# Chapter 7

## Exercises for Section 7.2.1

**1.** **(a)** To halve sd($\overline{X}$) we need $2^2 = 4$ times as many observations which is 40 in total or 30 in addition to the 10 we already have.

**(b)** To reduce sd($\overline{X}$) to one third of its original size we need $3^2 = 9$ times as many observations which is 90 in total or 80 in addition to the 10 we already have.

**(c)** To reduce sd($\overline{X}$) to one 9th of its original size we need $9^2 = 81$ times as many observations which is 810 in total or 800 in addition the 10 we already have.

**2.** Let $X$ be the monthly profit. Then $X \sim$ Normal($\mu = 10, \sigma = 3.5$) so that $\overline{X} \sim$ Normal($\mu_{\overline{X}} = 10, \sigma_{\overline{X}} = \frac{3.5}{\sqrt{6}} = 1.4289$). Using these values for $\mu_{\overline{X}}$ and $\sigma_{\overline{X}}$, pr($\overline{X} > 8.5$) = 0.8531 (computer).

## Exercises for Section 7.2.2

**1.** We can assume approximate Normality as $n = 50$. Hence $\overline{X}$ is approximately Normally distributed with $\mu_{\overline{X}} = 100$ and $\sigma_{\overline{X}} = \frac{15}{\sqrt{50}} = 2.1213$ giving pr($\overline{X} < 97$) $\approx 0.07865$. (8%)

**2.** Let $\overline{X}$ be the average service time. We can assume approximate Normality as $n = 50$.

**(a)** As $\overline{X}$ is approximately Normally distributed with $\mu_{\overline{X}} = 3.1$ and $\sigma_{\overline{X}} = \frac{1.2}{\sqrt{50}} =$ 0.16971), pr($\overline{X} < 3.3$) $\approx 0.8807$.

**(b)** Total = $T = 50\overline{X}$ is approximately Normally distributed with mean $\mu_T = 50 \times 3.1 = 155$ and standard deviation $\sigma_T = \sqrt{50} \times 1.2 = 8.4853$. Thus, pr($T < 150$) $\approx 0.2778$.

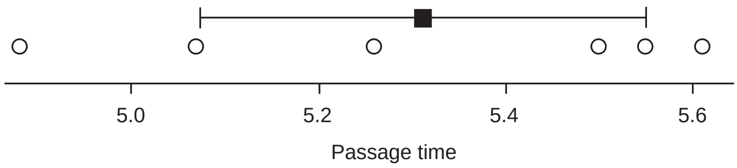
**3.** **(a)** Although $n$ is only 28 we shall assume we can use the Normal approximation. Then, $\overline{X} \sim$ Normal($\mu_{\overline{X}} = 620.6, \sigma_{\overline{X}} = \frac{241.5}{\sqrt{28}} = 45.6392$) so that pr($\overline{X} \geq 795.1$) $\approx$ 0.0001.
No, we do not believe they are the same. Testosterone levels seem to be higher in smokers. We used the mean and standard deviation for nonsmokers in our calculations. The probability is almost zero that a sample of nonsmokers of this size would have a mean testosterone level as high as we observed in our sample of smokers.

**(b)** Assuming that the serum level $X$ for nonsmokers has, approximately, a Normal ($\mu = 620.6, \sigma = 241.5$) distribution, the proportion above 795.1 is given by pr($X \geq 795.1$) = 0.2350.

**(c)** No, as these answers deal with different quantities. In (a) we are dealing with a sample mean (average) of measurements taken from 28 men. If we repeatedly took such samples, the sample mean would almost never be 795.1 or bigger (approximately 9999 samples in every 10,000 taken would give a value of $\overline{x}$ *smaller* than 795.1). In (b) we are dealing with the behavior of single individuals. Single individuals quite often give a value of 795.1 or bigger (in fact 23.5% of individuals do). The reason why this is happening is that individual measurements are more variable than averages are.

## Exercises for Section 7.2.3

**1.** **(a)**




**(b)** $\text{se}(\overline{x}) = \frac{s_X}{\sqrt{n}} = 0.1195$. Now $\overline{x} \pm 2\text{se}(\overline{x}) = 5.3117 \;\pm\; 2 \times 0.1195$, or $[5.07, 5.55]$. This has been added to plot above.

**(c)** Since $\frac{5.8 - 5.3117}{0.1195} = 4.09$, the value 5.8 is 4.09 standard errors above the sample mean from the data.

**2.** **(a)** $\text{se}(\overline{x}) = 57.696$. The two-standard-error interval is $\overline{x} \pm 2\text{se}(\overline{x}) = 795.1 \pm 2 \times 57.696$, or $[679.7, 910.5]$. It is a fairly safe bet that the true value is somewhere between 680 and 911 ng/dL.

**(b)** No, the value 620.6 ng/dL is not plausible as, since $\frac{620-6-795.1}{57.696} = -3.14$, this value is more than 3 standard errors below the estimate we obtained from our data. Alternatively, we could reach this conclusion by noting that 620.6 lies outside our 2-standard-error interval for the true mean.

**3.** Using a 2 standard-error interval, $\overline{x} \pm 2\text{se}(\overline{x}) = \overline{x} \pm 2\frac{s_X}{\sqrt{n}} = 250 \pm 2 \times \frac{50}{\sqrt{40}}$, i.e., $[234.2, 265.8]$. It is a fairly safe bet that the true average daily rate lies somewhere between \$234 and \$266.

## Exercises for Section 7.3.1

**1.** **(a)** Since $\widehat{p} \pm 2\text{se}(\widehat{p}) = 0.39 \pm 2\sqrt{\frac{0.39 \times 0.61}{90}}$, or $[0.29, 0.49]$, it is a fairly safe bet that the true percentage of music students with fathers in the highest socioeconomic group lies somewhere between 29% and 49%.

**(b)** No, the value 0.23 or 23% is not plausible as, since $\frac{0.23 - 0.39}{0.05141} = -3.1$, the value 0.23 is 3.1 standard errors below our data estimate.
Alternatively, we could reach the same conclusion by noting that 0.23 lies outside our two-standard-error interval for the true value.

**2.** *Professionals*: $0.32 \pm 2\sqrt{\frac{0.32 \times 0.68}{2280}}$, i.e., $[0.30, 0.34]$. It is a fairly safe bet that the true percentage of Yahoo users who are professionals lies somewhere between 30% and 34%.
*University*: $0.4 \pm 2\sqrt{\frac{0.4 \times 0.6}{2280}}$, i.e., $[0.38, 0.42]$. It is a fairly safe bet that the true percentage of Yahoo users who have been to university lies somewhere between 38% and 42%. [All of this assumes that we are making inferences from a random sample of Yahoo users.]

**3.** We could work from a two-standard-error interval, namely, $\widehat{p} \pm 2\text{se}(\widehat{p}) = 0.48 \pm 2\sqrt{\frac{0.48 \times 0.52}{825}}$, or $[0.445, 0.515]$, and then argue that there are values of $p$ above 50%

(namely, 0.5 to 0.515) that are plausible values because they lie in the interval. Alternatively, we could argue that as 0.5 (50%) is only $(0.5-0.48)/\sqrt{0.48 \times 0.52/825} = 1.15$ standard errors above our data estimate of 0.48, some values of $p$ greater than 0.5 are plausible.

**4.** **(a)** The two-standard-error interval is $\widehat{p} \pm 2\text{se}(\widehat{p}) = 0.176 \pm 2\sqrt{\frac{0.176\times0.824}{8000}}$, or $[0.167, 0.185]$. It is a fairly safe bet that the true percentage of American women who had been raped lies somewhere between 16.7% and 18.5%.

**(b)** The two-standard error interval is $0.216 \pm 2\sqrt{\frac{0.216\times0.784}{1323}}$, or $[0.19, 0.24]$, indicating that somewhere between 19% and 24% of all American women who had been raped were under 12 when first raped. [Note that the interval in (b) is considerably wider than that in (a) indicating greater uncertainty about the true percentage.]

**(c)** The two-standard error interval is $0.558 \pm 2\sqrt{\frac{0.558\times0.442}{1323}}$, or $[0.531, 0.585]$, indicating that somewhere between 53% and 59% of all American women who had been raped were under 18 when first raped.

## Exercises for Section 7.5

**1.** **(a)** $\text{se}(\widehat{p}_{11} - \widehat{p}_{13}) = \sqrt{\text{se}(\widehat{p}_{11})^2 + \text{se}(\widehat{p}_{13})^2} = \sqrt{0.031^2 + 0.030^2} = 0.04314$.

**(b)** $\frac{0.474-0.303}{0.04314} = 3.96$. The two sample proportions are nearly 4 standard errors apart clearly signalling that the corresponding true proportions smoking at least once in grades 11 and 13 are different. There is a real drop off.

**(c)** The two-standard-error interval is $\widehat{p}_{11} - \widehat{p}_{13} \pm 2\text{se}(\widehat{p}_{11} - \widehat{p}_{13}) = 0.474 - 0.303 \pm 2 \times 0.04314$, or $[0.09, 0.26]$. It is a fairly safe bet that the true percentage of grade 11 students using cigarettes at least once was larger than the corresponding percentage for grade 13 students by somewhere between 9 and 26 percentage points.

**(d)** For discussion. Some possibilities include students tending to experiment in grade 11 and then not continuing to smoke, the possibility that the group of students who stop attending school between grades 11 and 13 includes a higher proportion of the smokers, or that there are different social dynamics in different years of students.

**2.** We will denote observations 1–6 as being from group 1 and observations 7–29 as being from group 2. Then, $\overline{x}_1 = 5.3117$, $s_1 = 0.2928$: $\overline{x}_2 = 5.4835$, $s_2 = 0.1904$.

**(a)** $\text{se}(\overline{x}_2 - \overline{x}_1) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{0.2928^2}{6} + \frac{0.1904^2}{23}} = 0.1260$.
As the two sample means are only $\frac{5.4835-5.3117}{0.126} = 1.36$ standard errors apart, we have no evidence that the corresponding true means differ. Thus we have no evidence that changing the wire changed the quantity the experiment was measuring.

**(b)** The two-standard-error interval is $\overline{x}_2 - \overline{x}_1 \pm 2\text{se}(\overline{x}_2 - \overline{x}_1) = 5.4835 - 5.3117 \pm 2 \times 0.126$, or $[-0.08, 0.42]$. The quantity being measured after changing the wire could be anywhere between 0.08 units smaller and 0.42 units bigger than the quantity being measured before the wire was changed. This includes the possibility of "no change."

**3.** $\widehat{p}_{before} - \widehat{p}_{after} = 0.31 - 0.20 = 0.11$. Then,

$$\text{se}(\widehat{p}_{before} - \widehat{p}_{after}) = \sqrt{\frac{\widehat{p}_{before}(1 - \widehat{p}_{before})}{n_{before}} + \frac{\widehat{p}_{after}(1 - \widehat{p}_{after})}{n_{after}}} = \sqrt{\frac{0.31 \times 0.69}{186} + \frac{0.2 \times 0.8}{97}} = 0.05291.$$

**(a)** As $\frac{0.31-0.20}{0.05291} = 2.08$, the sample proportions are more than 2 standard errors apart. It is a fairly safe bet that there was a real change.

**(b)** The two-standard-error interval is $0.11 \pm 2 \times 0.0529$, or $[0.004, 0.216]$. It is a fairly safe bet that the percentage having a one-time encounter in the past 3 months decreased by somewhere between about 0.4 and 22 percentage points.

**(c)** The population of people who use the clinic "frozen" at the two different points in time. We were assuming that the sets of people questioned were random samples from these two populations.

**(d)** We would tend to believe that it was the anouncement if there were no other changes we could think of which might have affected the types of people visiting the clinic or their behavior. We would want to investigate what else had changed over this period.

**4. (a)** In each case, $\overline{x} \pm 2\text{se}(\overline{x}) = \overline{x} \pm 2\frac{s_X}{\sqrt{n}}$ gives a two-standard-error interval for the true mean for the relevant group.
None: $620.6 \pm 2 \times \frac{241.5}{\sqrt{62}}$, or $[559, 682]$.
$1 - 30$ : $715.6 \pm 2 \times \frac{248.0}{\sqrt{31}}$, or $[627, 805]$.
$31 - 70$ : $795.1 \pm 2 \times \frac{305.3}{\sqrt{28}}$, or $[680, 910]$.

**(b) (i)** The two-standard-error interval is $\overline{x}_{31-70} - \overline{x}_{none} \pm 2\text{se}(\overline{x}_{31-70} - \overline{x}_{none}) = 795.1 - 620.6 \pm 2\sqrt{\frac{305.3^2}{28} + \frac{241.5^2}{62}}$, or $[44, 305]$, which places the true mean testosterone level for the 31–70 group as being somewhere between 44 and 305 ng/dL higher than the true mean for nonsmokers. It is not plausible that there is no difference.

**(ii)** The two-standard-error interval is $\overline{x}_{1-30} - \overline{x}_{none} \pm 2\text{se}(\overline{x}_{1-30} - \overline{x}_{none}) = 715.6 - 620.6 \pm 2\sqrt{\frac{248.0^2}{31} + \frac{241.5^2}{62}} = 95 \pm 108$, or $[-13, 203]$. The true mean testosterone level for the 1–30 group is somewhere between being 13 ng/dL below and 305 ng/dL higher than the true mean for nonsmokers. This includes the possibility that there is no difference at all. It is plausible that there is no difference.

**(c)** We can't use individual two-standard-error intervals for making comparisons when there is overlap as the combined variation is not taken into account properly – see Section 7.5.3.

**(d)** We have an observational study and not a controlled experiment. There could be some other variable related to both smoking and testosterone that is causing the relationship we see. The causal influence could even be in the other direction. Perhaps high-testosterone men are more likely to take up smoking.

**(e)** No, we cannot reach any such conclusion. The two-standard-error intervals refer only to the true *means* and say nothing about any other aspect of the distributions. We see from Fig. 7.5.2 that there is actually a great deal of overlap in the testosterone levels of the groups.

