475.102 SC
475.107 ST
475.108 SC

EXAMINATION FOR BA BCom BSc ETC 2000

STATISTICS

VERSION 1

Introduction to Statistics
Statistics for Social Science
Statistics for Science & Technology
Statistics for Commerce

(Time allowed: THREE hours)

INSTRUCTIONS:

* **This examination consists of 65 multiple-choice questions.**
* **All questions have a single correct answer.**
* **If you give more than one answer to any question, you will receive zero marks for that question.**
* **No mark is deducted for an incorrect answer.**
* **All questions carry the same mark value.**
* **Answers must be written on the special answer sheet provided.**
* **Calculators are permitted.**

INCLUSIONS:

* **Appendix A: Car Data,** for use in Questions **41 to 65.**
* **Formulae Appendix**

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Questions 1 to **4** refer to the following information.

Sports Foundation grants for sports which won the right to represent New Zealand at the Sydney Olympics are shown in Table 1 below.

No.	Sport	1998–1999	1999–2000	2000–2001
01	Archery	$16,500	$15,000	$25,000
02	Athletics	$515,340	$485,400	$299,000
03	Basketball	$133,100	$90,000	$40,000
04	Boxing	$166,950	$55,000	$44,350
05	Cycling	$678,500	$747,182	$688,140
06	Equestrian	$691,000	$717,000	$558,620
07	Gymnastics	$94,500	$34,500	$22,400
08	Hockey	$498,500	$478,460	$554,000
09	Judo	$153,650	$93,179	$124,500
10	Rowing	$533,100	$466,700	$707,265
11	Shooting	$327,000	$106,000	$405,616
12	Softball	$251,913	$425,259	$254,542
13	Swimming	$431,470	$205,000	$280,594
14	Table Tennis	$26,250	$3,000	$29,000
15	Triathlon	$343,110	$548,255	$86,300
16	Weightlifting	$98,900	$48,125	$79,500
17	Wrestling	$13,520	$8,000	$15,000
18	Yachting	$947,000	$1,131,000	$622,356

Table 1: Sports Foundation Grants

1. Suppose the purpose of this table was to convey the information so that the reader could make visual comparisons between different sports with respect to the size of the grant awarded. One change in the presentation of the data which would **not** be an improvement would be to:

(1) interchange the rows and columns in the table.

(2) round all grants to the nearest thousand dollars.

(3) list the sports in order of the amount of the grant received in the year 2000-2001.

(4) add a column on the right of the table for the 'Average Amount Awarded per Year (1998–2001)'.

(5) add a row at the bottom of the table for the 'Average Amount Awarded per Sport'.

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2. Figure 1 is a dot plot of the grants for each of the 18 sports in the year 2000–2001.

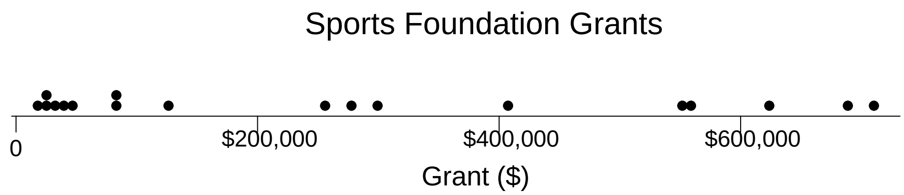

Figure 1: Sports Foundation Grants, 2000–2001

A **better** graph to highlight the difference between the grants obtained by different sports would be:

(1) side-by-side box plots with the same scaled x-axes.

(2) a labelled bar graph ordered by the size of the grant.

(3) a histogram with equal width class intervals for **Grants** on the x-axis.

(4) a scatter plot with **Grants** as the response variable and **Sport** as the explanatory variable.

(5) a pie chart with the sectors labelled and ordered by the size of the grant.

3. Suppose we wish to randomly select five of the sports listed in Table 1. The method for randomly selecting the five sports uses the number (in the **No.** column of Table 1) associated with each sport and random number digits. Use the row of random digits below to select a simple random sample of five sports. You must start at the beginning of the row and use consecutive pairs of digits.

```
09874    11018    39090    54804    17130
```

The five sports selected are:

(1) Judo, Rowing, Yachting, Judo, Cycling

(2) Judo, Shooting, Archery, Wrestling, Swimming

(3) Judo, Shooting, Archery, Cycling, Wrestling

(4) Judo, Rowing, Yachting, Cycling, Boxing

(5) Judo, Hockey, Gymnastics, Boxing, Archery

4. The five sports that received the highest grants in the year 2000–2001 are given in Table 2 below.

Sport	**Grant**
Rowing	\$707,265
Cycling	\$688,140
Yachting	\$622,356
Equestrian	\$558,620
Hockey	\$554,000

Table 2: Highest Five Grants Awarded, 2000–2001

The sample mean, $\overline{x}$, and sample standard deviation, s, for these five sports are:

(1) $\overline{x} = \$626,076$ $\quad s = \$71,068$

(2) $\overline{x} = \$622,356$ $\quad s = \$71,068$

(3) $\overline{x} = \$622,356$ $\quad s = \$63,565$

(4) $\overline{x} = \$622,000$ $\quad s = \$71,068$

(5) $\overline{x} = \$626,076$ $\quad s = \$63,565$

5. Let X and Y be independent continuous random variables. The mean of X is 5 and the standard deviation of X is 3. The mean of Y is -4 and the standard deviation of Y is 2. The mean, μ_W, and standard deviation, σ_W, of $W = 3X - 2Y$ are given by:

(1) $\mu_W = 23,$ $\quad \sigma_W = 9.85$

(2) $\mu_W = 23,$ $\quad \sigma_W = 8.06$

(3) $\mu_W = 7,$ $\quad \sigma_W = 8.06$

(4) $\mu_W = 23,$ $\quad \sigma_W = 97$

(5) $\mu_W = 7,$ $\quad \sigma_W = 9.85$

6. A recent study looked at the type of school a student attends and that student's performance in bursary examinations. It was found that students at single-sex schools perform significantly better in examinations than those students at co-educational schools. The results from this study alone should not be used to argue the case that single-sex schooling, generally, gives rise to better student performance in examinations **mainly** because:

(1) there may be a difference between male and female performance in examinations.

(2) there are many more co-educational schools than single-sex schools in New Zealand.

(3) the designers of this study would not have been able to use any form of blinding.

(4) the designers of this study have not used a control group.

(5) the designers of this study did not allocate each student to the school attended.

7. For which **one** of the following experiments is the Binomial distribution an appropriate model for the distribution of X?

(1) An archer in an archery competition has 10 shots aiming to hit the bulls-eye (ie the centre of the target). The archer uses the outcome of a shot to adjust her sights, if necessary, for the next shot. X is the number of times the archer hits the bulls-eye in 10 shots.

(2) A student studies the Poisson distribution using computer-assisted instruction. The computer then presents 10 problems which are of differing degrees of difficulty. X is the number of problems that the student solves correctly.

(3) In total for this year, 2643 students enrolled in a Stage I statistics paper. 993 of these students are BCom students. 100 students are randomly selected from these 2643 students. X is the number of BCom students in this random sample of 100 Stage I statistics students.

(4) A consignment of 15 boxes contains 3 boxes which are damaged. A simple random sample of 10 boxes is selected from the consignment of 15 boxes. X is the number of damaged boxes in the sample of 10.

(5) A record is kept in Auckland each day as to whether rain has fallen or not. X is the number of wet days in a calendar month.

Questions 8 to **10** refer to the following information.

Earlier this year it was estimated that 53.3% of Internet-users in New Zealand are male (compared with 61% in the UK and 49% in the USA). A group of 30 New Zealand Internet-users meets regularly. Assume that this group is a random sample of New Zealand Internet-users and that 53.3% is a reliable estimate.

Use the MINITAB output in Table 3 to answer Questions 8 and 9.

```
Binomial with n = 30 and p = 0.5330
```

x	Pr(X=x)	Pr(X<=x)
11.00	0.0281	0.0501
12.00	0.0507	0.1008
13.00	0.0802	0.1810
14.00	0.1111	0.2922
20.00	0.0508	0.9521
21.00	0.0276	0.9797
22.00	0.0129	0.9926
23.00	0.0051	0.9977

Table 3: MINITAB output for Binomial distribution

8. The probability that 13 or 14 of this group are male is approximately:

(1) 0.1913
(2) 0.4732
(3) 0.1111
(4) 0.1802
(5) 0.0802

9. The probability that at least 12, but less than 22, of this group are male is approximately:

(1) 0.9425
(2) 0.8116
(3) 0.8789
(4) 0.9296
(5) 0.8918

10. Throughout New Zealand there are 53 such groups — each group has 30 members. Assume that each group is a random sample of New Zealand Internet-users and that the groups themselves are independent of each other. The distribution of the average (mean) number of males per group is:

(1) approximately Binomial, with a mean of 15.99, and a standard deviation of 0.3754.

(2) approximately Normal, with a mean of 15.99, and a standard deviation of 0.3754.

(3) approximately Binomial, with a mean of 15.99, and a standard deviation of 2.733.

(4) approximately Normal, with a mean of 15.99, and a standard deviation of 2.733.

(5) approximately Binomial, with a mean of 116.41, and a standard deviation of 2.733.

11. For which **one** of the following experiments does the Poisson distribution have the **best** potential as an appropriate model for the distribution of X?

(1) There are 1.3 million people in New Zealand who have access to the Internet. In April, it was estimated that 635,000 of these people logged-on to the Internet. X is the percentage of New Zealanders with Internet access who log-on in any given month.

(2) The average time spent Internet-surfing in April was 8hr 46min 29sec for males and 6hr 0min 7sec for females. X is the time a randomly chosen Internet-user spends Internet-surfing per month.

(3) The average time spent during a single Internet-surfing session is 30min 2sec. X is the time spent during a randomly chosen Internet-surfing session.

