarding significance testing is **false**?

- A highly significant test result means that the size of the difference between the estimated value of the parameter and the hypothesised value of the parameter is significant in a practical sense.
- A  $P$ -value of less than 0.01 is often referred to as a highly significant test result.
- A nonsignificant test result does not necessarily mean  $H_0$  is true.
- A two-tailed test of  $H_0: \theta = \theta_0$  is significant at the 5% significance level if and only if  $\theta_0$  lies outside a 95% confidence interval for  $\theta$ .
- Testing at the 5% level of significance means that the null hypothesis is rejected whenever a  $P$ -value smaller than 5% is obtained.

3. Which **one** of the following statements is **false**?

- In hypothesis testing, a nonsignificant result implies that  $H_0$  is true.
- In hypothesis testing, a two-tailed test should be used when the idea for doing the test has been triggered by the data.
- In surveys, the nonsampling error is often greater than the sampling error.
- Larger sample sizes lead to smaller standard errors.
- In hypothesis testing, statistical significance does not necessarily imply practical significance.

4. Which **one** of the following statements regarding significance testing is **false**?

- Formal tests can help determine whether effects we see in our data may just be due to sampling variation.
- The  $P$ -value associated with a two-sided alternative hypothesis is obtained by doubling the  $P$ -value associated with a one-sided alternative hypothesis.
- The  $P$ -value says nothing about the size of an effect.
- The data should be carefully examined in order to determine whether the alternative hypothesis needs to be one-sided or two-sided.
- The  $P$ -value describes the strength of evidence against the null hypothesis.