## Review Exercises 7

**1.** The first 2 parts of this question concern ideas in Section 6.4.4 (see the paragraph entitled *Independent individuals versus Clones*).

**(a)** Here $Y = Sum = \sum_{i=1}^{7} X_i$ so that $\mu_Y = n\,\mu_X = 7 \times 87 = 609$ and $\sigma_Y = \sqrt{n}\,\sigma_X = \sqrt{7} \times 23 = 60.85$.

**(b)** Here $W = 7X$ so that $\mu_W = 7 \times \mu_X = 7 \times 87 = 609$ and $\sigma_W = 7 \times \sigma_X = 7 \times 23 = 161$.

**(c)** For a sample mean from a random sample of size $n = 36$, $\mu_{\overline{X}} = \mu_X = 87$ and $\sigma_{\overline{X}} = \frac{\sigma_X}{n} = \frac{23}{6} = 3.83$.

**(d)** Normal; the central limit theorem.

**(e)** It would have the same mean and smaller spread. More technically, as $144 = 4 \times 36$, the standard deviation of $\overline{X}$ for a sample of size 144 is one half as large as it is for a sample of size 36. If the original distribution was extremely non-Normal the distribution might also be more Normal looking.

**2.** Let $X$ = score for a single die. $\text{pr}(X = i) = \frac{1}{6}$ for $i = 1, 2, 3, 4, 5, 6$.

**(a)** Theory tells us that $\text{E}(\overline{X}) = \text{E}(X) = \sum x\text{pr}(x) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + \ldots + 6 \times \frac{1}{6} = 3.5$.

**(b)** Taking the median and mean of the 3 observations given for each experiment gives the following table:

| Expt. No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Medians, $M$ | 4 | 5 | 4 | 3 | 5 | 2 | 3 | 5 | 3 | 2 |
| Means, $\overline{x}$ | 4 | $4\frac{1}{3}$ | 4 | $3\frac{1}{3}$ | 4 | 3 | $2\frac{2}{3}$ | 4 | $3\ \frac{1}{3}$ | 3 |

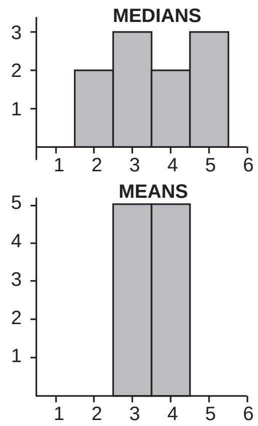
The sample mean and standard deviation of the 10 sample medians are respectively $\overline{x}_{med} = 3.6$ and $s_{med} = 1.174$.

The sample mean and standard deviation of the $n = 10$ sample means are $\overline{x}_{mean} = 3.567$ and $s_{mean} = 0.5676$.

**(c)** The frequencies for the medians and means are as follows:

| Interval | 1.5 - 2.5 | 2.5 - 3.5 | 3.5 - 4.5 | 4.5 - 5.5 |
|---|---|---|---|---|
| Frequencies for medians | 2 | 3 | 2 | 3 |
| Frequencies for means | 0 | 5 | 5 | 0 |

Histograms:




**(d)** From (b), we see that the standard deviation for the set of means from the 10 different samples is less than $\frac{1}{2}$ of that for the medians. Thus the median is subject to a higher degree of sampling variation than the mean. This conclusion is backed up by our histograms: the data for the means is all contained in the interval 2.5 - 4.5. but that for the medians is spread over 1.5 - 5.5 (which is twice as wide).

**3.** **(a)** $X \sim \text{Binomial}(n = 870, p = 0.79)$.

**(b)** Suppose the true value of $p$ really was $p = 0.79$. Then $\text{sd}(\widehat{p}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.79\times0.21}{870}} = 0.01381$. Now $\frac{0.3897-0.79}{0.01381} \approx -29$. The observed data value is 29 standard deviations below 0.79. This would virtually never happen if the selection was random. We do not believe that the jury drawing was at random.

**(c)** Suppose the true value of $p$ really was $p = 0.06$. Then $\text{sd}(\widehat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.06\times0.94}{405}} = 0.01180$. Since $\frac{0.037-0.06}{0.0118} = -1.95$ the observed data value is almost 2 standard deviations below 0.06. Values this far away would seldom occur if hiring was at random.

**(d)** The first recommendation questions the randomness of the jury selection process whereas the second questions whether a particular factor, race in this case, was involved in the teacher-hiring process. The first does not attempt to apportion blame to any factor in the event that the selection process was not random; the second does tend to attribute blame on racial discrimination as a primary factor in the event that the selection process was not random. The first limits "suspect" to applying to a specific group, namely social scientists while the second makes no such limitation.

**(e)** A prima facie case is a case considered strong enough that it must be answered in court. While the occurrence of gross statistical disparities is an indication that the selection of teachers is not random, the factors that are used as criteria in

the hiring process may not be necessarily based on any desire to discriminate.
*Reasons for the decision*: Past experience in other areas may have pointed to the practice of discrimination, and the strong opinion is an expression of the Court's disapproval in Law. Discrimination is hard to prove so that the opinion expressed tends to shift the burden onto proving nondiscrimination.
*Reasons against*: In the absence of further evidence, it is dangerous to blame just one factor in the actual hiring process.
Suppose, for example, that hiring was done blindly on the basis of qualifications. It might be that several factors including past discriminatory practices at other levels may have led to black teachers having lower qualifications on average. Basing hiring decisions on qualifications would then lead to a lower proportion of black teachers being hired. Would this make using qualifications discriminatory? Should the problem of black teachers being under-represented be attacked using the hiring process or at other levels? Political processes have long struggled with issues like this without reaching solutions that everybody can live with. So we leave it for you to decide!

**4. (a)** Using $\text{sd}(\widehat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.25}{n}}$, we obtain **(i)** 0.050 **(ii)** 0.01581 **(iii)** 0.005.

**(b) (i)** As $\widehat{P}$ is approximately Normal with $\mu_{\hat{P}} = 0.5$ and $\sigma_{\hat{P}}[= \text{sd}(\widehat{P})] = 0.050$,

$\text{pr}(0.49 \le \widehat{P} \le 0.51) \approx 0.1585$.

**(ii)** As $\widehat{P}$ is approximately Normal with $\mu_{\hat{P}} = 0.5$ and $\sigma_{\hat{P}} = 0.0158110$,

$\text{pr}(0.49 \le \widehat{P} \le 0.51) \approx 0.4729$.

**(iii)** As $\widehat{P}$ is approximately Normal with $\mu_{\hat{P}} = 0.5$ and $\sigma_{\hat{P}} = 0.005$,

$\text{pr}(0.49 \le \widehat{P} \le 0.51) \approx 0.9545$.

***(c)** Because of the symmetry of the approximate Normal distribution of $\widehat{P}$ about $\mu_{\hat{P}} = 0.5$, if $\text{pr}(0.499 \le \widehat{P} \le 0.501) = 0.95$ then $\text{pr}(\widehat{P} \le 0.501) = 0.975$. For the standard Normal $\text{pr}(Z \le 1.9600) = 0.975$. This tells us that 0.501 must be 1.96 standard deviations above the mean, i.e., $0.501 = 0.5 + 1.96\sqrt{\frac{0.25}{n}}$ so that $0.001 = 1.96\sqrt{\frac{0.25}{n}}$. Solving algebraically for $n$ we obtain $n = \left(\frac{1.96}{0.001}\right)^2 \times 0.25 =$ 960,400 which is almost a million.

**(d)** If $X$ is the number of heads, then $X$ has a Binomial distribution with $\text{E}(X) = np$ and $\text{sd}(X) = \sqrt{np(1-p)}$. Now the difference between the number of heads and the number of tails is $D = X - (n - X) = 2X - n$ so that, with $p = \frac{1}{2}$, $\text{E}(D) = 2\,\text{E}(X) - n = 2np - n = 2 \times n \times 0.5 - n = 0$ and $\text{sd}(D) = 2\,\text{sd}(X) = 2\sqrt{n \times 0.5 \times 0.5} = \sqrt{n}$.

**(e)** The difference between the number of heads and the number of tails becomes *more variable* as number of tosses made increases.

**5. (a)** Let $\widehat{P}$ be the proportion of voters who voted for her. Then $\widehat{P}$ is approximately Normally distributed with $\mu_{\hat{P}} = p = 0.6$ and $\sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6 \times 0.4}{n}}$.

**(b)** When $n = 200$, $\sigma_{\hat{P}} = \sqrt{\frac{0.6 \times 0.4}{200}} = 0.03464$ so that $\text{pr}(\widehat{P} > 0.5) = 0.998$.

**6. (a)** Let $\widehat{P}$ be the proportion of defects in the sample and $p$ be the true proportion in the whole batch. Then $\widehat{P}$ is approximately Normally distributed with $\mu_{\hat{P}} = p$ and $\sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{400}}$. If $p = 0.05$ then $\mu_{\hat{P}} = 0.05$ and $\sigma_{\hat{P}} = \sqrt{\frac{0.05\times 0.95}{400}} = 0.01090$. We want the cutoff value $c$ so that $\text{pr}(\widehat{P} > c) = 0.10$, or equivalently, $\text{pr}(\widehat{P} \le c) = 0.90$. This is an inverse-probability problem (cf. Fig. 6.2.6). Using our values for the mean and standard deviation, programs return $c = 0.0640$. If $c = 0.0640$ is the sample proportion above which we want to stop accepting batches, the number of defectives in a sample at which we refuse the batch should be $M = 400 \times 0.0640 = 25.6$. However, $M$ must be a whole number so we will take $M = 25$.

**(b)** If the true proportion of defects is 0.02, then $\mu_{\hat{P}} = 0.02$ and $\sigma_{\hat{P}} = \sqrt{\frac{0.02\times 0.98}{400}} = 0.007$. Using these values, $\text{pr}(\widehat{P} > \frac{25}{400} = 0.0625)$ is approximately 0 (it is much less than 1 in a million).

***(c)** Let $C$ be the cutoff value. For a standard Normal $\text{pr}(Z < -1.28155) = \text{pr}(Z > 1.28155) = 0.10$. Thus our cutoff must be 1.28155 standard deviations below the mean when $p = 0.05$, giving

$$C = 0.05 - 1.28155\sqrt{\frac{0.05 \times 0.95}{n}} \quad \text{simplified to} \quad \sqrt{n}(C - 0.05) = -0.27931.$$

It must also be 1.28155 standard deviations above the mean when $p = 0.02$ giving

$$C = 0.02 + 1.28155\sqrt{\frac{0.02 \times 0.98}{n}} \quad \text{simplified to} \quad \sqrt{n}(C - 0.02) = +0.17942.$$

Solving these 2 equations algebraically for both $C$ and $n$ we get $C = 0.0317$ and $n = 234$.

**7. (a)** $\mu_X = \text{E}(X) = \sum x\,\text{pr}(x) = 1 \times \frac{20}{38} + (-1) \times \frac{18}{38} = \frac{2}{38} = 0.05263$.
$\sum(x - \mu_X)^2 pr(x) = (1 - 0.05263)^2 \times \frac{20}{38} + (-1 - 0.05263)^2 \times \frac{18}{38} = 0.99723$.
$\sigma_X = \text{sd}(X) = \sqrt{0.99723} = 0.9986 \approx 1$.

**(b)** **(i)** We are working with a sum from a random sample of size $n = 50$ from this distribution so $\text{E}(Sum) = n\,\mu_X = 50 \times 0.05263 = 2.6315$ and $\text{sd}(Sum) = \sqrt{n}\,\sigma_X = \sqrt{50} \times 0.9986 = 7.0612$.
**(ii)** $\text{E}(\overline{X}) = \mu_X = 0.05263$ and $\text{sd}(\overline{X}) = \frac{\sigma_X}{\sqrt{n}} = \frac{0.9986}{\sqrt{50}} = 0.14122$.

**(c)** Basically already done in (b). **(i)** $\mu_{Sum} = n \times 0.05263$ and $\sigma_{Sum} = \sqrt{n} \times 0.9986$.
**(ii)** $\mu_{\overline{X}} = 0.05263$ and $\sigma_{\overline{X}} = \frac{0.9986}{\sqrt{n}}$.

**(d)** The casino makes money if the average winnings from the 50 bets exceeds \$0. The gambler makes money if the average is less than \$0. When $n = 50$ we have $\overline{X} \sim \text{Normal}(\mu_{\overline{X}} = 0.05263, \sigma_{\overline{X}} = 0.14122)$ so that $\text{pr}(\overline{X} < 0) = 0.3547$. Thus, after 50 bets, approximately 35% of gamblers have made money.

**(e)** $\overline{X} \sim \text{Normal}\ (\mu_{\overline{X}} = 0.05263, \sigma_{\overline{X}} = \frac{0.9986}{\sqrt{n}})$. When $n = 1000$, $\sigma_{\overline{X}} = 0.031579$ and $\text{pr}(\overline{X} < 0) = 0.0478$. After 1000 bets, approximately 1 person in 20 has made money. When $n = 100,000$, $\sigma_{\overline{X}} = 0.003158$ and $\text{pr}(\overline{X} < 0) \approx 10^{-62}$. Essentially no one has made money (except the casino of course). The probability is even smaller after a million bets.

**(f)** We have drastically rounded the data in the following table as we only want to give a broad picture. We have used 3 standard deviations about the mean each time so we are getting the range of values within which the result will fall 99.7% of the time. These intervals relate to the casino's winnings.

| $n$ | | Average | Total |
|---|---|---|---|
| | $\mu_X \pm 3\frac{\sigma_X}{\sqrt{n}}$ | | $n\mu \pm 3\sqrt{n}\sigma_X)$ |
| 50 | $0.0526 \pm 0.4237$ | [-0.37,0.48] | [-18.5,24] |
| 1000 | $0.0526 \pm 0.0947$ | [-0.04,0.15] | [-40,150] |
| 100,000 | $0.0526 \pm 0.0095$ | [0.043,0.062] | [4300,6200] |
| 1,000,000 | $0.0526 \pm 0.0030$ | [0.050,0.056] | [50,000, 56,000] |

**8.** **(a)** Using a random sample, as we have here, the sample proportion is an unbiased estimate of the population proportion.