(4) A 'typical' Internet-user accesses the Internet, on average, 15 times per month. X is the number of times this Internet-user accesses the Internet in a randomly chosen month.

(5) Internet-users visited, on average, 19 unique sites in April. Ten of these Internet-users are selected at random. X is the number of users in this group of ten who visited more than 19 unique sites.

12. Which **one** of the following statements is **false**?

(1) Blinding and double blinding are techniques often used by researchers when people are used as experimental units.

(2) Blocking is used in experiments to ensure fair comparisons with respect to factors the experimenter believes are important.

(3) In an experiment, the control group always receives no treatment.

(4) The placebo effect is the response caused in human subjects by the idea that they are being treated.

(5) Randomisation in experiments allows the calculation of the likely size of sampling errors.

13. Which **one** of the following statements is **false**?

(1) A relationship between two quantitative variables may look weak because it has been plotted over only a limited range of x-values.

(2) When exploring the relationship between two quantitative variables, precise prediction cannot be made from a weak relationship.

(3) If we wish to explore the relationship between a qualitative and a quantitative variable, we plot the values of the quantitative variable for each group against the same scale.

(4) Cross-tabulation is a process of recording count data when we have two qualitative variables.

(5) In regression the explanatory variable is the variable explained by the response variable.

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14. Which **one** of the following options results in a **true** statement?

A reported margin of error in the media often appreciably **overstates** the true value of the error:

(1) in a percentage which has a true value of 50%.

(2) in a percentage which has a true value very close to 0% or 100%.

(3) when considering a percentage associated with some subgroup of the whole sample.

(4) when considering the difference between two percentages.

(5) in a percentage which has a true value between 30% and 70%.

15. Which **one** of the following statements about study design is **false**?

(1) A paired design experiment where all subjects receive two treatments and the order in which each subject receives the treatments is randomised, is called a crossover design.

(2) Medical experimenters often use a paired design by forming "matched pairs"; that is, by matching people as closely as possible on a set of variables.

(3) Paired designs are ineffective when members of the same pair are not more similar with respect to the variable of interest than individuals from different pairs.

(4) Pairing is beneficial when the variability between pairs is small compared with the variability within pairs.

(5) A completely randomised design results in independent samples and a paired design results in a single sample of differences.

Questions 16 to **20** refer to the following information.

Four single-sex and two co-educational schools in Melbourne, Australia, were asked to participate in a recent study designed to examine adolescents' attitudes towards confidentiality in the school counselling situation. All six schools were private schools. Three of the single-sex schools agreed to take part; one of the single-sex schools and both of the co-educational schools declined to take part in the study.

The students were advised that participation was voluntary and anonymous, and that they were free to withdraw from the study at any time.

Questionnaires were completed in school. Some results from the study are given in Table 4 below. It shows the percentage of students (aged 14–18 years) agreeing, disagreeing, or unsure as to whether the school counsellor should tell parents in situations of contraceptive use, and/or pregnancy.

There were 221 male respondents and 174 female respondents.

Situation	**Response**			**Sample size**
	Agree %	**Disagree %**	**Unsure %**	
Contraception				
males	33	52	15	221
females	13	79	8	174
Pregnancy				
males	41	43	16	221
females	15	74	11	174

Table 4: Adolescents' Attitudes Towards Confidentiality

Let p_{agree} be the proportion of all Australian **male** secondary school students (aged 14–18 years) who agree that a counsellor should tell parents in situations of pregnancy and $p_{disagree}$ be the corresponding proportion who disagree.

The results from the study are used to conduct a 2-tailed test for no difference between p_{agree} and $p_{disagree}$.

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16. An estimate of the difference between p_{agree} and $p_{disagree}$ is:

(1) -1.9

(2) -0.02

(3) -0.2

(4) -0.59

(5) -0.19

17. For the purpose of calculating $\text{se}(\widehat{p}_{agree} - \widehat{p}_{disagree})$, the sampling situation can be described as:

(1) one sample of size 395, several response categories.

(2) one sample of size 395, many yes/no items.

(3) two independent samples of sizes 221 and 174.

(4) one sample of size 221, several response categories.

(5) one sample of size 221, many yes/no items.

18. The expression for evaluating the test statistic for the null hypothesis, $H_0 : p_{agree} - p_{disagree} = 0$, is:

(1) $\dfrac{\widehat{p}_{agree} - \widehat{p}_{disagree}}{\text{se}(\widehat{p}_{agree} - \widehat{p}_{disagree})}$

(2) $\dfrac{\widehat{p}_{agree} - \widehat{p}_{disagree}}{\sqrt{\text{se}(\widehat{p}_{agree})^2 - \text{se}(\widehat{p}_{disagree})^2}}$

(3) $\dfrac{p_{agree} - p_{disagree}}{\text{se}(\widehat{p}_{agree}) + \text{se}(\widehat{p}_{disagree})}$

(4) $\dfrac{p_{agree} - p_{disagree}}{\text{se}(\widehat{p}_{agree} - \widehat{p}_{disagree})}$

(5) $\dfrac{\widehat{p}_{agree} - \widehat{p}_{disagree}}{\text{se}(\widehat{p}_{agree}) + \text{se}(\widehat{p}_{disagree})}$

19. Let p_{contra} and p_{preg} be the proportions of Australian **female** students (aged 14–18 years) who **disagree** that a counsellor should tell parents in situations of contraceptive use, and pregnancy, respectively. Information from Table 4 is used to construct a 95% confidence interval for the difference $p_{contra} - p_{preg}$.

The formula for the standard error of the estimate, $\text{se}(\widehat{p}_{contra} - \widehat{p}_{preg})$, is:

(1) $\sqrt{\dfrac{\widehat{p}_{contra}(1 - \widehat{p}_{contra})}{174} + \dfrac{\widehat{p}_{preg}(1 - \widehat{p}_{preg})}{174}}$

(2) $\sqrt{\dfrac{\widehat{p}^2_{contra}}{174} - \dfrac{\widehat{p}^2_{preg}}{174}}$

(3) $\sqrt{\dfrac{(1 - \widehat{p}_{contra}) + (1 - \widehat{p}_{preg}) - (\widehat{p}_{contra} - \widehat{p}_{preg})^2}{174}}$

(4) $\sqrt{\dfrac{(\widehat{p}_{contra} + \widehat{p}_{preg}) - (\widehat{p}_{contra} - \widehat{p}_{preg})^2}{174}}$

(5) $\sqrt{\dfrac{\widehat{p}^2_{contra}}{174} + \dfrac{\widehat{p}^2_{preg}}{174}}$

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20. There has been debate in New Zealand recently over whether school counsellors should tell parents in situations of student contraception and pregnancy. Suppose the conclusions from the analysis of the Australian study are applied to the population of New Zealand students.

Which of the following errors listed is/are potential sources of nonsampling error?

I	Transferring findings
II	Selection bias
III	Nonresponse bias

(1) I, II, III (all of them)

(2) III only

(3) I and III only

(4) I and II only

(5) II and III only

21. When using a t-procedure to construct a confidence interval for a population mean, the confidence interval is constructed using the formula:

$$\text{estimate} \ \pm t \text{ standard errors}$$

Which **one** of the following statements is **false**?

(1) The margin of error is the quantity added to, and subtracted from, the estimate to construct the interval.

(2) The standard error used to construct the interval will be identical for all samples of the same size.

(3) A confidence interval is preferred to a point estimate because the interval summarises the uncertainty due to sampling variation.

(4) The size of the multiplier, t, depends on both the sample size and the desired confidence level.

(5) The process of using sample data to construct an interval estimate for a population mean is an example of *statistical inference*.

Questions 22 to **24** refer to the following information.

A recent study was designed to investigate the abundance and size of snapper in the Cape Rodney – Okakari Point Marine Reserve and in an adjacent non-reserve region. Fishing surveys were conducted in both regions.

The data on the lengths of the snapper caught have been explored. We have decided to model the distribution of the length of a **Reserve** snapper, X_R, and the distribution of the length of a **non-Reserve** snapper, X_{NR}, as follows:

$$X_R \sim \text{ Normal}(\mu_R = 360.18\text{mm}, \, \sigma_R = 94.47\text{mm})$$
$$X_{NR} \sim \text{ Normal}(\mu_{NR} = 257.09\text{mm}, \, \sigma_{NR} = 59.35\text{mm})$$

22. Use the following MINITAB output (Table 5) in this question.

```
Normal with mean = 360.18000 and standard deviation = 94.470000

               x     P( X <= x)
          0.2000         0.0001
          0.8000         0.0001
        239.1118         0.1000
        280.6720         0.2000
        439.6880         0.8000
        481.2482         0.9000
```

Table 5: MINITAB output

80% of Reserve snapper are longer than:

(1) 280.7mm

(2) 239.1mm

(3) 360.2mm

(4) 481.2mm

(5) 439.7mm

For **Questions 23** and **24** recall:

$$X_R \sim \text{ Normal}(\mu_R = 360.18\text{mm},\, \sigma_R = 94.47\text{mm})$$

$$X_{NR} \sim \text{ Normal}(\mu_{NR} = 257.09\text{mm},\, \sigma_{NR} = 59.35\text{mm})$$

23. Which **one** of the following statements is **false**?

(1) The proportion of **Reserve** snapper longer than 400mm is smaller than the proportion of **non-Reserve** snapper longer than 300mm.

(2) Almost all **non-Reserve** snapper are between 80mm and 435mm in length.

(3) Approximately two-thirds of **Reserve** snapper are between 265mm and 455mm.

(4) On average, **Reserve** snapper are longer than **non-Reserve** snapper.

(5) The chances of a randomly selected **Reserve** snapper being shorter than 200mm and a randomly selected **non-Reserve** snapper being longer than 360mm are approximately the same.

24. Assume that *catching* a snapper in a region is the same as *randomly selecting* a snapper from that region. If a snapper is caught in each region, then the chances that the **Reserve** snapper is longer than the **non-Reserve** snapper can be determined by evaluating the probablity $\text{pr}(D > 0)$ where the random variable D is Normally distributed with:

(1) $\mu_D = 103.09\text{mm}, \quad \sigma_D = 12.40\text{mm}$

(2) $\mu_D = -103.09\text{mm}, \quad \sigma_D = 35.12\text{mm}$

(3) $\mu_D = 103.09\text{mm}, \quad \sigma_D = 111.57\text{mm}$

(4) $\mu_D = 103.09\text{mm}, \quad \sigma_D = 35.12\text{mm}$

(5) $\mu_D = -103.09\text{mm}, \quad \sigma_D = 111.57\text{mm}$

Questions 25 to **30** refer to the following information.

The University of Otago Injury Prevention Research Unit recently published a report titled *Road traffic practices among a cohort of young adults in New Zealand*. The aim of the study was to describe the road safety practices of young adults in New Zealand. Face-to-face interviews were conducted with 21-year-olds who were born in Dunedin. The report concluded that unsafe road practices, especially among males, were unacceptably high.

One area of the study investigated the wearing of seat belts. Some results are given in Table 6, a two-way table of counts for seat belt usage by rear seat passengers.

	Usage				
Gender	Always	Nearly Always	Sometimes	Never	**Total**
Female	138	79	139	107	463
Male	103	66	152	161	482
Total	241	145	291	268	945

Table 6: Self-reported seat belt usage by rear seat passengers

We used MINITAB to conduct a Chi-square test to investigate any differences between females and males for the Usage distribution. The output is given in Table 7 below. Some values have been removed and replaced with an asterisk (∗).

```
Expected counts are printed below observed counts

          Always   Nearly Always  Sometimes  Never   Total
  Female   138         79            139      107      463
           118.08       *            142.57    *

  Male     103         66            152      161      482
           122.92       *            148.43    *

Total      241        145            291      268      945

Chi-Sq =     *    +      *     +     0.090  +  4.499 +
          3.229   +   0.856    +     0.086  +  4.322 = 17.335
DF = *, P-Value = 0.001
```

Table 7: MINITAB output

25. For this investigation the null hypothesis is:

(1) H_0: The distribution of **Usage** is different for females and males.

(2) H_0: The factors **Gender** and **Usage** are associated.

(3) H_0: $p_1 = p_2 = p_3 = p_4$ where p_i is the proportion of 21-year-olds in each **Usage** group.

(4) H_0: The distribution of **Usage** is the same for females and males.

(5) H_0: The factors **Female** and **Male** are independent.

26. The expected cell count, under the null hypothesis, for those 21-year-old **males** who **never** wear a rear seat belt is:

(1) 137.08

(2) 130.29

(3) 136.76

(4) 131.31

(5) 136.69

27. The degrees of freedom for this Chi-square test is:

(1) 6

(2) 4

(3) 8

(4) 3

(5) 2

28. Consider the cell for **Female** and **Always**. This cell's contribution to the Chi-square test statistic value of 17.335 is:

(1) 2.875

(2) 0.144

(3) 1.645

(4) 0.169

(5) 3.360

29. Which **one** of the following statements regarding the *P-value* of 0.001 is **true**?

(1) Such a small *P-value* indicates that there must be a big difference between the Female and Male Usage distributions.

(2) Such a small *P-value* indicates that the alternative hypothesis must be true.

(3) The probability that the null hypothesis is false is 0.001.

(4) If the null hypothesis for this test is true, then the probability of getting a test statistic at least as large as 17.335 is 0.001.

(5) The probability that the null hypothesis is true is 0.001.

30. Which **one** of the following statements is **false**?

(1) One of the main reasons for such a small *P-value* in this test is because of the relatively small number of **Males** who said that they were **Always** users of rear seat belts.

(2) If the Chi-square test statistic had been 27.000 instead of 17.335, then the resulting *P-value* would have been smaller than 0.001.

(3) One of the main reasons for such a small *P-value* in this test is because of the relatively large number of **Males** who said that they were **Sometimes** users of rear seat belts.

(4) If one of the cells had an expected count of less than 1, then it would have been unwise to interpret the output from this test.

(5) The sum of the expected counts for **males** is 482 and the sum of the expected count for **females** is 463.

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Questions 31 and **32** refer to the following information.

The results of a British study investigating how diet affects the health of new mothers and their babies were recently reported in the *New Zealand Herald* (14 August, 2000). The study involved 5942 new born babies. Assume the 5942 babies form a random sample of new born British babies. Two hundred and fifty of the babies in the study were born to women who were vegetarians. In the sample of babies with vegetarian mothers, there were 81.2 baby boys for every 100 baby girls. Suppose that there were 3057 baby boys and 2885 baby girls in the study.

31. In Britain, for every 100 baby girls born, the number of boys born is approximately:

(1) 90

(2) 108

(3) 110

(4) 94

(5) 106

32. In the study, the number of baby girls with non-vegetarian mothers was approximately:

(1) 2121

(2) 2747

(3) 2600

(4) 2351

(5) 2564

Questions 33 to **36** refer to the following information.

This year's test for the Statistics course 475.340ST consisted of two sections, each worth a maximum of 15 marks. Fifty-five students sat the test and their marks for each section were recorded. The variable **Diff** represents the Section A mark minus the Section B mark. Figure 2 shows dot plots of the marks for each section and for the difference in marks.

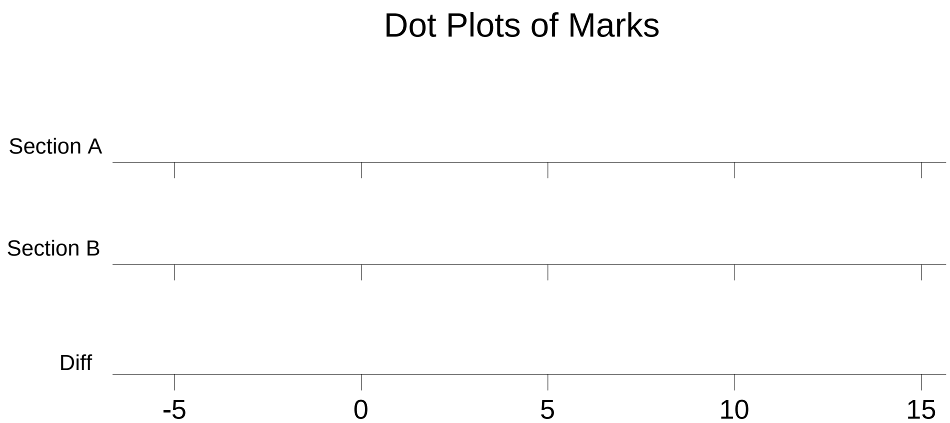

Figure 2: Dot plots of marks, 475.340ST test

33. The course lecturers want to determine whether one section is easier than the other. The **most** appropriate test to use is a:

(1) one sample t-test.

(2) two sample t-test.

(3) Mann-Whitney (Wilcoxon rank-sum) test.

(4) Chi-square test for homogeneity.

(5) Sign test.

34. Suppose a Sign test on the differences is the most appropriate test to use. (**Note:** This may not be true.) The MINITAB output in Table 8 shows the results of applying the Sign test. The result under P has been removed and replaced with an asterisk ($*$).

Sign Test for Median

```
Sign test of median = 0.00000 versus  not =  0.00000

            N    Below   Equal    Above       P     Median
Diff       55       13       6       36       *      2.000
```

Table 8: MINITAB output for Sign test

The *P-value* for this Sign test is:

(1) $2 \times \text{pr}(Y \geq 36)$ where $Y \sim$ Binomial$(n = 55, p = 0.5)$

(2) $2 \times \text{pr}(Y \geq 13)$ where $Y \sim$ Binomial$(n = 49, p = 0.5)$

(3) $\text{pr}(Y \leq 13)$ where $Y \sim$ Binomial$(n = 55, p = 0.5)$

(4) $2 \times \text{pr}(Y \leq 13)$ where $Y \sim$ Binomial$(n = 49, p = 0.5)$

(5) $\text{pr}(Y \geq 36)$ where $Y \sim$ Binomial$(n = 49, p = 0.5)$

CONTINUED

35. The lecturers also wanted to investigate the strength of the linear relationship between the Section A marks and the Section B marks. Figure 3 shows a scatter plot of the data.

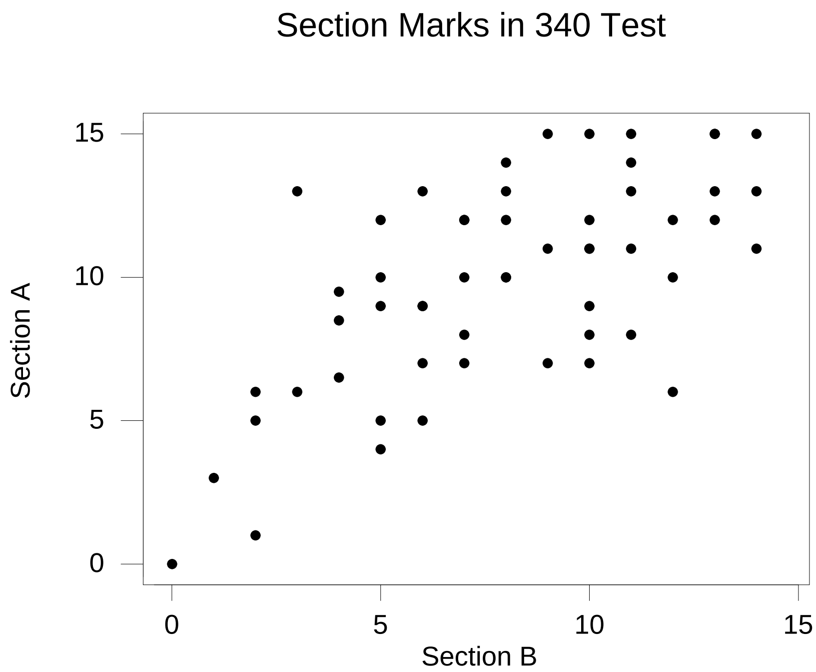

Figure 3: Scatter plot of marks in 475.340ST test

The sample correlation coefficient for the relationship between Section A marks and Section B marks is $r = 0.653$. Which **one** of the following statements is the correct interpretation of this value of r?