**(b)** This is not the population we sampled. People's opinions could change from July to September.

**(c)** This is not the population we sampled. "Public eating establishments" also includes places other than restaurants.

**9.** We use $\text{sd}(\widehat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4\times0.6}{n}}$. We have to think about what $n$ should be.

**(a)** Units are couples and we have $n = 100$ of them so $\text{sd}(\widehat{P}) = \frac{0.4\times0.6}{\sqrt{100}} = 0.024$.

**(b)** Units are individuals and we have $n = 200$ of them so $\text{sd}(\widehat{P}) = \frac{0.4\times0.6}{\sqrt{200}} = 0.0170$.

**(c)** The reality will be between the two extremes of (a) and (b). Individuals within a married couple do not necessarily think alike but they are more likely to think alike than are any two random individuals. Thus the variability in $\widehat{P}$ values will be larger than predicted by (b) and smaller than predicted by (a). We would tend to favor (a) which overstates the uncertainty.

**10.** An opportunity sample (like Kinsey's) will almost certainly provide heavily biased estimates of things like population proportions. Having a large sample size does not help – see Section 1.1. We would always trust a smaller random sample over a larger opportunity sample.
The quotation is perpetuating the myth that we need larger samples to investigate larger populations and also the myth that bigger samples are more reliable. Where populations are large so that samples constitute a small part of the population, a sample of 1000 say provides equally precise estimates for proportions of a population of size 100,000,000 as it does for 100,000. They have ignored the critical issue of how the sample was obtained.

**11.** **(a)** $\text{se}(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{(0.66)(0.34)}{943}} = 0.0154$.

**(b)** $\text{se}(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{0.5\times0.5}{172}} = 0.0381$.

**(c)** $\frac{0.0381}{0.0154} = 2.47$ times bigger.

**(d)** $\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}} = \sqrt{\frac{0.67\times 0.33}{221} + \frac{0.5\times 0.5}{172}} = 0.0495.$

**(e)** $\sqrt{\frac{0.67\times 0.33}{221}} = 0.0316$. The ratios are $\frac{0.0495}{0.0316} = 1.57$ times bigger and $\frac{0.0495}{0.0381} = 1.30$ times bigger.

**(f)** It will be too small for looking at proportions calculated from subsets of the data and for looking at differences between proportions. The poll will appear to be more accurate than it really is in these situations. See Section 8.5.3 for a detailed discussion of these issues.

**12. (a)** $\text{se}(\overline{x}_1 - \overline{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{1.82^2}{53} + \frac{1.53^2}{60}} = 0.3186.$

**(b)** The two-standard-error interval is $7.90 - 4.30 \pm 2 \times 0.3186 = [2.96, 4.24]$.

**(c)** As this interval is well away from zero, sexual content seems to make a difference. The true mean number of correctly remembered brands under these conditions is likely to be greater when sexual content is present by somewhere between 3.0 and 4.2 brands than when it is absent. There is too little information for us to criticize the experiment. We would hope, however, that the complete set of students was split into two groups at random.

**13. (a)** The proportion of the sample of downtown cap wearers who wear their caps backwards is $\widehat{p}_{dntn} = \frac{174}{407} = 0.4275$. The corresponding two-standard-error interval is $0.4275 \pm 2\sqrt{\frac{0.4275\times 0.5725}{407}}$ giving $[0.378, 0.477]$. This suggests that the true proportion of downtown cap wearers who wore their caps backwards at this time was somewhere between about 38% and 48%.

**(b)** For business school cap wearers, the proportion of the sample who wore their caps backward is $\widehat{p}_{bus} = \frac{107}{319} = 0.3354$. The standard error of the difference in sample proportions, $\text{se}(\widehat{p}_{dntn} - \widehat{p}_{bus}) = \sqrt{\frac{0.4275\times 0.5725}{407} + \frac{0.3354\times 0.6646}{319}} = 0.03606$. As $\frac{(0.4275-0.3354)}{0.03606} = 2.55$, the two sample proportions are more than 2 $\text{se}(\widehat{p}_{dntn} - \widehat{p}_{bus})$ apart suggesting that the true proportions are in fact different.

**(c)** No. They do not conflict as $\text{se}(\widehat{p}_{dntn} - \widehat{p}_{bus}) < \text{se}(\widehat{p}_{dntn}) + \text{se}(\widehat{p}_{bus})$. See Section 7.5.3 for a discussion.

**(d)** Yes. As the individual intervals do not overlap we know that the two-standard-error interval for the difference will not contain zero in each case (see Section 7.5.3).

**(e)** If we just observe individuals moving past some location, we will often get people in groups who are likely to do similar things like wearing their caps the same way, thus violating the independence assumption. We might also be getting a biased sample. The types of people walking past the locations we choose to observe may tend to behave differently from those found at other locations. Locations would have to be sampled carefully to avoid this.

**14. (a)** Looking across rows, the main trend we see is that these students tend to overestimate their grades. Overestimation appears to be most pronounced for low-GPA students where 33 of the 41 students overestimate with only 4 underestimating and 4 predicting correctly. The effect is less pronounced with high GPA students.

**(b)** *Low*: $\widehat{p}_L = \frac{33}{41} = 0.8049$, *high*: $\widehat{p}_H \frac{22}{44} = 0.4773$. The standard error of the difference in sample proportions is $\text{se}(\widehat{p}_L - \widehat{p}_H) = \sqrt{\frac{0.8049\times0.1951}{41} + \frac{0.4773\times0.5227}{44}} = 0.09747$. As $(0.8049 - 0.4773)/0.09747 = 3.361034$, $\widehat{p}_L$ is more than 3 standard errors larger than $\widehat{p}_H$. It is a fairly safe bet, therefore, that the true value of $p_L$ is larger than the true value of $p_H$, i.e., that low GPA students are more likely to overestimate their grades than are high GPA students.
Alternatively, we could argue by constructing a two-standard-error interval for the true difference $p_L - p_H$, namely $0.8049 - 0.4773 \pm 2 \times 0.09747$, or $[0.13, 0.52]$. For all values in this interval, the difference is greater than zero, i.e., $p_L$ is larger than $p_H$.

**(c)** Suppose we have grades ranging from A=best, B, C, D=worst. We will consider two extreme cases. If you usually get D's it is going to be hard to underestimate. All available choices for a prediction apart from D will usually correspond to an overestimate. On the other hand, if you usually get A's all predictions except A will usually correspond to an underestimate.

**15. (a)** $\text{se}(\overline{x}_{bottle} - \overline{x}_{breast}) = \sqrt{\frac{15.18^2}{90} + \frac{17.39^2}{210}} = 2.00$.
Since $\frac{(103.0-92.8)}{2.00} = 5.1$, the sample mean for breast-fed babies $\overline{x}_{breast}$ is more than 5 standard errors bigger than the sample mean for bottle-fed babies, so we conclude that the true mean IQ for breast-fed babies is larger than the true mean IQ for bottle-fed babies.

**(b)** The two-standard-error interval for the true difference $\mu_{breast} - \mu_{bottle}$ is given by $(103.0 - 92.8) \pm 2 \times 2.00$, or $[6.2, 14.2]$. This suggests that the true mean IQ for breast-fed babies is larger than that for bottle-fed babies by somewhere between 6 and 14 units.

**(c)** Preterm, low-birth-weight babies in the catchment areas of these special-care units fitting the profiles that trigger referral to these units.

**(d)** No, because this is observational data. Mothers chose to breast feed or not to breast feed. There was no random assignment. It may be, for example, that higher IQ mothers are more likely to choose to breast feed than lower IQ mothers and it is this (or one of a host of other possible differences) that leads to the observed IQ differences in the babies.

**(e)** Yes, as it partially answers the objection we raised in (d). We might expect mothers who wanted to breast-feed but could not (for physical reasons) to have similar IQs to those that wanted to and could. We note that the IQs of the babies of the wanted-to-but-couldn't group were similar to the IQs of the bottle-fed babies, and not to the breast-fed babies. This strengthens the impression that it is the breast-feeding itself that is causing the difference we are seeing.

**16. (a)** We would expect positive contrasts to increase the self-ratings and negative contrasts to decrease the self-ratings (for both males and females).

**(b)** Yes. We notice that the effect of the contrasts seems to be smaller for females than it is for males. (The shorter intervals also suggests less variation among females than among males.)

**(c)** $21.16 - 18.28 \pm 2\sqrt{\frac{4.31^2}{22} + \frac{3.26^2}{20}}$, or $[0.53, 5.2]$. This interval does not contain zero so that there is some evidence of a difference between the true means. Overlapping confidence intervals do not necessarily tell us that no difference has been established as the true standard error of the difference is less than the sum of the individual standard errors (see Section 7.5.3).

**(d)** (Reasoning from non-overlapping intervals)
*Males*: Self-ratings following positive contrasts are higher on average than those following control conditions and negative contrasts.
*Females*: Self-ratings for positive contrasts are higher on average than for negative contrasts.

**(e)** *Females*: The difference in sample means is $18.28 - 15.15 = 3.13$ and the standard error associated with that difference is $\sqrt{\frac{3.26^2}{20} + \frac{2.96^2}{21}} = 0.9740$.

**(f)** *Males*: The difference in sample means is $21.16 - 14.16 = 7.00$ and the standard error associated with that difference is $\sqrt{\frac{4.31^2}{22} + \frac{4.64^2}{22}} = 1.3502$.

***(g)** *Males–Females*: Our estimated difference is $7.00 - 3.13 = 3.87$ and the standard error associated with that difference is se $= \sqrt{0.9740^2 + 1.3502^2} = 1.665$ [using $\text{se}(\widehat{\theta}_1 - \widehat{\theta}_2) = \sqrt{\text{se}(\widehat{\theta}_1)^2 + \text{se}(\widehat{\theta}_2)^2}$]. The resulting two-standard-error interval is $[0.54, 7.2]$. Because the values in the interval for the difference in contrast effects between males and females are all positive, the interval does support the idea that the effect for males is bigger than it is for females. How much bigger? By somewhere between 0.5 and 7.2 units on average.

**(h)** Some ideas are: everyone should see a photograph of themselves, samples should be bigger, each person in a particular group should see the same photograph (not clear from our discussion of the study).

**(i)** Several questions are of interest. For example, once a student has been tested it might be an idea to show them a picture of the opposite type and see if they want to change their score. It would interesting to know how each student is rated in terms of attractiveness by their fellow students. This measure could be used as a "blocking" variable (see Section 1.2).

**17. (a)** The three answers are: (i) Those in the "humane" group should score higher than those in the "inhumane" group.
(ii) In the humane group, there should be a trend downwards from "typical" to "atypical" (we would be more affected by what we saw if we thought it was typical). In the inhumane group, the reverse should hold. (iii) We would expect the "control" group reactions would fall in between those of the humane and inhumane "no-information group".

**(b)** We will react to sample means more than two standard errors apart as providing evidence that differences between the corresponding true means exist. When sample means are less than 2 standard errors apart it is plausible that no true differences exist. In such situations we will say that we have not demonstrated a true difference.
We have not demonstrated a true difference between any of the three subgroups shown the inhumane portrayal.
We have not demonstrated a true difference between any of the subgroups shown

the humane portrayal.
We can conclude that the control group scored lower on average than the "typical" and "no-information" subgroups in the humane group and scored higher than the typical subgroup of the inhumane group.
Under "no-information" conditions, those seeing the humane portrayal have been demonstrated to score higher on average than those shown the inhumane portrayal.
Under typical conditions, those seeing the humane portrayal have been demonstrated to score higher on average than those shown the inhumane portrayal.
Even when told that the behavior is atypical, those seeing the humane portrayal have been demonstrated to score higher on average than those shown the inhumane portrayal.
The data supports contention (i) in (a), fails to demonstrate (ii), and partly supports (iii).

**(c)** Use larger samples with groups for each sex.

**(d)** Are there sex or age differences in the participants? What effect does the sex of the guard have? etc.

***18.** The two intervals are $[\widehat{\theta}_i - 2\text{se}(\widehat{\theta}_i), \widehat{\theta}_i + 2\text{se}(\widehat{\theta}_i)]$, i = 1,2.
Suppose $\widehat{\theta}_1$ is less than $\widehat{\theta}_2$. Then, since the intervals do not overlap,
$\widehat{\theta}_1 + 2\text{se}(\widehat{\theta}_1) < \widehat{\theta}_2 - 2\text{se}(\widehat{\theta}_2)$ and
$\widehat{\theta}_2 - \widehat{\theta}_1 > 2[\text{se}(\widehat{\theta}_1) + \text{se}(\widehat{\theta}_2)] \geq 2\sqrt{se(\widehat{\theta}_1)^2 + \text{se}(\widehat{\theta}_2)^2} = 2\,\text{se}(\widehat{\theta}_2 - \widehat{\theta}_1)$.
We have used the fact that $a^2 + b^2 + 2ab = (a+b)^2$, i.e., $\sqrt{a^2+b^2} \leq a + b$ when $a > 0$, $b > 0$.

***19. (a)** Our estimate of the number visiting the doctor in Canada is $\hat{t}_{can} = 0.1 \times 27 \times 10^6$ (proportion times population size). Similarly, $\hat{t}_{usa} = 0.2 \times 250 \times 10^6$ and $\hat{t}_{mex} = 0.6 \times 90 \times 10^6$. Thus the total number visiting the doctor is $0.1 \times 27 \times 10^6 + 0.2 \times 250 \times 10^6 + 0.6 \times 90 \times 10^6$.
The total population size is $27 \times 10^6 + 250 \times 10^6 + 90 \times 10^6 = 367 \times 10^6$. The proportion visiting the doctor is thus

$$\widehat{p} = \frac{0.1 \times 27 \times 10^6 + 0.2 \times 250 \times 10^6 + 0.6 \times 90 \times 10^6}{367 \times 10^6}.$$

This gives $\frac{106.70}{367.00} = 0.297$.