(1) The linear relationship between Section A marks and Section B marks is so weak it is not worth studying.

(2) The linear relationship between Section A marks and Section B marks is positive and very strong.

(3) The linear relationship between Section A marks and Section B marks is positive and weak to moderate.

(4) Each increase of one mark in Section B is associated with an increase of 0.653 marks in Section A.

(5) The linear relationship between Section A marks and Section B marks is negative and weak to moderate.

36. Suppose that on further investigation it was found that the student who scored 13 marks in Section A and 3 marks in Section B was ill during the test and had to leave without completing Section B. It was decided to remove this observation from the analysis and recalculate the sample correlation coefficient.

Which **one** of the following statements is **true**?

(1) It is impossible to determine how the recalculated sample correlation coefficient would compare with the original value of 0.653.

(2) The recalculated sample correlation coefficient would increase because the slope of the new fitted line would be greater than the slope of the original fitted line.

(3) The recalculated sample correlation coefficient would decrease because the slope of the new fitted line would be less than the slope of the original fitted line.

(4) The recalculated sample correlation coefficient would increase because the data would more closely fit a straight line with a positive slope.

(5) The recalculated sample correlation coefficient would decrease because the data would more closely fit a straight line with a negative slope.

CONTINUED

Questions 37 and **38** refer to the following information.

The paper "Family Planning: Football Style. The Relative Age Effect in Football." investigated the relationship between month of birth and achievement in sports for men. Birth dates were collected on all players in teams competing in the 1990 World Cup soccer games, and they are summarised in Table 9 below.

Birthdays by Quarter	**Frequency**
Quarter 1: Aug–Oct	150
Quarter 2: Nov–Jan	138
Quarter 3: Feb–April	140
Quarter 4: May–July	100
Total	528

Table 9: Birth dates, 1990 World Cup Soccer Players

The paper claims that the distribution of players' birth dates is not random and that the number of players is related to the "Quarters of the football year". The claim is based on the results of a Chi-square test for goodness-of-fit.

37. The hypotheses for such a test are:

(1) H_0: Over a year, the greatest proportion of players are born in Quarter 1: Aug–Oct.

H_1: Over a year, the greatest proportion of players are **not** born in Quarter 1: Aug–Oct.

(2) H_0: 25% of all players are born in each Quarter.

H_1: There are at least two Quarters in which the proportion of all players born is not 25%.

(3) H_0: The proportion of players born in each Quarter is different for each Quarter.

H_1: The proportion of players born in each Quarter is the same for each Quarter.

(4) H_0: The proportion of players born in each Quarter is approximately 0.28, 0.26, 0.27, and 0.19.

H_1: The proportions of players born in each Quarter are **not** those given in H_0.

(5) H_0: 25% of all players are born in each Quarter.

H_1: There is **no** Quarter in which the proportion of all players born is 25%.

CONTINUED

38. Under H_0, the expected count for the number of players born in Quarter 1: Aug–Oct is:

(1) 528

(2) 138

(3) 132

(4) 117

(5) 150

39. Which **one** of the following statements about data in tables of counts is **false**?

(1) A Chi-square test of homogeneity on the column distributions can be used on a single random sample cross-classified by two response factors.

(2) A Chi-square test of goodness-of-fit can be used on a single random sample classified into categories of the response factor.

(3) A Chi-square test of independence can be used on several random samples each classified into the same categories of the response factor.

(4) A Chi-square test of homogeneity on the row distributions can be used on a single random sample cross-classified by two response factors.

(5) A Chi-square test of independence can be used on a single random sample cross-classified by two response factors.

40. Which **one** of the following statements about simple linear regression analysis is **false**?

(1) The least-squares regression line is found by choosing the line that minimises the sum of the squared prediction errors.

(2) When a least-squares regression line is fitted to the data, the sum of the prediction errors is zero.

(3) For a particular x-value, the 95% prediction interval for the next actual Y-value is generally narrower than the 95% confidence interval for the mean of Y.

(4) For a particular x-value, the standard error used to calculate the prediction interval for Y allows for uncertainty about the true values of the intercept and the slope of the line, as well as the uncertainty due to random scatter about the line.

(5) When data from a well designed, well executed, controlled experiment indicate a strong relationship between the two variables, we could have reliable evidence of causation.

Questions 41 to **65** refer to Appendix A: Car Data. See Pages 41–50.

Questions 41 to **46** refer to the following additional information.

A researcher is interested in the engine size of models of new cars on the market in New Zealand.

41. Figure 4 (page 43) shows a stem-and-leaf plot for engine size. Using Figure 4, the median engine size of the sample is:

(1) 2000cc.

(2) 2300cc.

(3) 2200cc.

(4) 2250cc.

(5) 2100cc.

42. Using the stem-and-leaf plot in Figure 4 (page 43), for this sample which **one** of the following statements is **false**?

(1) 10% of the cars have an engine size greater than 3900cc.

(2) The engine sizes of the cars are negatively (left) skewed.

(3) The upper quartile is 2800cc.

(4) Less than 25% of the cars have an engine size between 2100cc and 2600cc.

(5) The range is 4400cc.

43. The MINITAB output in Table 11 (page 43) shows a 95% confidence interval for μ_{Eng}, the true mean engine size of models of new cars on the market in New Zealand in May 2000. Which **one** of the following statements is **false**?

(1) In light of the data, the interval from 2185cc to 2685cc contains the plausible values for μ_{Eng}.

(2) If many such samples are taken and a 95% confidence interval for μ_{Eng} is calculated from each sample, then statements such as "μ_{Eng} is somewhere between the two confidence limits" are true, on average, 19 times out of 20.

(3) There is a probability of 0.95 that a randomly selected engine size is in the interval from 2185cc to 2685cc.

(4) With 95% confidence, the value of μ_{Eng} is estimated to be 2435cc with a margin of error of 250cc.

(5) If many such samples are taken and a 95% confidence interval for μ_{Eng} is calculated from each sample, then approximately 95% of these confidence intervals will contain μ_{Eng}.

44. Refer again to the confidence interval in Table 11 and the plot in Figure 4. Which **one** of the following statements is **true**?

(1) The validity of the confidence interval is not in doubt because, for a sample of this size, t-procedures work well even for clearly skewed data.

(2) The validity of the confidence interval is in doubt because the data suggest the underlying distribution is not unimodal.

(3) The validity of the confidence interval is in doubt because the data suggest the underlying distribution is severely skewed.

(4) To improve the validity of the confidence interval, the two observations of 5700cc should be removed from the data and the confidence interval recalculated.

(5) To improve the validity of using a confidence interval based on t-procedures to estimate μ_{Eng}, we should use a 99% confidence interval instead of a 95% confidence interval.

45. Table 11 shows a 95% confidence interval for μ_{Eng}, based on a sample of 60 models of new cars. Suppose that a second sample of 60 car models was taken and this sample was used to construct a 95% confidence interval for μ_{Eng} (the second confidence interval). Suppose also that, compared with the first sample, the second sample had a **smaller** sample mean but a **larger** sample standard deviation.

Which **one** of the following statements is **true**?

(1) The second confidence interval would be centred around a higher value than the original confidence interval.

(2) Both confidence intervals would have the same width because they are both 95% confidence intervals.

(3) It is not possible to compare the widths of the two confidence intervals because the two samples have different means.

(4) Both confidence intervals would be identical because they are both 95% confidence intervals for μ_{Eng}.

(5) The second confidence interval would be wider than the original confidence interval.

46. Suppose a sample of 15 cars, instead of the original sample of 60 cars, was taken from all models of new cars available in New Zealand in May 2000. If the engine sizes of these 15 cars were used to form a 95% confidence interval for μ_{Eng}, then which **one** of the following statements is **true**?

We would expect the confidence interval formed from the sample of 15 cars to be approximately:

(1) twice the width of the confidence interval formed from the sample of 60 cars.

(2) half the width of the confidence interval formed from the sample of 60 cars.

(3) one-quarter of the width of the confidence interval formed from the sample of 60 cars.

(4) the same width as the confidence interval formed from the sample of 60 cars.

(5) four times the width of the confidence interval formed from the sample of 60 cars.

CONTINUED

Questions 47 to **51** refer to the following additional information.

The researcher is also interested in whether there is a difference between the mean engine size of cars with different numbers of doors. After some exploratory analysis the researcher chose to investigate cars with 2, 4 and 5 doors.

A one-way analysis of variance (ANOVA) F-test was conducted. Figure 5 (page 44) shows a dot plot of the data and Table 12 (page 45) shows MINITAB output for the one-way analysis of variance.

47. The values for the degrees of freedom, `df1` and `df2`, for this F-test are:

(1) `df1`=3, `df2`=48

(2) `df1`=2, `df2`=51

(3) `df1`=3, `df2`=49

(4) `df1`=2, `df2`=50

(5) `df1`=2, `df2`=49

48. The value of the F-test statistic, `f0`, for this F-test is approximately:

(1) 0.053

(2) 0.774

(3) 1.670

(4) 18.960

(5) 1.292

49. Suppose that the 2, 4 and 5 door samples are independent random samples. Which **one** of the following statements is **true**?

(1) It is **not** appropriate to use the F-test because, the F-test is **not** sufficiently robust to withstand the departures from the assumption of equal group population standard deviations suggested by the samples.

(2) It is **not** appropriate to use the F-test because all three sample sizes are less than 30.

(3) It is **not** appropriate to use the F-test because the F-test is **not** sufficiently robust to withstand the departures from Normality suggested by the samples.

(4) It is **not** appropriate to use the F-test because there are only three samples.

(5) It is appropriate to use the F-test because the F-test is sufficiently robust to withstand both the departures from Normality and the departures from the assumption of equal group population standard deviations suggested by the samples.

50. Suppose it is appropriate to conduct an F-test. (**Note:** This may not be true.) Which **one** of the following is the **best** interpretation of the results of this F-test?

(1) There is strong evidence that the underlying mean engine size is the same for all three groups.

(2) There is no evidence of a difference in the underlying mean engine sizes of the three groups.

(3) There is no evidence that the underlying standard deviation for engine size is the same for all three groups.

(4) There is strong evidence that the sample mean engine size is the same for all three groups.

(5) There is no evidence that the sample mean engine size is the same for all three groups.

CONTINUED

51. Which **one** of the following statements about the pairwise comparisons in Table 12 (page 45) is **false**?

(1) With 95% confidence, the true mean engine size of the **4-door** group is between 277cc less than and 1217cc greater than the true mean engine size of the **5-door** group.

(2) This process of generating intervals for true differences will produce at least one interval that will not contain its true difference about 1.94% of the time.

(3) With 95% confidence, the true mean engine size of the **4-door** group is between 797cc less than and 910cc greater than the true mean engine size of the **2-door** group.

(4) At the 5% level of significance, there is no significant difference between the true mean engine sizes of the **2-door** and **5-door** groups.

(5) In a two-tailed significance test for no difference between the true mean engine sizes of the **2-door** and **4-door** groups, the *P-value* will be greater than 5%.

CONTINUED

Questions 52 to **55** refer to the following additional information.

The researcher investigated the differences in the price of car models made by Asian and European companies for the New Zealand market. Table 13 (page 46) shows MINITAB output of this analysis, Figure 6 (page 46) shows a dot plot of the data and Figure 7 (page 47) shows a box plot of the data.

Let μ_{Asia} and μ_{Eur} be the underlying means for the price of Asian and European models, respectively.

52. For the cars in the sample used for this analysis, which **one** of the following statements is **false**?

(1) There is at least one European car with a higher price than the most expensive Asian car.

(2) The least expensive Asian car costs more than the least expensive European car.

(3) The price of European cars has a greater spread than the price of Asian cars.

(4) The mean price of a European car is more than the mean price of an Asian car.

(5) Only one-quarter of the European cars have a price more than the median priced Asian car.

53. For the t-test shown in Table 13 (page 46), which **one** of the following statements is **false**?

(1) The test is significant at the 5% level of significance.

(2) Using the standard Normal distribution instead of the Student(df = 37) distribution would have resulted in a larger *P-value*.

(3) The difference in the sample means, $\overline{x}_{Asia} - \overline{x}_{Eur}$, is 3.20 standard errors below zero.

(4) Using the Student(df = 19) distribution instead of the Student(df = 37) distribution would have resulted in a larger *P-value*.

(5) The test is significant at the 1% level of significance.

CONTINUED

54. Which **one** of the following statements gives the **best** interpretation of the t-test shown in Table 13 (page 46)?

(1) There is some evidence against the hypothesis of **no** difference between μ_{Asia} and μ_{Eur}.

(2) There is no evidence against the hypothesis of **no** difference between μ_{Asia} and μ_{Eur}.

(3) There is very strong evidence against the hypothesis of a difference between μ_{Asia} and μ_{Eur}.

(4) There is no evidence against the hypothesis of a difference between μ_{Asia} and μ_{Eur}.

(5) There is very strong evidence against the hypothesis of **no** difference between μ_{Asia} and μ_{Eur}.

55. Suppose that the price of each car in the European sample was reduced by \$1000. This would cause **only one** change in the summary statistics in Table 13 — the **European** sample mean would change from \$72,351 to \$71,351.

Suppose also that a t-test was conducted on this altered data, giving a new value for the test statistic. (**Note:** The old test statistic is −3.20.) Which **one** of the following statements is **true**?

(1) The new test statistic value will be further from zero than −3.20 is, resulting in a *P-value* larger than 0.0028.

(2) The new test statistic value will be closer to zero than −3.20 is, resulting in a *P-value* larger than 0.0028.

(3) The new test statistic value will be further from zero than −3.20 is, resulting in a *P-value* smaller than 0.0028.

(4) The new test statistic value will be closer to zero than −3.20 is, resulting in a *P-value* smaller than 0.0028.

(5) It is not possible to determine whether the new test statistic value will be closer to, or further from, zero than −3.20 is.

Questions 56 to **59** refer to the following additional information.

The researcher also wants to investigate the differences in the price of car models made by Australian and USA companies for the New Zealand market. Figure 8 (page 47) shows a dot plot of the data with $n_{Aust} = 5$ and $n_{USA} = 7$.

Let μ_{Aust} and μ_{USA} be the underlying means for the price of Australian and USA models, respectively.

56. Which **one** of the following statements is the **most** appropriate to make about conducting a two sample t-test on these data?

- **(1)** A two sample t-test is valid because a two sample t-test is always valid when the two samples are independent.
- **(2)** A two sample t-test is invalid because both sample sizes are too small.
- **(3)** A two sample t-test is invalid because of the large difference between the sample means.
- **(4)** A two sample t-test is invalid because the USA car priced at $\$64,300$ is an outlier.
- **(5)** A two sample t-test is valid because there are no outliers and each sample is reasonably symmetrical.

57. Assume that t-procedures are appropriate for these data. (**Note:** This may not be true.) A 95% confidence interval for $\mu_{Aust} - \mu_{USA}$ is calculated to be (−\$13,844, \$25,258). Consider the two sample t-test with hypotheses:

$$H_0 : \mu_{Aust} = \mu_{USA}$$
$$H_1 : \mu_{Aust} \neq \mu_{USA}$$

Which **one** of the following statements is **false**?

- **(1)** The test is not significant at the 5% level of significance.
- **(2)** The *P-value* is greater than 0.05.
- **(3)** The test is not significant at the 1% level of significance.
- **(4)** There is strong evidence in the data against the hypothesis: $\mu_{Aust} = \mu_{USA}$.
- **(5)** We would not reject H_0 at the 5% level of significance.

CONTINUED

58. Recall that a 95% confidence interval for $\mu_{Aust} - \mu_{USA}$ is calculated to be ($-$\$13,844, \$25,258). Based on this confidence interval, which **one** of the following statements is **true**?

(1) With 95% confidence, μ_{Aust} is somewhere between \$13,844 higher than and \$25,258 lower than μ_{USA}.

(2) With 95% confidence, μ_{Aust} is somewhere between \$13,844 lower than and \$25,258 higher than μ_{USA}.

(3) With 95% confidence, μ_{Aust} is either \$13,844 lower than μ_{USA} or \$25,258 higher than μ_{USA}.

(4) With 95% confidence, μ_{Aust} is either \$13,844 higher than μ_{USA} or \$25,258 lower than μ_{USA}.

(5) No statement can be made about the relative sizes of μ_{Aust} and μ_{USA} because another sample of cars would give different estimates of μ_{Aust} and μ_{USA}.

59. The **most** appropriate nonparametric test to use to investigate the difference between the average prices of Australian and USA models is a:

(1) one sample t-test.

(2) Kruskal-Wallis test.

(3) Mann-Whitney (Wilcoxon rank-sum) test.

(4) Sign test.

(5) one-way analysis of variance F-test.

CONTINUED

Questions 60 to **64** refer to the following additional information.

The researcher believes that the engine size of cars with small to moderate sized engines (under 2500cc) could be used to predict the weight of a car. The results of a linear regression analysis using MINITAB and associated plots are shown in Figure 9 (page 48), Table 14 (page 48), Figure 10 (page 49) and Figure 11 (page 49).

60. One of the cars in the sample has an engine size of 1590cc and a weight of 1215kg. If a new car has an engine size of 1590cc, the regression equation predicts the car's weight to be approximately:

(1) 1215kg
(2) 1826kg
(3) 836kg
(4) 1321kg
(5) 1072kg

61. Another of the cars in the sample has an engine size of 1497cc and a weight of 940kg. Based on the regression equation, the residual for this car is approximately:

(1) -83kg
(2) 83kg
(3) 767kg
(4) 1023kg
(5) -767kg

62. Suppose that the engine sizes of two cars differ by 500cc. The regression equation predicts that the difference in the weights of these two cars will be:

(1) 498kg
(2) 139kg
(3) 263kg
(4) 117.5kg
(5) 504kg

63. In a test for no linear relationship between engine size and weight the hypotheses are:

(1) $H_0 : \beta_0 \neq 0$ $\quad$ $H_1 : \beta_0 = 0$

(2) $H_0 : \widehat{\beta}_0 = 0$ $\quad$ $H_1 : \widehat{\beta}_0 \neq 0$

(3) $H_0 : \widehat{\beta}_1 = 0$ $\quad$ $H_1 : \widehat{\beta}_1 \neq 0$

(4) $H_0 : \beta_1 = 0$ $\quad$ $H_1 : \beta_1 \neq 0$

(5) $H_0 : \beta_0 = 0$ $\quad$ $H_1 : \beta_0 \neq 0$

64. You may need to refer to Figure 9, Figure 10 and Figure 11 to help answer this question. Which **one** of the following statements about this linear regression analysis is **false**?

(1) It is reasonable to assume that the error terms have a constant underlying standard deviation.

(2) It would be difficult to have faith in a 95% prediction interval for an engine size of 2150cc because there are so few observations with a similar engine size.

(3) Engine size is a quantitative variable and weight is a continuous random variable.

(4) It would be unwise to use this data to predict the weight of a car with a 3000cc engine.

(5) It is believable that the error terms are Normally distributed with a mean of zero.