**(b)** Cancelling the $10^6$ terms in the displayed equation above, we see that $\widehat{p} = \frac{27}{367} \times 0.1 + \frac{250}{367} \times 0.2 + \frac{90}{367} \times 0.6$ which is of the form $a_{can}\widehat{p}_{can} + a_{usa}\widehat{p}_{usa} + a_{mex}\widehat{p}_{mex}$.

**(c)** $\text{sd}(\widehat{p}) = \sqrt{\text{sd}(a_{can}\widehat{p}_{can})^2 + \text{sd}(a_{usa}\widehat{p}_{usa})^2 + \text{sd}(a_{mex}\widehat{p}_{mex})^2}$
$= \sqrt{a_{can}^2\text{sd}(\widehat{p}_{can})^2 + a_{usa}^2\text{sd}(\widehat{p}_{usa})^2 + a_{mex}^2\text{sd}(\widehat{p}_{mex})^2}$
Thus,
$\text{se}(\widehat{p}) = \sqrt{a_{can}^2\text{se}(\widehat{p}_{can})^2 + a_{usa}^2\text{se}(\widehat{p}_{usa})^2 + a_{mex}^2\text{se}(\widehat{p}_{mex})^2}$
$= \sqrt{a_{can}^2\frac{\widehat{p}_{can}(1-\widehat{p}_{can})}{n_{can}} + a_{usa}^2\frac{\widehat{p}_{usa}(1-\widehat{p}_{usa})}{n_{usa}} a_{mex}^2\frac{\widehat{p}_{mex}(1-\widehat{p}_{mex})}{n_{mex}}}$
$= \sqrt{(\frac{27}{367})^2 \times \frac{0.1\times0.9}{1000} + (\frac{250}{367})^2 \times \frac{0.2\times0.8}{1000} + (\frac{90}{367})^2 \times \frac{0.6\times0.4}{1000}}$
$= 0.00944$

***20. (a)** $I = 100 \times \widehat{P} - 100 \times (1 - \widehat{P}) = 100(2\widehat{P} - 1) = 200\widehat{P} - 100$.

**(b)** $\text{sd}(I) = 200\,\text{sd}(\widehat{P})$.

**(c)** Solving $I = 100(2\widehat{P} - 1)$ we get $\widehat{P} = \frac{I}{100} - \frac{1}{2}$.

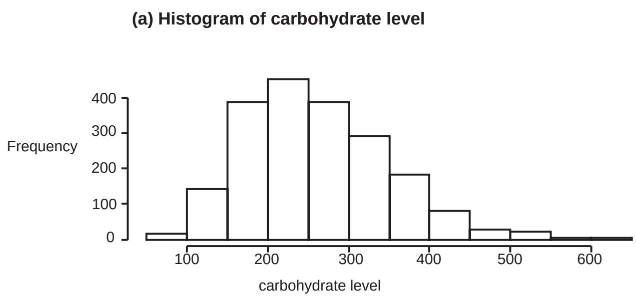
**(i)** When $I = 40$, $\widehat{p} = \frac{40}{200} + \frac{1}{2} = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$.

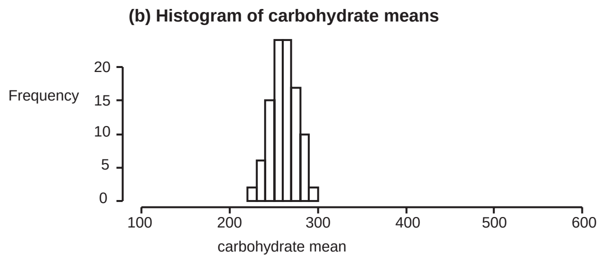
**(ii)** $\text{se}(I) = 200\,\text{se}(\widehat{p}) = 200 \times \sqrt{\frac{\widehat{p}(1-\widehat{p})}{1000}} = 200 \times \sqrt{\frac{0.7 \times 0.3}{1000}} = 2.90\%$.

**(d)** It is a measure of "confidence excess". It is positive when the majority of respondents are confident ($\widehat{p} > \frac{1}{2}$) and negative when only a minority are confident ($\widehat{p} < \frac{1}{2}$).

**Note about simulation exercises:** Simulation is a random process. Every time you do it, you get different answers. The results your simulations produce for questions 21–23 will differ from ours in detail but should be fairly similar.

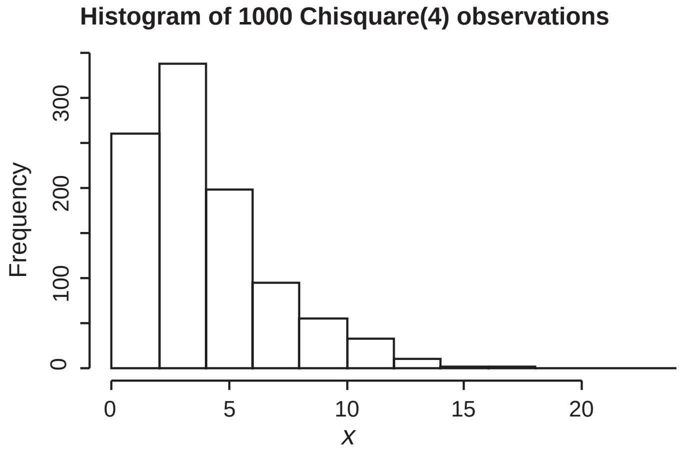
**21.** **(a)** Using the Chi-square distribution with 17 degrees of freedom, 1000 "carbohydrate levels" were generated and a histogram of the data follows. It is similar in shape to Fig. 6.1.1.

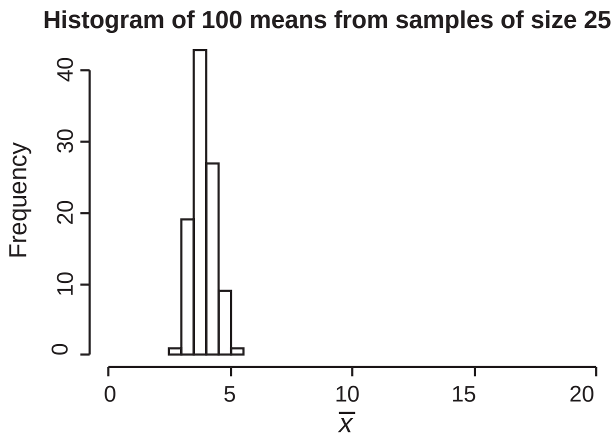



**(b)** 100 samples of size 25 were generated and a histogram of the 100 sample means is given above. It is plotted against the same horizontal scale as the plot for individual observations above it. We note that the histogram of sample means has a much smaller spread, showing that sample means are much less variable than individual observations. The histogram of the individual observations is very skewed while that of the sample means is much more bell shaped and Normal looking (a consequence of the central limit effect).

**(c)** Another 1000 carbohydrate levels were generated using a Chi-square distribution with 4 degrees of freedom. The histogram of the 1000 Chi-square random numbers and the histogram of the 100 sample means from samples of size 25 are given below. We observe the same sort of behavior as above.

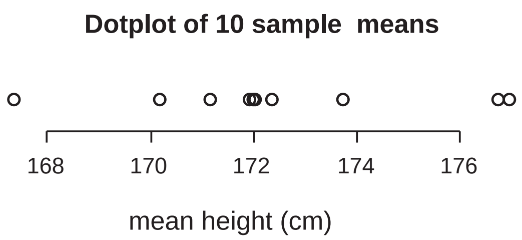


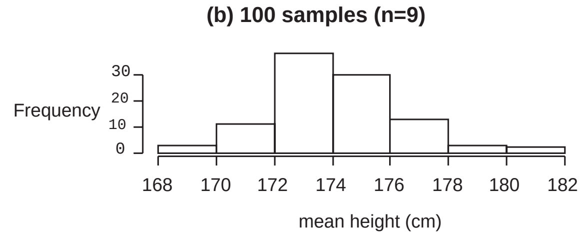

**22.** **(a)** 10 samples of 9 male heights were generated from a Normal($\mu$ = 174cm, $\sigma$ = 6.57cm) distribution giving the corresponding sample means:

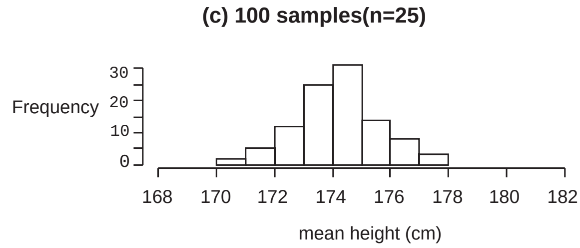
173.6 174.5 174.1 174.2 170.7 172.8 175.9 176.2 176.8 171.6

A dot plot of these sample means is given below. Each point on the graph represents the average height from a batch (sample) of 9 men. The smallest average we got was 170.7; the biggest was 176.2.




**(b)** A histogram of the 100 sample means follows.

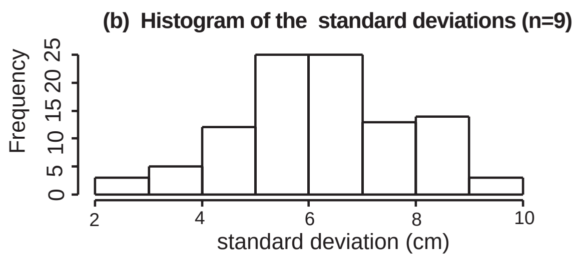


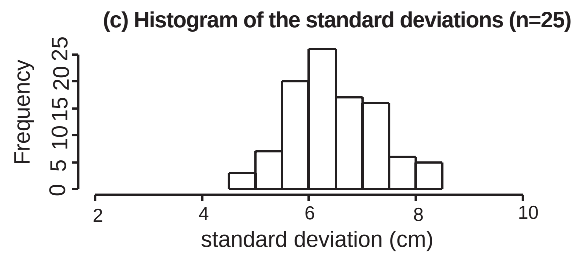




**(c)** 100 samples each of size 25 were generated and the histogram of the 100 sample means is given above.

**(d)** The sample standard deviation of the set of 100 sample means of size 9 in (b) was 2.222. From theory we would expect the standard deviation to be about $\frac{\sigma}{\sqrt{n}} = \frac{6.57}{\sqrt{9}} = 2.19$. The value we got was very close to this. The sample standard deviation of the set of 100 sample means of size 25 in (b) was 1.287, which may be compared with the theoretical standard deviation of $\frac{\sigma}{\sqrt{n}} = \frac{6.57}{\sqrt{25}} = 1.314$.

**(e)** The histograms of the 100 sample standard deviations for each sample of size 9 in (b), and for each sample of size 25 in (c) are given below. Both histograms appear to have similar centers. Ths histogram of standard deviations from the samples of size 25 shows a considerably smaller spread, but it is still distinctly skewed. (It is also strongly skewed for samples of size 1,000.)




**(f)** For each of the 100 samples in (b) we calculated a two-standard error interval $(\overline{x} \pm 2 \times \text{se}(\overline{x}))$ giving:

```
(169.8037, 183.5743); (170.6262, 178.7426); (173.0295, 178.7387);
  .....     .....     .....     .....     .....     .....
(176.7898, 183.0196); (171.9569, 181.6141); (170.1725, 176.0769).
```

The proportion of our intervals that contained the true mean (174) was $\frac{91}{100} = 0.91$. The average width of our intervals was 9.119093.

**(g)** For the samples of size 25, the proportion of our intervals that contained the true mean (174) was $\frac{95}{100} = 0.95$, and the average width of the intervals was 5.11662.

**23. (a)** For you to answer. Many people think that there should be no difference because the true percentage is 50% in both cases. This exercise is intended to bring home to you in a fairly concrete way that sample proportions from large samples are less variable than proportions from small samples, and thus are less likely to give a value as far away from the true proportion (0.5) as 0.7.

**(b)** By generating 10 Binomial($n = 1, p = 0.5$) random numbers, a simulation of the 10-question test follows:

0 0 0 1 0 0 1 0 1 0

**(c)** A simulation of 15 people guessing answers to the 10-question test is given below. Each column relates the results for one "person."

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

Adding the number of answers each person got right (i.e., counting the onesdown a column, or equivalently, adding down the column) gives

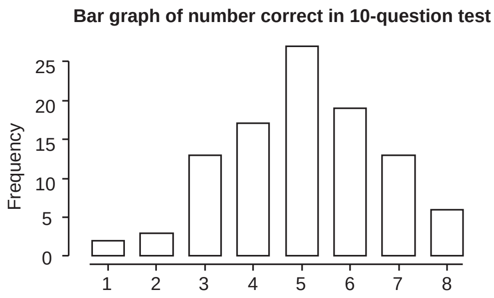
6 7 6 4 6 4 8 5 6 8 4 5 4 6 4

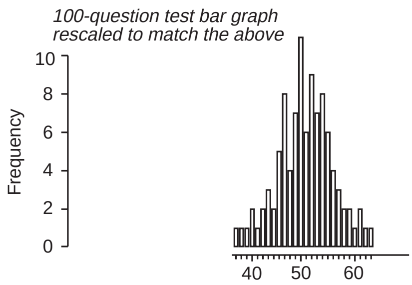
Thus, our 1st person got 6 correct, our 2nd got 7 correct, our 3rd got 6 correct, and so on.