65. The researcher also used all 60 cars (irrespective of engine size) to investigate if engine size could be used to predict the weight of a car. The residual plot in Figure 12 (page 50) was produced as part of this investigation. The **most** useful information provided by this plot about this linear regression model is that:

(1) the errors are not independent.

(2) the errors are not Normally distributed.

(3) the relationship between weight and engine size is not linear.

(4) the mean of the errors is not equal to zero.

(5) all of the assumptions underlying this regression model are satisfied.

ATTACHMENT FOLLOWS

INCLUSIONS:

- * **Appendix A: Car Data,** for use in Questions 41 to 65.
- * **Formulae Appendix**

CONTINUED

Appendix A: Car Data

Questions 41 to **65** refer to the information given in this appendix.

A random sample of 60 models of new cars was taken from all models on the market in New Zealand in May 2000. For each model of car, measurements were made on the following variables:

- **Reg**: The region where the manufacturing company is based.
- **Drs**: The number of doors.
- **Eng**: The engine size in cubic centimetres (cc).
- **Wt**: The weight in kilograms (kg).
- **Price**: The price in New Zealand dollars.

Some tables and figures follow.

Data Table for 60 Models of New Cars

No.	Reg	Drs	Eng	Wt	Price	No.	Reg	Drs	Eng	Wt	Price
1	Eur	3	1690	1150	14,995	31	Eur	5	1598	1135	44,900
2	USA	3	1299	871	18,995	32	Asia	5	2995	1415	45,500
3	Eur	5	1390	960	22,990	33	Eur	4	1997	1350	45,990
4	Asia	3	1590	1215	25,495	34	Asia	5	2457	1445	48,990
5	Asia	5	1343	935	25,500	35	Asia	5	2835	1930	49,950
6	Asia	4	1497	940	26,000	36	Asia	4	2438	1700	51,200
7	Eur	5	1589	1040	26,000	37	USA	5	3984	1608	51,500
8	Eur	5	1598	1000	26,990	38	USA	4	2499	1535	52,990
9	USA	5	1753	1125	27,850	39	Aust	5	5665	1702	57,895
10	Eur	3	1581	1050	27,995	40	Eur	4	2947	1470	58,990
11	Asia	3	1590	1145	28,000	41	Asia	4	1994	1320	59,990
12	Aust	4	1796	1189	28,295	42	Asia	5	1994	1320	59,990
13	Asia	5	1597	1102	29,195	43	Eur	4	1895	1360	63,900
14	Eur	3	1587	935	29,900	44	Aust	4	5665	1732	64,295
15	Asia	3	1995	1260	31,995	45	USA	4	4942	1525	64,300
16	Asia	2	1975	1230	33,950	46	Eur	4	2497	1515	67,000
17	Asia	4	1997	1378	33,990	47	Eur	4	2290	1585	76,700
18	Asia	5	2351	1440	34,350	48	Eur	5	1781	1365	80,800
19	Eur	5	1761	1145	34,990	49	Eur	2	2290	1290	82,600
20	Eur	3	1747	1100	34,995	50	Eur	4	2397	1420	86,800
21	USA	4	1998	1260	35,450	51	Eur	2	2793	1360	94,000
22	Eur	5	1998	1182	35,990	52	Eur	4	2295	1450	96,000
23	Asia	5	1991	1221	38,445	53	Eur	2	2295	1325	98,900
24	Asia	4	2164	1375	39,500	54	Eur	2	2435	1565	99,900
25	USA	2	3960	1575	41,990	55	Asia	4	2997	1650	100,500
26	Asia	2	1998	1270	42,995	56	Eur	2	2793	1440	108,000
27	Aust	5	2498	1416	42,995	57	Eur	2	3199	1495	118,500
28	Eur	2	2495	1698	43,000	58	Eur	2	2799	1760	165,000
29	Aust	4	3791	1551	44,395	59	Eur	4	3996	1775	165,000
30	Asia	4	3497	1518	44,550	60	Eur	2	3199	1780	175,000

Table 10: New car data, May 2000

CONTINUED

Stem-and-Leaf Plot of Engine Size

Units: 2 | 1 = 2100cc

$n = 60$

1	3 3 4
1	5 6 6 6 6 6 6 6 6 7 7 8 8 8 8 9
2	0 0 0 0 0 0 0 0 0 0 2 3 3 3 3 4 4 4 4
2	5 5 5 5 5 8 8 8 8 9
3	0 0 2 2
3	5 8
4	0 0 0
4	9
5	
5	7 7

Figure 4: Engine sizes of new cars

T Confidence Intervals

```
Variable      N        Mean      StDev     SE Mean          95.0 % CI
Eng          60        2435        968         125       (2185,  2685)
```

Table 11: MINITAB output, confidence interval for population mean engine size

CONTINUED

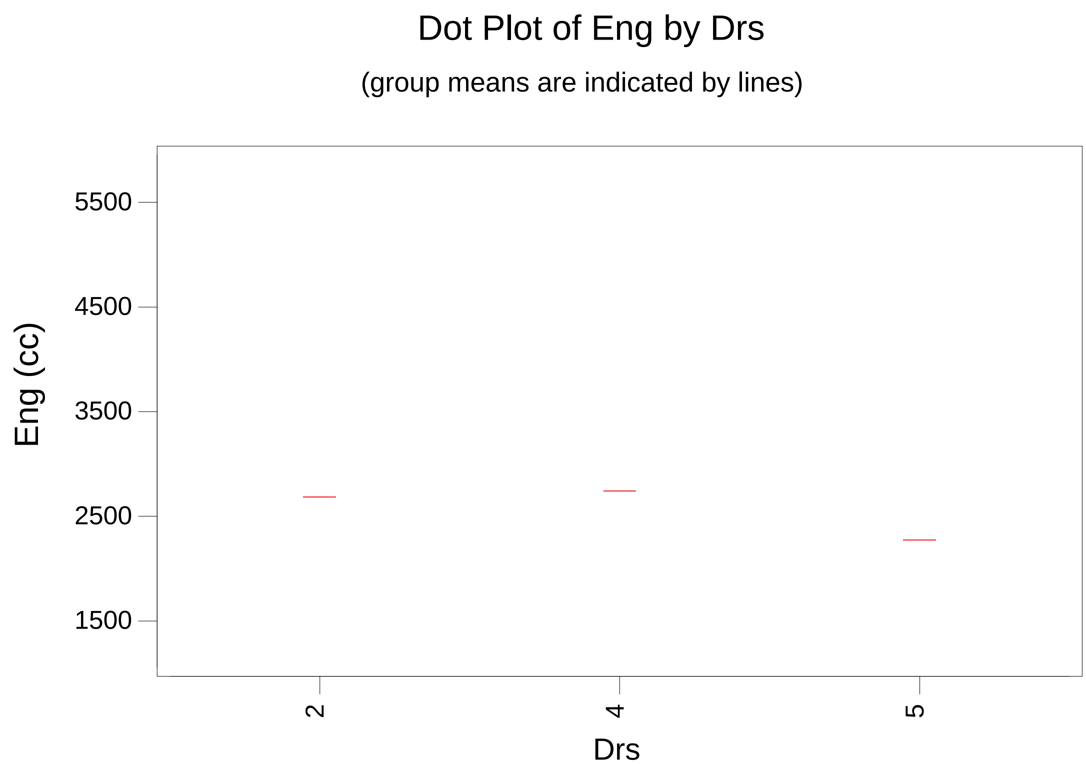

Figure 5: Dot plot of engine size by number of doors for 2, 4 and 5 door cars

CONTINUED

One-way Analysis of Variance

```
Analysis of Variance for Eng
Source     DF        SS        MS        F        P
Drs       df1   2458309   1229155       f0    0.284
Error     df2  46610338    951231
Total      df  49068648
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  -----+---------+---------+---------+-
2          12    2685.9     569.9          (-------------*-------------)
4          21    2742.3    1076.8               (----------*---------)
5          19    2272.5    1050.1   (----------*----------)
                                   -----+---------+---------+---------+-
Pooled StDev =    975.3              2000      2400      2800      3200

Tukey's pairwise comparisons

    Family error rate = 0.0500
Individual error rate = 0.0194

Critical value = 3.42

Intervals for (column level mean) - (row level mean)

                 2           4

       4       -910
                797

       5       -456        -277
               1283        1217
```

Table 12: MINITAB output, one-way analysis of variance for engine size

CONTINUED

Two Sample T-Test and Confidence Interval

```
Two sample T for Price

Reg           N       Mean      StDev    SE Mean
Asia         20      42504      17393       3889
Eur          28      72351      44889       8483

95% CI for mu (Asia) - mu (Eur ): ( -48755,  -10938)
T-Test mu (Asia) = mu (Eur ) (vs not =): T = -3.20  P = 0.0028  DF = 37
```

Table 13: MINITAB output, confidence interval and t-test for difference between population means for price

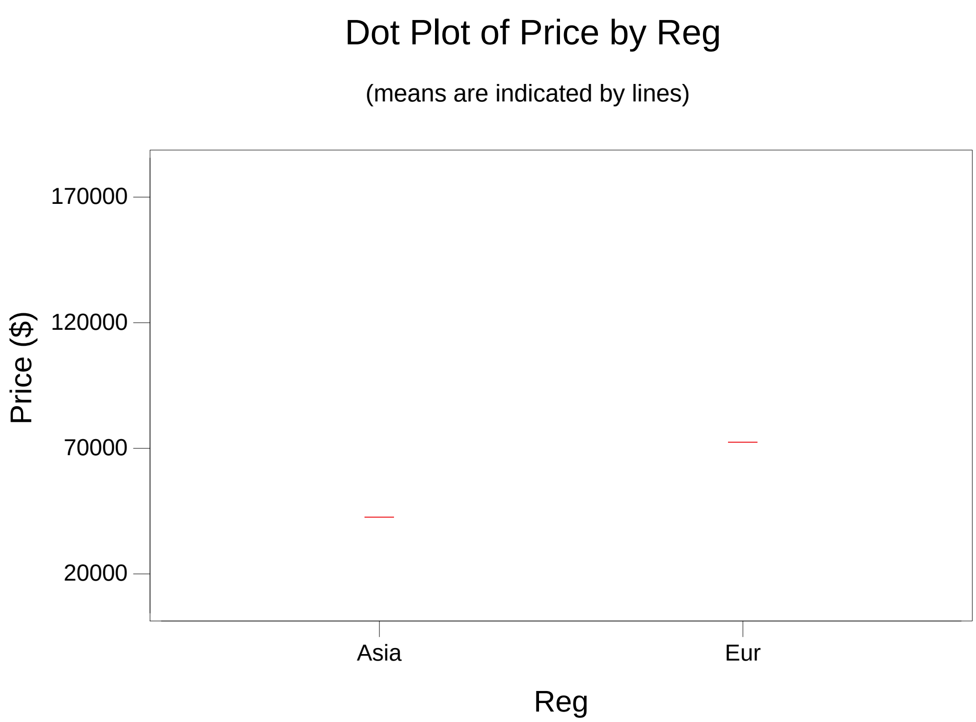

Figure 6: Dot plot of prices of new Asian and European cars

CONTINUED

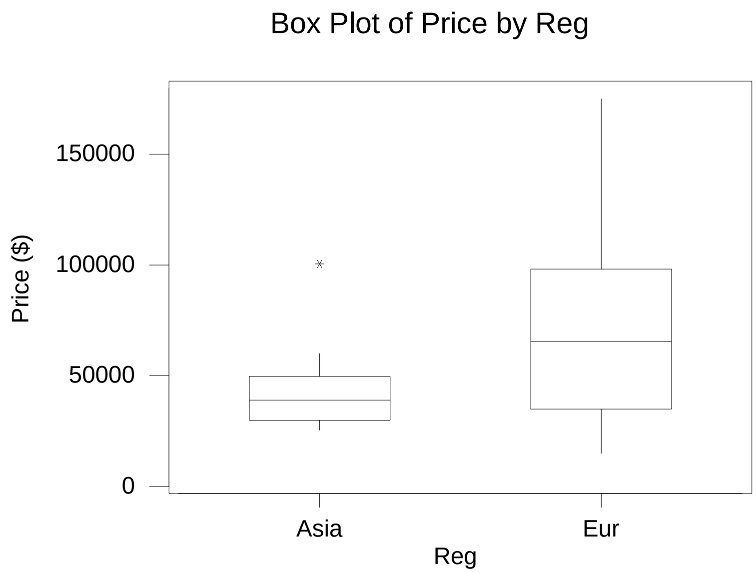

Figure 7: Box plot of prices of new Asian and European cars

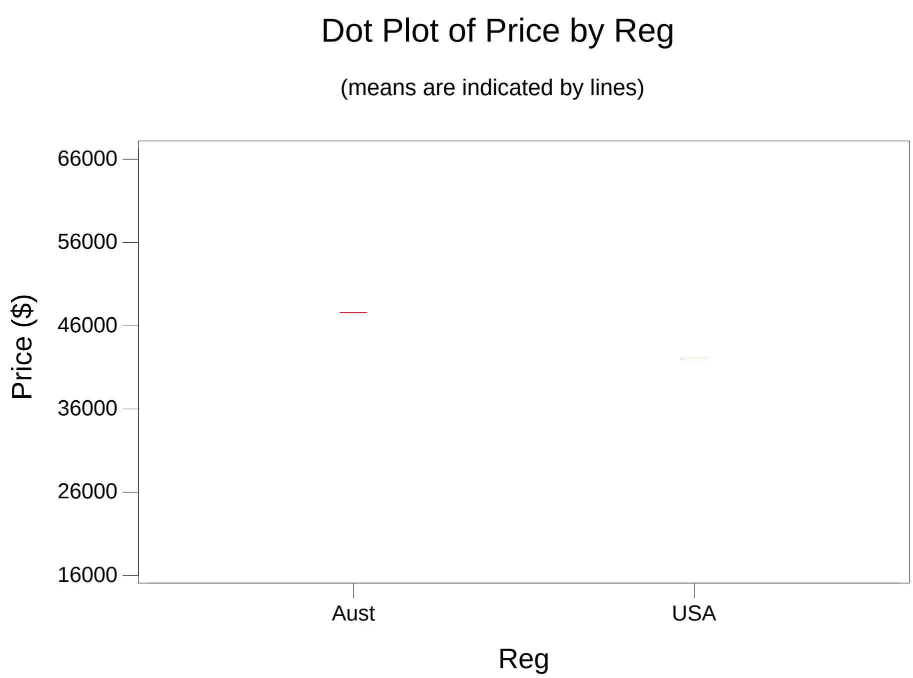

Figure 8: Dot plot of prices of new Australian and USA cars

CONTINUED

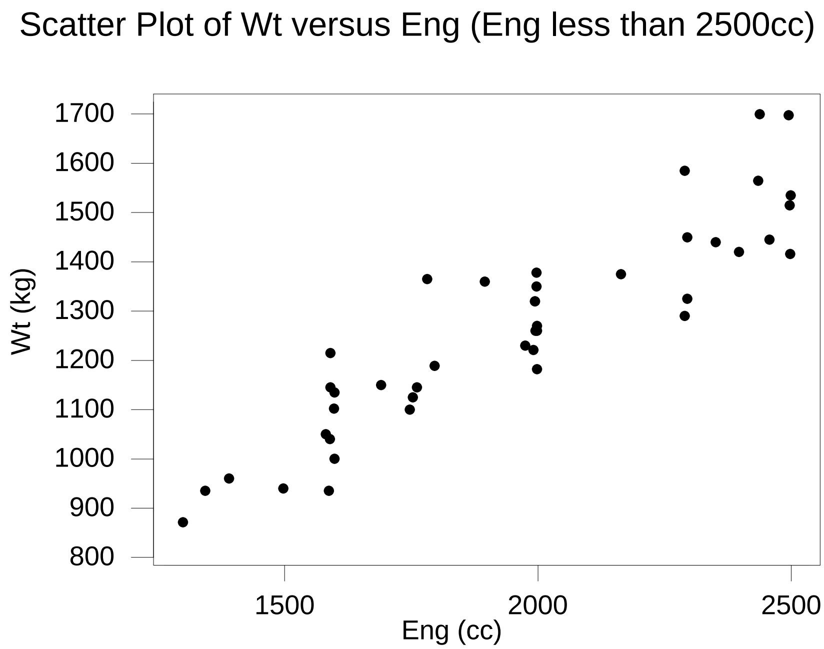

Figure 9: Scatter plot of weight versus engine size for cars with engines smaller than 2500cc

Regression Analysis

```
The regression equation is
Wt = 235 + 0.526 Eng

Predictor        Coef      StDev          T          P
Constant       235.41      73.68       3.19      0.003
Eng           0.52594    0.03710      14.18      0.000

S = 86.24      R-Sq = 83.1%    R-Sq(adj) = 82.6%
```

Table 14: MINITAB output, linear regression analysis of the relationship between weight and engine size

CONTINUED

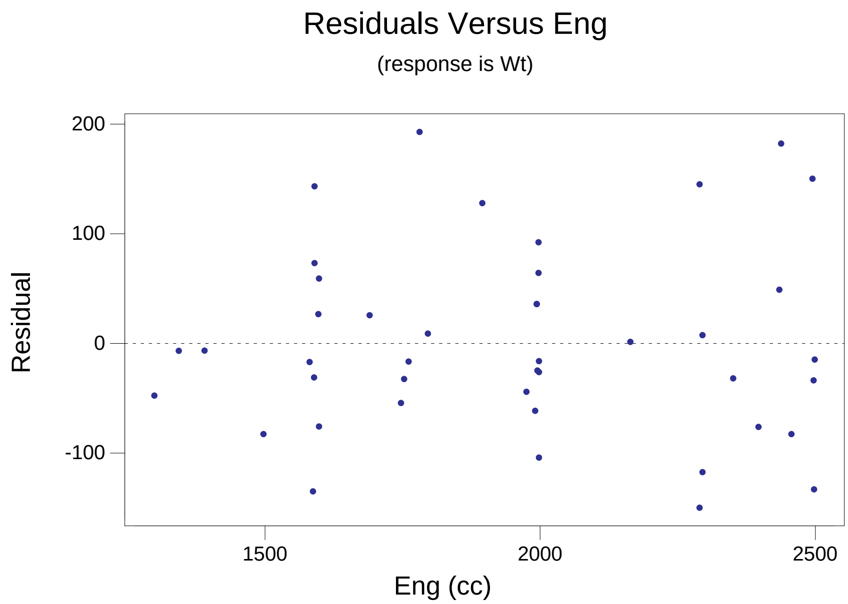

Figure 10: Scatter plot of residuals versus engine size for cars with engines smaller than 2500cc