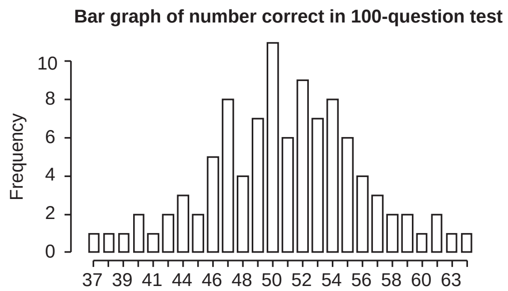
**(d)** The four conditions required for the Binomial model to be valid are satisfied. By generating 15 Binomial($n = 10, p = 0.5$) random numbers, the number of correct answers for 15 people randomly guessing in the 10-question test follows:

6 6 6 6 8 5 5 5 4 3 4 6 6 7 4

**(e)** Simulations of 100 people taking the 10-question test and the 100-question test are displayed in the bar graphs below.




The proportion of people who got at least 7 of the 10 questions correct was $\frac{19}{100} = 0.19$ and the proportion of people who got at least 70 of the 100 questions correct was $\frac{0}{100} = 0$. (The highest mark we observed was 64.)

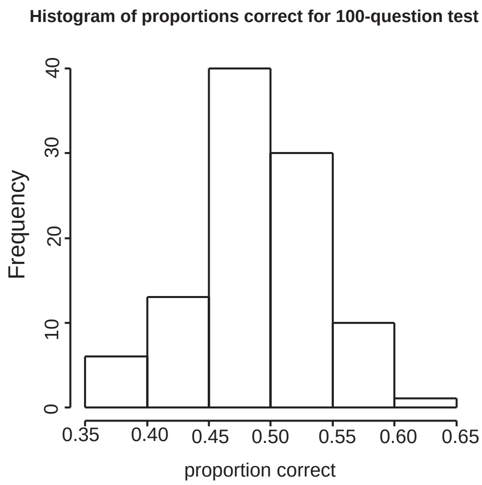
**(f)** The theoretical result is $\text{sd}(\widehat{P}) = \sqrt{\frac{p(1-p)}{n}}$ which is proportional to $\frac{1}{\sqrt{n}}$. This tells us that the variability in the values of $\widehat{P}$ decreases as the sample size increases.

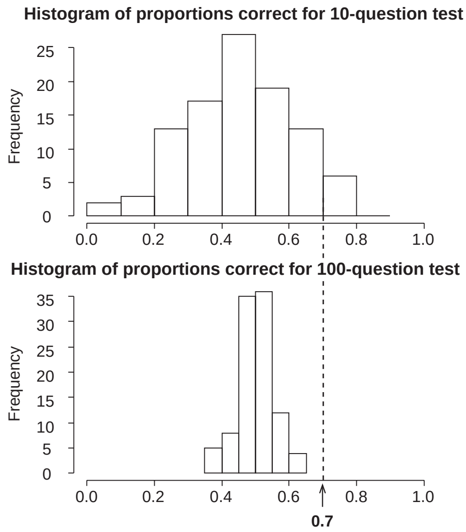
**(g)** For the people doing the 100-question test in (e) the 100 values of "proportion correct" are as follows:

```
0.55 0.50 0.57 0.51 0.48 0.56 0.49 0.51 0.41 0.57 0.47 0.50 0.38 0.64 0.52
0.50 0.51 0.46 0.52 0.49 0.63 0.50 0.53 0.52 0.47 0.50 0.49 0.47 0.53 0.50
0.59 0.59 0.54 0.58 0.55 0.49 0.40 0.61 0.37 0.54 0.49 0.58 0.52 0.54 0.44
0.49 0.51 0.51 0.46 0.55 0.44 0.48 0.40 0.55 0.53 0.39 0.48 0.50 0.50 0.56
0.61 0.42 0.52 0.52 0.55 0.52 0.57 0.42 0.46 0.54 0.56 0.52 0.47 0.53 0.47
0.46 0.54 0.47 0.44 0.50 0.53 0.60 0.45 0.48 0.53 0.55 0.49 0.47 0.54 0.56
0.51 0.45 0.50 0.47 0.54 0.46 0.52 0.50 0.53 0.54
```

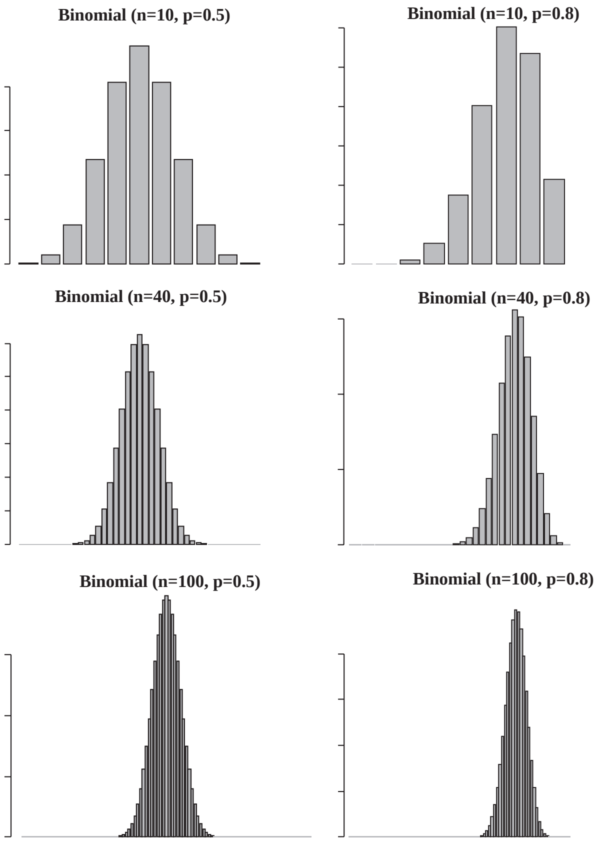
The standard deviation of these sample proportions is 0.053. This is very similar to the theoretical standard deviation of $\widehat{P}$ when $p = 0.5$, namely $\text{sd}(\widehat{P}) = \sqrt{\frac{0.5(1-0.5)}{100}} = 0.05$.

**(h)** A histogram of the proportions from (g) is given on the left below. The histogram is bell shaped but not completely symmetrical. [More interesting histograms are given on the right. These show the decrease in variability in the $\widehat{P}$ values with the larger sample size (number of questions). As the variability about the true value of 0.5 or 50% contracts, the proportion of values above 0.7 (70%) decreases.]




**(i)** Bar graphs of Binomial probabilities of the Binomial($n, p = 0.5$) for $n = 10$, $n = 40$, and $n = 100$ are given below. All are symmetrically bell shaped. We note the decreasing spread as $n$ increases. On the right we have the same thing for $p = 0.8$. The bar graph is quite skewed when $n = 10$ but is symmetrical by the time $n = 100$.



# Chapter 8

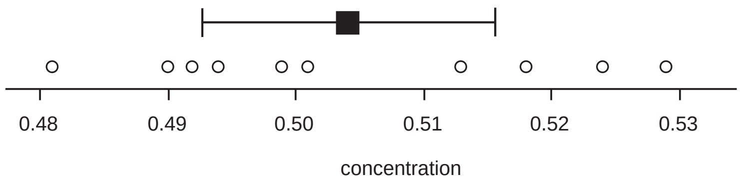
## Exercises for Section 8.2

For all of these problems, we will be using the following formula $\overline{x} \pm t\text{se}(\overline{x}) = \overline{x} \pm t\frac{s_X}{\sqrt{n}}$ to construct our confidence intervals. "Confidence interval" is frequently abbreviated to "CI." The confidence intervals in problems 1 and 2 can be also be generated automatically using a package like Minitab or Excel (see Section 10.1.1). Of greatest importance is learning to interpret the intervals in the context of the particular data set.

**1.** We have $df = n - 1 = 5$ so that the $t$ multiplier for a 95% CI is $t = 2.5706$. The 6 observations have sample mean and standard deviation given by $\overline{x} = 5.3117$ and $s_X = 0.2928$ respectively. The resulting 95% CI is $5.3117 \pm 2.5706 \times \frac{0.2928}{\sqrt{6}}$ or approximately [5.00, 5.62]. From this data, we can say with 95% confidence that the true mean density of the earth is somewhere between 5.0 g/cm$^3$ and 5.6 g/cm$^3$.

**2.** **(a)** We have $df = n - 1 = 9$ so that the $t$ multiplier for a 95% CI is $t = 2.2622$. The 10 observations have sample mean and standard deviation given by $\overline{x} = 0.5041$ and $s_X = 0.0160$ respectively. The resulting 95% CI is $0.5041 \pm 2.2622 \times \frac{0.0160}{\sqrt{10}}$ or approximately [0.493, 0.516]. With 95% confidence, the true nitrate ion concentration is somewhere between 0.49 $\mu$g/mL and 0.52 $\mu$g/mL.

**(b)**




**3.** **(a)** **(i)** Here $df = n - 1 = 61$ so for a 95% CI, $t = 1.9996$. The resulting CI is $620.6 \pm 1.9996 \times \frac{241.5}{\sqrt{62}}$ or approximately [559, 682]. With 95% confidence, the true or population mean testosterone level for nonsmokers is somewhere between 559 ng/dL and 682 ng/dL.

**(ii)** Here $df = n - 1 = 27$ so for a 95% CI, $t = 2.0518$. The resulting CI is $795.1 \pm 2.0518 \times \frac{305.3}{\sqrt{28}}$ or approximately [677, 913]. With 95% confidence, the true mean testosterone level for the 31–70 per day group is somewhere between 677 ng/dL and 913 ng/dL.

**(b)** When $df = 27$, $t = 1.7033$ for a 90% CI and $t = 2.7707$ for a 99% CI. The resulting CIs for the true mean testosterone level are:

**(i)** (90% CI) $795.1 \pm 1.7033 \times \frac{305.3}{\sqrt{28}}$, or [697, 893].

**(ii)** (99% CI) $795.1 \pm 2.7707 \times \frac{305.3}{\sqrt{28}}$, or [635, 955].

## Exercises for Section 8.3

**1.** In Example 8.3.1, $n = 200$ and $\widehat{p} = 0.7$. Our CI formula is $\widehat{p} \pm z\text{se}(\widehat{p}) = \widehat{p} \pm z\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$. The value of the multiplier $z$ depends upon the confidence level.

**(a)** (90% CI) $0.7 \pm 1.6449 \times \sqrt{\frac{0.7\times 0.3}{200}}$, or [0.647, 0.753].

**(b)** (99% CI) $0.7 \pm 2.5758 \times \sqrt{\frac{0.7\times 0.3}{200}}$, or [0.617, 0.783].

**2.** The 95% CI is $0.36 \pm 1.96 \times \sqrt{\frac{0.36\times 0.64}{139}}$, or approximately [0.280, 0.440]. With 95% confidence, the true (or population) proportion of Hispanic people who have been pulled over on the roads by the police is somewhere between 28% and 44%.

## Exercises for Section 8.4

We are using the confidence interval formula for a difference between two means from independent samples, namely $\overline{x}_1 - \overline{x}_2 \pm t\,\text{se}(\overline{x}_1 - \overline{x}_2) = \overline{x}_1 - \overline{x}_2 \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$.

**1.** **(a)** We will use $df = min(5, 22) = 5$. For a 95% CI the $t$ multiplier is $t = 2.5706$. Our 95% CI is $5.3117 - 5.4835 \pm 2.5706 \times \sqrt{\frac{0.2928^2}{6} + \frac{0.1904^2}{23}}$, or [−0.496, 0.152].

**(b)** The two-standard-error interval is $5.3117 - 5.4835 \pm 2 \times \sqrt{\frac{0.2928^2}{6} + \frac{0.1904^2}{23}}$, or [−0.424, 0.080], which is narrower.

**(c)** From either interval there is no evidence of a difference between the two true means, as the interval contains zero.

**2.** We use $df = min(30, 27) = 27$. For a 95% CI, $t = 2.0518$. Our 95% CI is $715.6 - 795.1 \pm 2.0518 \times \sqrt{\frac{248^2}{31} + \frac{305.3^2}{28}}$, or [−229, 70]. With 95% confidence, the true mean testosterone level for 1–30 per day smokers falls somewhere between smaller than that for 31–70 per day smokers by 229 ng/dL and larger by 70 ng/dL. This includes the possibility that there is no difference at all.

## Exercises for Section 8.5.1 and 8.5.2

The confidence intervals asked for in these exercises are all for differences between two proportions and are of the form $\widehat{p}_1 - \widehat{p}_2 \pm z\,\text{se}(\widehat{p}_1 - \widehat{p}_2)$. The formula used for the standard error is as given in Table 8.5.5. We will not repeat these formulas here but simply indicate whether we are dealing with a sampling situation (a), (b), or (c) as depicted in Fig. 8.5.1.

**1.** **(a)** Situation (c). **(b)** Situation (a). **(c)** Situation (a). **(d)** Situation (b). **(e)** Situation (a).

**2.** **(a)** $0.51 - 0.21 \pm 1.96 \times \sqrt{\frac{0.51+0.21-(0.51-0.21)^2}{500}}$, or [0.23, 0.37]. With 95% confidence, the population percentage of 15- to 17-year-olds who know a student who sells illegal drugs is bigger than the percentage who know a teacher who uses illegal drugs by somewhere between 23 and 37 percentage points.

**(b)** $0.51 - 0.22 \pm 1.96 \times \sqrt{\frac{0.51\times 0.49}{500} + \frac{0.22\times 0.78}{500}}$, or [0.23, 0.35]. With 95% confidence, the population percentage of 15- to 17-year-olds who know a student who sells illegal drugs is bigger than the corresponding percentage for 12- to 14-year-olds by somewhere between 23 and 35 percentage points.

**(c)** $0.35 - 0.23 \pm 1.96 \times \sqrt{\frac{0.35\times0.65}{822} + \frac{0.23\times0.77}{500}}$, or [0.07, 0.17].
With 95% confidence, the population percentage of principals who think students can use marijuana every weekend and still do well at school is bigger than the corresponding percentage for 15- to 17-year-olds by somewhere between 7 and 17 percentage points.