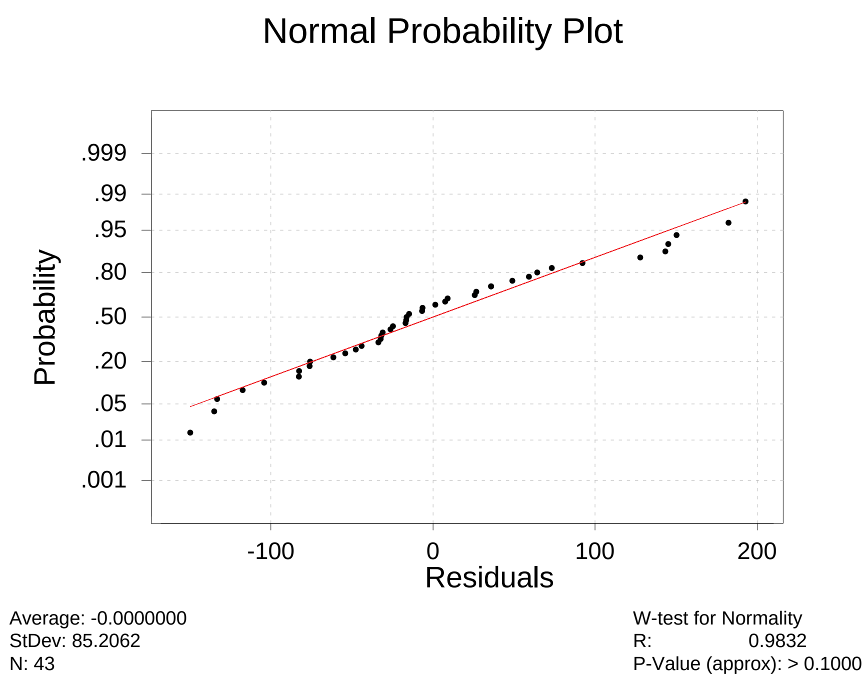

Figure 11: Normal probability plot of residuals for cars with engine size smaller than 2500cc

CONTINUED

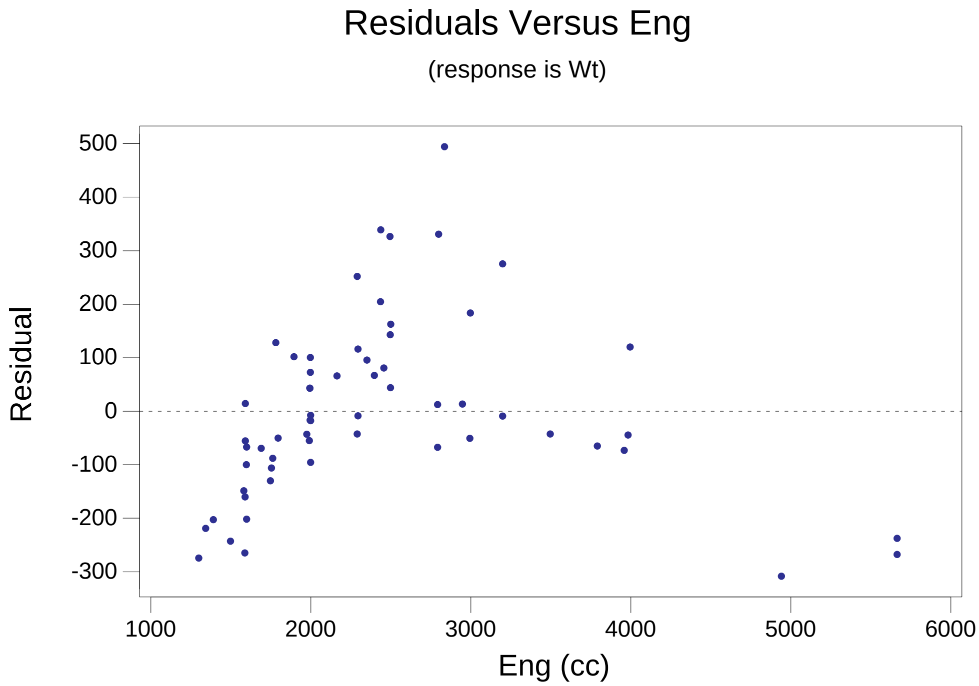

Figure 12: Scatter plot of residuals versus engine size for all cars

ANSWERS:

1. (1)	2. (2)	3. (4)	4. (1)	5. (1)
6. (5)	7. (3)	8. (1)	9. (4)	10. (2)
11. (4)	12. (3)	13. (5)	14. (2)	15. (4)
16. (2)	17. (4)	18. (1)	19. (3)	20. (1)
21. (2)	22. (1)	23. (1)	24. (3)	25. (4)
26. (5)	27. (4)	28. (5)	29. (4)	30. (3)
31. (5)	32. (2)	33. (1)	34. (4)	35. (3)
36. (4)	37. (2)	38. (3)	39. (3)	40. (3)
41. (4)	42. (2)	43. (3)	44. (1)	45. (5)
46. (1)	47. (5)	48. (5)	49. (5)	50. (2)
51. (2)	52. (5)	53. (2)	54. (5)	55. (2)
56. (5)	57. (4)	58. (2)	59. (3)	60. (5)
61. (1)	62. (3)	63. (4)	64. (2)	65. (3)

CONTINUED

FORMULAE

Median Position $= \dfrac{n+1}{2}$

Distributions

In general: $\mathrm{sd}(X) = \sqrt{\mathrm{E}(X-\mu_X)^2}$

If X is a **discrete random variable**:

$$\mu_X = \mathrm{E}(X) = \sum x_i \mathrm{pr}(X = x_i) \qquad \mathrm{sd}(X) = \sqrt{\sum (x_i - \mu_X)^2 \mathrm{pr}(X = x_i)}$$

$\boldsymbol{X} \sim$ **Binomial** (n, p) $\quad \mathrm{E}(X) = np \quad \mathrm{sd}(X) = \sqrt{np(1-p)}$

$\boldsymbol{X} \sim$ **Poisson** (λ) $\quad \mathrm{E}(X) = \lambda \quad \mathrm{sd}(X) = \sqrt{\lambda}$

$\boldsymbol{X} \sim$ **Normal** (μ, σ) $\quad \mathrm{E}(X) = \mu \quad \mathrm{sd}(X) = \sigma$

Combining random variables

For any constants a and b:

$$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b \qquad \mathrm{sd}(aX + b) = |a|\mathrm{sd}(X)$$

If X_1 and X_2 are independent random variables:

$$\mathrm{E}(a_1X_1 + a_2X_2) = a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2)$$

$$\mathrm{sd}(a_1X_1 + a_2X_2) = \sqrt{a_1^2\mathrm{sd}(X_1)^2 + a_2^2\mathrm{sd}(X_2)^2}$$

If $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with mean μ and standard deviation σ:

$$\mathrm{E}(X_1 + X_2 + \cdots + X_n) = n\mu$$

$$\mathrm{sd}(X_1 + X_2 + \cdots + X_n) = \sqrt{n}\sigma$$

Sampling distributions

$\mathrm{E}(\overline{X}) = \mu, \quad \mathrm{sd}(\overline{X}) = \dfrac{\sigma}{\sqrt{n}}$

$\mathrm{E}(\widehat{P}) = p, \quad \mathrm{sd}(\widehat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$

CONTINUED

Standard error of a difference (independent estimates)

$$\text{se}(\widehat{\theta}_1 - \widehat{\theta}_2) = \sqrt{\text{se}(\widehat{\theta}_1)^2 + \text{se}(\widehat{\theta}_2)^2}$$

Confidence intervals and t-tests

Confidence interval: $\quad$ *estimate* $\pm$ t *standard errors*

$$\widehat{\theta} \pm t_{df}(\alpha/2)\text{se}(\widehat{\theta})$$

t-test statistic: $\quad t_0 = \dfrac{\textit{estimate} - \textit{hypothesised value}}{\textit{standard error}}$

$$t_0 = \frac{\widehat{\theta} - \theta_0}{\text{se}(\widehat{\theta})}$$

Applications

Mean $\boldsymbol{\mu_X}$: $\quad \widehat{\theta} = \overline{x}, \quad \text{se}(\overline{x}) = \dfrac{s_X}{\sqrt{n}}, \quad df = n - 1$

Proportion $\boldsymbol{p}$: $\quad \widehat{\theta} = \widehat{p}, \quad \text{se}(\widehat{p}) = \sqrt{\dfrac{\widehat{p}(1-\widehat{p})}{n}}, \quad df = \infty$

Difference between two means $\boldsymbol{\mu_1 - \mu_2}$: $\quad \widehat{\theta} = \overline{x}_1 - \overline{x}_2$ (independent samples),

$$\text{se}(\overline{x}_1 - \overline{x}_2) = \sqrt{\text{se}(\overline{x}_1)^2 + \text{se}(\overline{x}_2)^2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \quad df = \text{Min}(n_1 - 1, n_2 - 1)$$

Difference in proportions $\boldsymbol{p_1 - p_2}$: $\quad \widehat{\theta} = \widehat{p}_1 - \widehat{p}_2$ with

(a) ***Proportions from two independent samples*** of sizes n_1, n_2 respectively

$$\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}} \qquad df = \infty$$

(b) ***One sample of size n, several response categories***

$$\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{\widehat{p}_1 + \widehat{p}_2 - (\widehat{p}_1 - \widehat{p}_2)^2}{n}} \qquad df = \infty$$

(c) ***One sample of size n, many yes/no items***

$$\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{\text{Min}(\widehat{p}_1 + \widehat{p}_2, \widehat{q}_1 + \widehat{q}_2) - (\widehat{p}_1 - \widehat{p}_2)^2}{n}} \qquad df = \infty$$

where $\widehat{q}_1 = 1 - \widehat{p}_1$ and $\widehat{q}_2 = 1 - \widehat{p}_2$

CONTINUED

The *F*-test (ANOVA)

$$f_0 = \frac{s_B^2}{s_W^2} \quad df_1 = k - 1 \quad df_2 = n_{\text{tot}} - k$$

The Chi-square test

$$x_0^2 = \sum_{\text{all cells in the table}} \frac{(\text{observed} \ - \ \text{expected})^2}{\text{expected}}$$

For one-way tables:

$$df = J - 1$$

For two-way tables:

$$\text{Expected count in cell } (i, j) = \frac{R_i C_j}{n}$$

$$df = (I - 1)(J - 1)$$

Regression

Fitted least-squares regression line: $\widehat{y} = \widehat{\beta}_0 + \widehat{\beta}_1 x$

Inference about the intercept, β_0, and the slope, β_1: $df = n - 2$

ANSWER SHEET FOLLOWS