**(d)** $0.21 - 0.13 \pm 1.96 \times \sqrt{\frac{0.21+0.13-(0.21-0.13)^2}{500}}$, or [0.03, 0.13].
With 95% confidence, the population percentage of 12- to 14-year-olds who are most likely to hang out with friends after school is bigger than the percentage who go home and watch TV by somewhere between 3 and 13 percentage points.

**(e)** $0.22 - 0.16 \pm 1.96 \times \sqrt{\frac{0.22\times0.78}{500} + \frac{0.16\times0.84}{500}}$, or [0.01, 0.11].
With 95% confidence, the population percentage of 12- to 14-year-olds who are most likely to hang out with friends after school is bigger than the corresponding percentage of 15- to 17-year-olds by somewhere between 1 and 11 percentage points.

**3. (a)** [Situation (b)] $0.59 - 0.25 \pm 1.96 \times \sqrt{\frac{0.59+0.25-(0.59-0.25)^2}{1000}}$, or [0.29, 0.39].
With 95% confidence, the population percentage of New York voters who supported Clinton was bigger than the percentage who supported Dole by somewhere between 29 and 39 percentage points.

**(b)** [Situation (a)] $0.33-0.29\pm1.96\times\sqrt{\frac{0.33\times0.67}{1000} + \frac{0.29\times0.71}{1000}}$, or [−0.001, 0.08]. With 95% confidence, the population percentage of voters who supported Dole in New Jersey was somewhere between being the same as the percentage in Connecticut and being larger by 8 percentage points than in Connecticut.

**(c)** [Situation (a)] $0.28 - 0.2 \pm 1.96 \times \sqrt{\frac{0.28\times0.72}{1000} + \frac{0.2\times0.8}{1000}}$, or [0.04, 0.12].
With 95% confidence, the population percentage of Americans worried about difficulties in getting health care was larger than the corresponding percentage for Canadians by somewhere between 4 and 12 percentage points.

**(d)** [Situation (c)] $0.38 - 0.32 \pm 1.96 \times \sqrt{\frac{0.38+0.32-(0.38-0.32)^2}{1000}}$, or [0.01, 0.11]. With 95% confidence, the population percentage of New Zealanders who think recent changes have harmed the quality of health care was larger than the percentage who believe the system should be rebuilt by somewhere between 1 and 11 percentage points.

**(e)** The people in the UK, which spends least on health care, seem happiest with their system.

## Exercises for Section 8.6

**1. (a)** Take $n \geq \left(\frac{1.96}{.02}\right)^2 \times 0.5 \times 0.5 \approx 2401$.

**(b)** Take $n \geq \left(\frac{1.96}{.02}\right)^2 \times 0.15 \times 0.85 \approx 1225$.

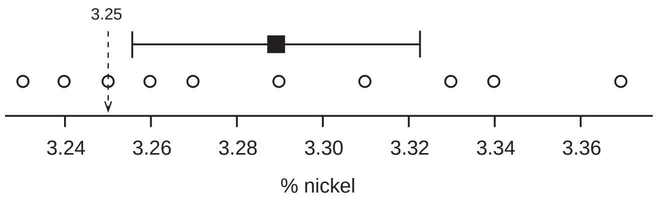
**(c)** Take $n \geq \left(\frac{1.96}{.02}\right)^2 \times 0.85 \times 0.15 \approx 1225$.

**2. (a)** Take $n \geq \left(\frac{1.96\times.07559}{.025}\right)^2 \approx 35$.

**(b)** Take $n \geq \left(\frac{1.96 \times .44049}{.025}\right)^2 \approx 1193$.
The difference is so big because thiol measurements are much more variable in the rheumatoid population than they are in the normal population so substantially more people must be sampled from the rheumatoid population to estimate the population mean to the same level of precision.

## Review Exercises 8

**1. (a)**




No, the dot plot looks well behaved.

**(b)** $\overline{x} = 3.289$, $s_X = 0.04701$

**(c)** With $df = n - 1 = 9$, $t = 2.262$ for a 95% CI. The resulting 95% CI for the true percent nickel content is $3.289 \pm 2.262 \times \frac{0.04701}{\sqrt{10}}$, or [3.26%, 3.32%]. Yes, there is evidence that this batch differs from previous batches as the usual mean nickel content for previous batches (3.25%) lies outside the 95% confidence interval for the true mean nickel content of this batch.

**(d)** Added to the plot in (a) above.

The interval is now $3.289 \pm 2.022691 \times \frac{0.04701064}{\sqrt{40}}$, or [3.27, 3.30].

If the multiplier did not change, the width of the confidence interval would halve. The multiplier also gets slightly smaller with the increase in *df*, so with more significant figures, you will see that the width of the new interval for $n = 40$ is slightly less than half the width of the interval for $n = 10$.

***(e)** Take $n \geq \left(\frac{1.96 \times 0.04701064}{.015}\right)^2 \approx 38$

**2. (a)** The more enthusiastic people show higher scores, on average, on all scales. Scores for non-volunteers look less variable on "support".

**(b)** In each case we use the following formula for a 95% CI for a difference between two true means $\mu_1 - \mu_2$, namely,

$$\overline{x}_1 - \overline{x}_2 \pm t\,\text{se}(\overline{x}_1 - \overline{x}_2) \;=\; \overline{x}_1 - \overline{x}_2 \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

We use $df = \min(n_1 - 1, n_2 - 1)$ to obtain the multiplier $t$ for the 95% CI.

**(i)** $df = 27$, 95% CI $= 3.82 - 3.32 \pm 2.0518 \times \sqrt{\frac{0.729^2}{38} + \frac{0.723^2}{28}}$, or [0.13, 0.87].
With 95% confidence, the true mean for enthusiastic volunteers is larger than that for reluctant volunteers by somewhere between 0.13 and 0.87.

**(ii)** $df = 12$, 95% CI $= 3.82 - 3.15 \pm 2.1788 \times \sqrt{\frac{0.729^2}{38} + \frac{0.689^2}{13}}$, or [0.18, 1.16]. With 95% confidence, the true mean for enthusiastic volunteers is larger than that for nonvolunteers by somewhere between 0.18 and 1.16.

**(iii)** $df = 12$, 95% CI $= 3.32 - 3.15 \pm 2.1788 \times \sqrt{\frac{0.723^2}{28} + \frac{0.689^2}{13}}$, or [−0.34, 0.68]. With 95% confidence, the true mean for reluctant volunteers could be anywhere between being smaller than that for nonvolunteers by 0.34 and larger by 0.68. This includes the possibility that there is no difference at all between the true means.

**(c)** We have evidence that the enthusiastic volunteers have the higher average scores than nonvolunteers for all three characteristics (goal emphasis, support and team building). For goal emphasis, we have evidence that enthusiastic volunteers score higher on average than reluctant volunteers, but could not demonstrate a difference between reluctant volunteers and nonvolunteers. For both support and team building, however, we have evidence that reluctant volunteers score higher on average than nonvolunteers, but could not demonstrate a difference between enthusiastic volunteers and reluctant volunteers. We have thus been able to confirm most, but not all, of what we observed in (a).

**(d)** The group studied were a specific group of people in a municipal department and were therefore not necessarily representative of people in general. We need to investigate whether these trends carry over to other types of people. Also the tendency to return a questionnaire may not be a very good indicator of the tendency to volunteer. It would be good to design an experiment where people are asked directly to volunteer to take part in some activity. We would also be interested in whether there were be sex or cultural differences in the relationship between voluntarism and goal emphasis etc.

**3. (a)** A single piece of paper may look like it would take less time to answer. If there was a difference, we would expect the single sheet version would have a higher response rate.

**(b)** We have independent samples (situation (a) in Fig. 8.5.1) so we use the corresponding formula in Table 8.5.5 for se($\widehat{p}_1 - \widehat{p}_2$). We have $\widehat{p}_1 = 0.36$, $\widehat{p}_2 = 0.3$, $n_1 = 220$, and $n_2 = 220$, giving $\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{0.36\times0.64}{220} + \frac{0.30\times0.70}{220}} = 0.044742$.

(90% CI) $0.36 - 0.30 \pm 1.6449 \times 0.044742$, or [−0.014, 0.13]. With 90% confidence, the true response rate for the one-sheet version is somewhere between being 1.4 percentage points lower than for the two-sheet version and 13 percentage points higher.

(95% CI) $0.36 - 0.30 \pm 1.96 \times 0.044742$, or [−0.03, 0.15]. When we change to a 95% CI our interval becomes wider (less precise). With 90% confidence, the true response rate for the one-sheet version is somewhere between being 3 percentage points lower than for the two-sheet version and 15 percentage points higher.

**(c)** Since both intervals contain zero, the change in printing format may make no difference to the response rate. We would be inclined to use the two-sided version in accordance with our intuition and the slight suggestion of an increased response rate given by the data. We note that both response rates were quite low.

**4.** All confidence intervals asked for in this problem are confidence intervals for a difference in proportions from independent samples, i.e., sampling situation (a) of Fig. 8.5.1, so we use the corresponding formula in Table 8.5.5 for se($\widehat{p}_1 - \widehat{p}_2$). All are 95% CIs, so we use $z = 1.96$ as our multiplier.

**(a)** Our intervals are for $p_{1st} - p_{33rd}$, the difference between the response rate when the question is asked first and when it is asked 33rd. We do this for various for various subgroups.

**(i)** *All respondents*:
$0.804 - 0.887 \pm 1.96 \times \sqrt{\frac{0.804\times0.196}{337} + \frac{0.887\times0.113}{328}}$, or $[-0.14, -0.03]$. With 95% confidence, somewhere between 3% and 14% more people expressed an opinion when the question was asked 33rd than when it was 1st.

**(ii)** *Less Educated*:
$0.795 - 0.916 \pm 1.96 \times \sqrt{\frac{0.795\times0.205}{224} + \frac{0.916\times0.084}{214}}$, or $[-0.19, -0.06]$. With 95% confidence, somewhere between 6% and 19% more people expressed an opinion when the question was asked 33rd than when it was 1st.

**(iii)** *More Educated*:
$0.842 - 0.832 \pm 1.96 \times \sqrt{\frac{0.842\times0.158}{101} + \frac{0.832\times0.168}{113}}$, or $[-0.09, 0.11]$. For the subgroup consisting of the more educated people, because zero is in the interval, we cannot say in which direction the true difference lies or whether one exists.

**(b)** Our 95% CI for the true difference in approval ratings among those who would respond is $0.528 - 0.515 \pm 1.96 \times \sqrt{\frac{0.528\times0.472}{271} + \frac{0.515\times0.485}{291}}$, or $[-0.07, 0.10]$. Although zero is in the interval so that it is quite plausible that ordering makes no difference to approval ratings, the data does not demonstrate that there is no difference. The data is consistent with true differences of up to 7% in one direction or 10% in the other.

**(c)** In the study, the intervening questions did not directly relate to the actions of the President as they did in the Johnson polls.

***5.** **(a)** Take $n \geq \left(\frac{2.5758}{.04}\right)^2 \times 0.5 \times 0.5 \approx 1037$.

**(b)** Take $n \geq \left(\frac{2.5758}{.04}\right)^2 \times 0.4 \times 0.6 \approx 995$ (which is only slightly smaller).

**(c)** Take $n \geq \left(\frac{2.5758}{.04}\right)^2 \times 0.6 \times 0.4 \approx 955$ (which is the same as for (b)).

**(d)** Practical questions including how to sample students from the many schools in a city. Definitional questions such as how to handle students whose parents were never legally married but have now split. (Whether to include them depends on the purpose of the survey. Are you interested in legalities or whether the students are living with both parents?)

**6.** **(a)** We have $\widehat{p}_{TM} = 0.719$ and $n = 362$. Assuming a random sample, our 90% CI for $p_{TM}$ is $= 0.719 \pm 1.644854 \times \sqrt{\frac{0.719\times0.281}{362}}$, or [0.68, 0.76]. If we believed this interval it would be telling us that the true 5-year disease-free rate under the TM treatment at this time was somewhere between 68% and 76%. The considerations discussed under part (b) of the question, show that in fact the uncertainty about the true value is greater than this interval would suggest.

**(b)** Here, $\widehat{p}_{TM} = 0.719$, $\widehat{p}_{SM} = 0.681, n_{TM} = 362$ and $n_{SM} = 390$. We have independent samples (situation (a) in Fig. 8.5.1) so, assuming ordinary random samples, our 95% CI for $p_{TM} - p_{SM}$ is $0.719 - 0.681 \pm 1.96 \times \sqrt{\frac{0.719\times0.281}{362} + \frac{0.681\times0.319}{390}}$, or $[-0.03, 0.10]$. This would tell us that the true 5-year disease-free rate under the TM treatment could be anywhere between being lower than that for the SM treatment by 3% and higher by 10%.

**(c)** In (b), we have worked with calculated standard errors of the form $\text{se}(\widehat{p}) = \sqrt{\widehat{p}(1-\widehat{p})/n}$ for each of our sample proportions. We have now been told that these are not appropriate given the way the proportions were calculated. Each comparison we make still compares proportions from independent groups of women so we can use $\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\text{se}(\widehat{p}_1)^2 + \text{se}(\widehat{p}_2)^2}$. However, we will now substitute the real $\text{se}(\widehat{p})$ values given in the question into this equation. The resulting 95% CIs are as follows.

For $p_{SM} - p_{TM}$: $0.681 - 0.719 \pm 1.96 \times \sqrt{0.035^2 + 0.035^2}$, or $[-0.135, 0.059]$.
For $p_{SM+R} - p_{TM}$: $0.814 - 0.719 \pm 1.96 \times \sqrt{0.029^2 + 0.035^2}$, or $[0.006, 0.184]$
For $p_{SM+R} - p_{SM}$: $0.814 - 0.681 \pm 1.96 \times \sqrt{0.029^2 + 0.035^2}$, or $[0.044, 0.222]$.

**(d)** No difference has been demonstrated between TM and SM (zero is within the interval), but SM + R shows significant improvement over both TM and SM.

**7. (a)** We would plot the data using box plots to compare groups (as these groups are quite large). We would also look at stem-and-leaf plots or histograms to look at distributional shape.

**(b)** We use $df = \min(n_1 - 1, n_2 - 1) = 89$ in determining the size of the multiplier $t$. The only difference between the 95% confidence interval and the two-standard-error interval we calculated in problem 15(b) in Review Exercises 7 is that we are now using $t = 1.9870$ standard errors rather than 2 standard errors. Not surprisingly, we get virtually identical intervals. Our 95% CI for the true difference in means is $103.0 - 92.8 \pm 1.9870 \times \sqrt{\frac{17.39^2}{210} + \frac{15.18^2}{90}}$, or [6.2, 14.2]. Breast-fed babies have IQs that are higher on average than bottle-fed babies by somewhere between 6 and 14 points.

***(c)** Take $n \geq (\frac{1.96\times17.39}{1})^2 \approx 1162$.

**(d)** The babies were all pre-term and very small, and only from special care units in several areas in England. The results may be special to this population.

**(e)** It is an observational study in which mothers chose whether to breast feed. The study does not demonstrate that the effect is causal.

**(f)** The CI would change to $103.0 - 92.8 \pm 2.144787 \times \sqrt{\frac{17.39^2}{210} + \frac{15.18^2}{15}}$, or [1.4, 19.0]. The interval has become more than twice as wide.

**(g)** This problem is very similar to Example 6.4.2. We want $\text{pr}(X < Y)$ where $X \sim \text{Normal}(\mu_X = 103.0, \sigma_X = 17.39)$ and $Y \sim \text{Normal}(\mu_Y = 92.8, \sigma_Y = 15.18)$. $\text{pr}(X < Y) = \text{pr}(X - Y < 0) = \text{pr}(W < 0)$. Here $W = X - Y$ has mean $\mu_W = 103.0 - 92.8 = 10.2$ and standard deviation $\sigma_W = \sqrt{17.39^2 + 15.18^2} = 23.0834$. Using these values, $\text{pr}(W < 0) = 0.3293$. For any two randomly selected babies, there is approximately 1 chance in 3 that the bottle-fed baby will have a higher IQ. The confidence interval is only talking about the difference between the means and says nothing about any other aspect of the distribution. In fact, there is substantial overlap between the IQ distributions for both groups.

**8. (a) (i)** The overall sample proportion with TB is $\widehat{p} = \frac{556+36}{984+90} \approx 0.55$.

**(ii)** The sample proportion of intravenous drug users with TB is $\widehat{p}_{Int.} = \frac{361}{629} \approx 0.57$.

**(iii)** The sample proportion of "Whites" with TB is $\widehat{p}_W = \frac{496}{886} \approx 0.56$.

The sample proportion of "Gypsies" with TB is $\widehat{p}_G = \frac{74}{152} \approx 0.49$.

The sample proportion of "Others" with TB is $\widehat{p}_O = \frac{22}{36} \approx 0.61$.

We see that the "Other" group had the highest sample rate of TB.

**(b)** We are comparing proportions from independent samples (situation (a) in Fig. 8.5.1. We have $\widehat{p}_{male} = \frac{556}{984} = 0.56504 \approx 0.57$, $\widehat{p}_{female} = \frac{36}{90} = 0.4$. Our 95% CI for the true difference is $0.56504 - 0.4 \pm 1.96 \times \sqrt{\frac{0.56504 \times 0.43496}{984} + \frac{0.4 \times 0.6}{90}}$, or [0.06,0.27]. With 95% confidence, the population percentage of males with TB is higher than that for females by between 6 and 27 percentage points.

**(c)** We are again comparing proportions from independent samples (situation (a) in Fig. 8.5.1). We have $\widehat{p}_{HIV} = \frac{186}{294} = 0.63265 \approx 0.57$, $\widehat{p}_{NoHIV} = \frac{406}{780} = 0.5205128$. Our 95% CI for the true difference is $0.63265 - 0.52051 \pm 1.96 \times \sqrt{\frac{0.63265 \times 0.36735}{294} + \frac{0.52051 \times 0.47948}{780}}$, or [0.05, 0.18]. With 95% confidence, the population percentage of prisoners with HIV who have TB is higher than that for prisoners without TB by between 5 and 18 percentage points.

**(d)** We are again comparing proportions from independent samples (situation (a) in Fig. 8.5.1). We have $\widehat{p}_W = \frac{496}{886} = 0.55982$ and $\widehat{p}_G = \frac{74}{152} = 0.48684$. Our 95% CI for the true difference is $0.55982 - 0.48684 \pm 1.96\sqrt{\frac{0.55982 \times 0.44018}{886} + \frac{0.48684 \times 0.51316}{152}}$, or [−0.01, 0.16]. The 90% CI (i.e., using $z = 1.6449$ standard errors) is [0.001, 0.145]. At the 90% confidence level, the difference is positive thus suggesting an ethnic difference, but the 95% interval contains zero. We therefore have some evidence confirming a link, but it is not particularly strong.

**(e)** We need to be able to compare the proportions of males and females with TB among HIV-positive prisoners, and also among HIV-negative prisoners.

**(f)** The study is on prisoners – a very special subset of the population in a very special environment in which TB contraction patterns may be different from in society at large.

**(g)** We had not intended that calculations be done here. We just wanted to give some practice with classifying sampling situations.

**(i)** Situation (c). Since the total number of prisoners with TB in the sample is 592, the sample proportions to be compared are $\widehat{p}_1 = 361/592$ and $\widehat{p}_2 = 186/592$ using $n = 592$.

**(ii)** Situation (b). The sample proportions to be compared are $\widehat{p}_1 = 74/592$ and $\widehat{p}_2 = 496/592$ using $n = 592$.

**(iii)** Situation (a). This is the same comparison as in (d).

**9. (a)** The data suggests that ex-smokers have healthier eating patterns on average than smokers, both when we look within manual workers and when we look within non-manual workers. Similarly, non-manual workers seem to have healthier eating patterns on average than manual workers, both when we look within smokers and within ex-smokers.

**(b)** All confidence intervals calculated here are 95% CIs for differences between proportions from independent samples (situation (a) in Fig. 8.5.1). All are calculated using $\widehat{p}_1 - \widehat{p}_2 \pm 1.96\,\mathrm{se}(\widehat{p}_1 - \widehat{p}_2)$ where $\mathrm{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}}$.
Using only non-manual workers, the following are 95% confidence intervals for differences in true proportions between ex-smokers who consume the item (e.g., breakfast) and smokers who consume the item. The sample sizes are always $n_1 = 517$ and $n_2 = 404$.
*Breakfast*: $\widehat{p}_1 = 0.824$, $\widehat{p}_2 = 0.629$, and the CI is [0.14, 0.25].
*Brown bread*: $\widehat{p}_1 = 0.536$, $\widehat{p}_2 = 0.345$, and the CI is [0.12, 0.25].
*Fresh Fruit*: $\widehat{p}_1 = 0.776$, $\widehat{p}_2 = 0.594$, and the CI is [0.12, 0.24].
*Fried food*: $\widehat{p}_1 = 0.162$, $\widehat{p}_2 = 0.282$, and the CI is [−0.17, −0.07].
We have clearly demonstrated that the ex-smokers do better than the non-smokers when it comes to both having breakfast and having a healthy breakfast. For example, with 95% confidence, the true percentage of exsmokers consuming breakfast is larger than that for smokers by somewhere between 14 and 25 percentage points. The other intervals are all read similarly. The only exception is the last interval which tells us that, with 95% confidence, the true percentage of exsmokers consuming fried food is smaller than that for smokers by somewhere between 7 and 17 percentage points.

**10. (a)** The percentages are listed as follows:

| *Make* | *91–93 trouble-free %* | *94–96 trouble-free %* |
|---|---|---|
| Honda | 53.9 | 54.1 |
| Mazda | 51.8 | 58.2 |
| Mitsubishi | 45.1 | 51.4 |
| Nissan | 42.3 | 51.9 |
| Subaru | 50.7 | 62.9 |
| Toyota | 52.0 | 58.6 |

For the period 1991–93, Honda appears the most reliable and Nissan the least reliable. For the period 1994–96, Subaru appears the most reliable and Mitsubishi the least reliable.

**(b)** For all other years except 1996, our data concerns problems in the last year. None of the 1996 cars had been in use for a full year. Also, as cars age they tend to become less reliable. Very new cars often have teething problems. Comparisons between makes where the age-distribution of cars in use is different may be biased.

**(c)** Using the formula $\widehat{p} \pm 1.96\,\mathrm{se}(\widehat{p})$ where $\mathrm{se}(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$ we get the following.
Toyota: $n = 408$, $\widehat{p} = 212/408$ and the 95% CI is [0.47, 0.57]. With 95% confidence the population proportion of 1991–1993 Toyota owners who experienced "trouble free" motoring is somewhere between 47% and 57%. Similarly, we have:

*Honda*: $n = 152$, $\widehat{p} = 82/152$, and the 95% CI is [0.46, 0.62];
*Mazda*: $n = 85$, $\widehat{p} = 44/85$, and the 95% CI is [0.41, 0.62];
*Mitsubishi*: $n = 244$, $\widehat{p} = 110/244$, and the 95% CI is [0.39, 0.51];
*Nissan*: $n = 208$, $\widehat{p} = 88/208$, and the 95% CI is [0.36, 0.49];
*Subaru*: $n = 73$, $\widehat{p} = 37/73$, and the 95% CI is [0.39, 0.62].

All confidence intervals calculated in (d) and (e) are 95% CIs for differences between proportions from independent samples (situation (a) in Fig. 8.5.1). All are calculated using $\widehat{p}_1 - \widehat{p}_2 \pm 1.96\,\text{se}(\widehat{p}_1 - \widehat{p}_2)$ where $\text{se}(\widehat{p}_1 - \widehat{p}_2) = \sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}}$.

**(d)** We want to estimate, for the 91–93 models, $p_{Toy} - p_{Nis}$ denoting the difference between the true proportions of Toyotas and Nissans that were trouble-free. We have $n_{Toy} = 408$, $\widehat{p}_{Toy} = 212/408$, $n_{Nis} = 208$, and $\widehat{p}_{Nis} = 88/208$. The 95% CI for the true difference is approximately $[0.014, 0.18]$. The true percentage of Toyotas that were trouble-free is greater than that for Nissans by between 1.4 and 18 percentage points.

**(e)** The sample proportion of 94–96 Nissans that were trouble free was $\widehat{p}_{Nis.94} = 80/154$ with $n_{Nis.94} = 154$. The sample proportion of 91–93 Nissans that were trouble free was $\widehat{p}_{Nis.91} = 88/208$ with $n_{Nis.91} = 208$. The 95% CI for the difference in true proportions is approximately [−0.007, 0.20]. With 95% confidence, the percentage of 94–96 Nissans that are trouble free is somewhere between essentially the same as for 91–93's and 20 percentage points higher than for 91–93's. Confidence intervals making the equivalent comparisons for the other makes (expressed in percentage terms) are : Honda- [−11%, 11%]; Mazda- [−9%, 22%]; Mitsu.- [−3%, 16%]; Sub.- [−8%, 32%]; Toy.- [−2%, 15%]. Since all these intervals contain zero, we cannot demonstrate that the older cars are less reliable.

**(f)** Changes in design and technology. Also older cars might be treated with less respect than new cars.

**(g)** If there was an aging effect, there would be a bias in favor of Subaru, i.e., towards making Subaru look better.

**(h)** Cars under the same name in different markets are not necessarily identical cars. They may contain different parts manufactured using different processes. Differing climatic conditions may change what cars are more reliable (some may be more affected by extremes of heat or cold) as may driving habits. We would need to know about some of these things.

**(i)** $\widehat{p}_{Hon.94} = \frac{148}{799} = 0.1852315$. A 95% CI for Honda's market share is approximately [0.158, 0.212] telling us that Honda's market share was somewhere between about 16% and 21%. Equivalent 95% CIs for the market shares of other companies are (expressed as percentages): Mazda- [7.8%, 12.0%]; Mitsu.- [18.8%, 24.5%]; Nis. - [16.5%, 22.0%]; Sub.- [3.0%, 5.8%]; Toy. - [23.2%, 29.3%].

**(j)** This comparison is of the form situation (b) shown in Fig. 8.5.1. The sample size is $n = 799$. The sample market share proportions are (Toyota) $\widehat{p}_{Toy.94} = 210/799 \approx 0.0.2628$ and (Honda) $\widehat{p}_{Hon.94} = 148/799 \approx 0.1852$. The resulting 95$ CI is $0.2628 - 0.1852 \pm 1.96 \times \sqrt{\frac{0.2628+0.1852-(0.2628-0.1852)^2}{799}}$, or [0.031, 0.124]. With 95% confidence, Toyota's market share was greater than Honda's by somewhere between about 3% and 12%.

**(k)** We are comparing proportions from independent samples (situation (a) in Fig. 8.5.1) – see formulas just prior to the answer to part (d). For 94–96, $n_{94} = 799$ and our estimate of Honda's share from the sample is $\widehat{p}_{Hon.94} = \frac{148}{799} \approx 0.1852$. For 91–93, $n_{91} = 1150$ and our estimate of Honda's share from the sample is $\widehat{p}_{Hon.94} = \frac{152}{1170} \approx 0.1299$. Our 95% CI for the true difference is $0.1852 - 0.1299 \pm 1.96 \times \sqrt{\frac{0.1852\times0.8148}{799} + \frac{0.1299\times0.8701}{1150}}$, or [0.022, 0.089]. With 95% confidence, the increase in market share for Honda was somewhere between 2.2 and 8.9 percentage points. Equivalent 95% CIs for the changes in the market shares of other companies are (expressed as percentages): Mazda- [0%, 5.2%]; Mitsu.- [−2.9%, 4.5%]; Nis.- [−2.0%, 5.0%]; Sub. -[−3.9%, 0.1%]; Toy.-[−12.7%, −4.5%].

**(l)** You might think so from the figures, but in fact a market for used cars imported direct from Japan opened up, increasing the numbers of older used cars.

**11. (a)** 95% CI: $0.48 \pm 1.96 \times \sqrt{\frac{0.48\times0.52}{2700}}$, or [0.461, 0.500]. With 95% confidence, the true percentage of Independents who voted Republican was somewhere between 46% and 50%.

**(b)** We are comparing proportions from independent samples (situation (a) in Fig. 8.5.1). Our 95% CI for $p_{94} - p_{98}$ is $0.55 - 0.48 \pm 1.96 \times \sqrt{\frac{0.55\times0.45}{2700} + \frac{0.48\times0.52}{2700}}$, or [0.043, 0.097]. With 95% confidence, the true percentage of Independents who voted Republican in 1998 was smaller than that in 1994 by somewhere between about 4 and 10 percentage points.

**(c)** We are comparing proportions from independent samples (situation (a) in Fig. 8.5.1). Our 95% CI for $p_{4year} - p_{postgrad}$ is
$0.53 - 0.45 \pm 1.96 \times \sqrt{\frac{0.53\times0.47}{2700} + \frac{0.45\times0.55}{1800}}$, or [0.050, 0.110]. With 95% confidence, the true percentage of 4-year college graduates who voted Republican was larger than that for people who have done postgraduate study by somewhere between about 5 and 11 percentage points.

**(d)** 95% CIs for population proportions of ethnic group voting Republican:
White [0.539, 0.561]; Black [0.09, 0.13]; Hispanic [0.31, 0.39]; Asian [0.32, 0.52].
The 95% CI for difference between population proportions of Asians and Hispanics voting Republican (situation (a) comparison):
$0.42 - 0.35 \pm 1.96 \times \sqrt{\frac{0.42\times0.58}{100} + \frac{0.35\times0.65}{500}}$, or [−0.04, 0.18].

**12. (a)** Of all the issues discussed, money problems stand out as the most common causes of stress. Of the groups, single parents seem most likely to be stressed by these issues, most notably by money problems, living relationships and relationships with partners (presumably residual problems from failed relationships).

Unhealthy life styles: Binge drinking seems less prevalent than the other two types – most markedly so for those living with a partner and child. Smoking and unhealthy eating practices seem most common among single parents, and binge drinking among those in shared accommodation.

There appears to be a tendency for those living with parents to be in lower weight categories and those with partner and child to be in higher weight categories. (This may be partially explainable in terms of age).

**(b)** Differences between groups might be due to to age differences rather than living-status differences.

**(c)** The study also asked about stresses stemming from work, study, health, and other types of relationships. Other possibilities include major life changes such as bereavements.

**(d)** 95% CI: $0.173 \pm 1.96 \times \sqrt{\frac{0.173\times 0.827}{3125}}$, or [0.160, 0.186]. With 95% confidence, among young women living in shared accommodation, the true percentage who were stressed by their living arrangements lies somewhere between about 16 and 19%.

**(e)** We are comparing proportions using independent samples (situation (a) in Fig. 8.5.1). A 95% CI for the difference in proportions stressed by relationships with boyfriends between those living alone and those in shared accommodation $p_{alone} - p_{share}$ is given by $0.145 - 0.116 \pm 1.96 \times \sqrt{\frac{0.145\times 0.855}{875} + \frac{0.116\times 0.884}{3125}}$, or [0.003, 0.055]. With 95% confidence, the true percentage stressed by boyfriends among those living alone lies somewhere between being very similar to the corresponding percentage among those living in shared accommodation and being about 6 percentage points greater.

**(f)** This is a situation (c) comparison in Fig. 8.5.1. We are only considering people living alone of which we have $n = 875$ in our sample. A conservative 95% CI for the difference between the true (or population) proportion stressed by money problems and the proportion stressed by living arrangements $p_{money} - p_{living}$ is given by
$0.298 - 0.162 \pm 1.96 \times \sqrt{\frac{0.298+0.162-(0.298-0.162)^2}{875}}$, or [0.09, 0.18]. With 95% confidence, the true percentage stressed by money problems is greater than the percentage stressed by living arrangements by somewhere between 9 and 18 percentage points.

**(g)** This is a situation (b) comparison in Fig. 8.5.1. We are only considering people living with a partner and child, of which we have $n = 915$ in our sample. A 95% CI for the difference between the true proportion in the underweight category and the true proportion in the overweight category $p_{uw} - p_{ow}$ is given by
$0.253 - 0.216 \pm 1.96 \times \sqrt{\frac{0.253+0.216-(0.253-0.216)^2}{915}}$, or [−0.007, 0.081]. With 95% confidence, the true percentage in the underweight category lies somewhere between being very similar to the percentage in the overweight category and being larger by about 8 percentage points.

**13.** The actual margin of error associated with $\widehat{p} = 0.03$ is $1.96 \times \sqrt{\frac{.03\times .97}{1000}} \approx 0.0106$, which is very close to 1%.

**14.** **(a)** When $df = n - 1 = 19$, for a 95% CI we use $t = 2.093$. The resulting 95% CI is given by $\overline{x} \pm t\text{se}(\overline{x}) = \overline{x} \pm t\,\frac{s_X}{\sqrt{n}} = 16.98 \pm 2.093 \times \frac{2.85}{\sqrt{20}}$, or [15.6, 18.3]. With 95% confidence, the true mean rating for females under control conditions lies somewhere between 15.6 and 18.3.

**(b)** We will use $df = \min(n_1 - 1, n_2 - 1) = 21$, so for a 95% CI we use $t = 2.080$. The resulting 95% CI is given by $\overline{x}_1 - \overline{x}_2 \pm t\text{se}(\overline{x}_1 - \overline{x}_2) = \overline{x} \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} =$

$17.18 - 14.16 \pm 2.080 \times \sqrt{\frac{4.72^2}{23} + \frac{4.64^2}{22}}$, or [0.1, 5.9]. With 95% confidence, the true mean rating for males under control conditions is larger than it is under negative-contrast conditions by somewhere between 0.1 and 5.9.

**(c)** Computation is as for (b). The resulting 95% CI is $21.16 - 17.18 \pm 2.079614 \times \sqrt{\frac{4.31^2}{22} + \frac{4.72^2}{23}}$, or [1.2, 6.8]. With 95% confidence, the true mean rating for males under positive-contrast conditions is larger than it is under control conditions by somewhere between 1.2 and 6.8.

**15. (a)** When $df = n - 1 = 38$, for a 95% CI we use $t = 2.024$. The resulting 95% CI is (using the formula in answer to 14(a)): $10.97 \pm 2.024 \times \frac{2.67}{\sqrt{39}}$, or [10.1, 11.8]. With 95% confidence the true mean score under control conditions lies somewhere between 0.1 and 11.8.

**(b)** We will use $df = \min(n_1 - 1, n_2 - 1) = 17$, so for a 95% CI we use $t = 2.110$. The resulting 95% CI is (using the formula in answer to 14(b)): $13.28 - 10.97 \pm 2.110 \times \sqrt{\frac{1.9^2}{18} + \frac{2.67^2}{39}}$, or [1.0, 3.6]. With 95% confidence, the true mean score under "humane/no info." conditions is larger than it is under control conditions by somewhere between about 1 and 6.

**(c)** Interval set up is as for (b). The resulting 95% CI is: $10.97 - 10.44 \pm 2.110 \times \sqrt{\frac{2.67^2}{39} + \frac{2.43^2}{18}}$, or [−0.98, 2.04]. As zero lies in this interval we cannot tell whether there is a true difference or in what direction such a difference lies. What we can say with 95% confidence is that the true mean score under control conditions lies somewhere between being smaller than it is under "inhumane/no info." conditions by approximately 1 and and being larger by 2.04.

***16. (a)** Since the true proportion is $p = \frac{M}{N}$ we have $N = \frac{M}{p}$. We can estimate $N$ by substituting an estimate $\widehat{p}$ of obtained $p$ from our sample. This gives $\widehat{N} = \frac{M}{\widehat{p}}$. For our sample $n = 321$ and $\widehat{p} = \frac{89}{321} = 0.2772586$. Thus $\widehat{N} = \frac{600}{0.2772586}$, or 2164.

**(b)** A 95% CI for the true value of $p$ is given by $0.27726 \pm 1.96 \times \sqrt{\frac{0.27726 \times 0.72274}{321}}$, or [0.22829, 0.32623].

**(c)** Recall that $N = \frac{M}{p}$. We have with 95% confidence that $p \geq 0.2282$ and $p \leq 0.32623$. If $p \geq 0.22829$ then $N \leq \frac{600}{0.22829} \approx 2628$, and if $p \leq 0.32623$ then $N \geq \frac{600}{0.32623} \approx 1839$. Our 95% confidence interval for $N$ is therefore [1839, 2628]. With 95% confidence, the population size lies somewhere between 1839 ants and 2628 ants.

***17 (a)** For independent samples (situation (a) in Fig.8.5.1), the margin of error is $z$ standard errors or $z\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}}$. When $n_1 = n_2 = n$, this becomes $\frac{z}{\sqrt{n}}\sqrt{\widehat{p}_1(1-\widehat{p}_1) + \widehat{p}_2(1-\widehat{p}_2)}$. Solving $\frac{z}{n}\sqrt{\widehat{p}_1(1-\widehat{p}_1) + \widehat{p}_2(1-\widehat{p}_2)} \leq w$ for $n$ gives the desired expression.

**(b)** Since taking $\widehat{p} = 0.5$ maximizes $\widehat{p}(1 - \widehat{p})$, we should use $\widehat{p}_1 = 0.5$ and $\widehat{p}_2 = 0.5$.

**(c)** Using these values we get $n \geq \left(\frac{z}{w}\right)^2 \times 0.5 = \frac{1}{2}\left(\frac{z}{w}\right)^2$.

**(d)** Arguing as in (a), we get get $n \geq \left(\frac{z}{w}\right)^2 \times [\widehat{p}_1 + \widehat{p}_2 - (\widehat{p}_1 - \widehat{p}_2)^2]$. The biggest value this can take also happens when $\widehat{p}_1 = 0.5$, and $\widehat{p}_2 = 0.5$ giving $n \geq \left(\frac{z}{w}\right)^2$.

***18** **(a)** The margin of error associated with $\widehat{p}$ is $z\sqrt{\widehat{p}(1-\widehat{p})/n}$.

**(b)** The margin of error for the difference between proportions from independent samples is $z\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}}$. When $n_1 = n_2 = n$ and $\widehat{p}_1 = \widehat{p}_2 = 0.5$, this reduces to $z/\sqrt{2n}$.

**(c)** By substituting $\widehat{p} = 0.5$ into (a), the margin of error for a single proportion is $z/\sqrt{4n}$. The ratio (difference/single) is $[z/\sqrt{2n}]/[z/\sqrt{4n}] = \sqrt{2} \approx 1.4$. The margin of error for a difference should be approximately 40% larger than that for a single proportion.

**(d)** The margin or error for situation (b) is $z\sqrt{\frac{\widehat{p}_1+\widehat{p}_2-(\widehat{p}_1-\widehat{p}_2)^2}{n}}$. This reduces to $z/\sqrt{n}$ when $\widehat{p}_1 = \widehat{p}_2 = 0.5$. The ratio (difference/single) is $[z/\sqrt{n}]/[z/\sqrt{4n}] = \sqrt{4} = 2$. The margin of error for a situation (b) difference should be approximately twice as large as that for a single proportion.

**19.** **(a)** 100 samples of 9 "male heights" following a $N(\mu = 174\text{cm}, \sigma = 6.57\text{cm})$ distribution were generated. For each sample, a 95% confidence interval for the true mean was obtained using the formula $\overline{x} \pm t\,\text{se}(\overline{x})$. Here $n = 9$ so that $df = 8$ and $t = 2.306$. The 100 95% confidence intervals were:

```
(172.3962, 178.6724); (174.4476, 183.1760); (173.8045, 178.3554);
  .....     .....       .....     .....       .....     .....
(169.0617,176.4371); (169.2078, 177.2485);  (170.8897, 182.4987).
```

The proportion of our intervals that contain the true mean 174 is $\frac{96}{100} = 0.96$. The average width of the 100 intervals is 9.66.

**(b)** This time 100 samples of 25 male heights were generated and, for each sample, a 95% confidence interval for the true mean was obtained. We now have $df = 24$ and $t = 2.064$. The proportion of the intervals that contain the true mean 174 is $\frac{94}{100} = 0.94$. The average width of the 100 intervals is 5.39, which is 4.27 less than than the average width of the intervals from (a).

[We expect the intervals to have approximately a 95% coverage no matter what the sample size is. The length of the intervals, however, is proportional to $\frac{1}{\sqrt{n}}$ (apart from a minor *df* effect), so the intervals get shorter as $n$ increases. (See Fig. 8.1.4).]

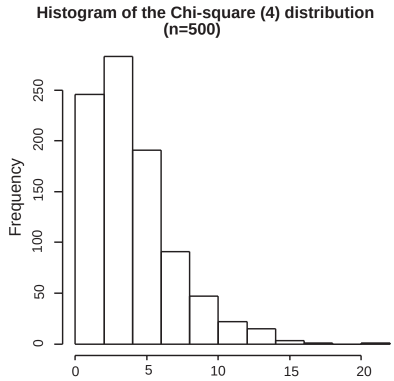
**20.** **(a)** The situation is just like tossing a biased coin $M$ times and counting the number of heads. Each "toss" consists of taking a sample and calculating an interval. Each results in one of only two possible outcomes ("heads" corresponds to covering the true value, "tails" corresponds to not covering the true value). The probability of getting a covering interval is always the same, namely 0.95. Independence comes from taking independent samples. Thus all the assumptions of the Binomial($n = M, p = 0.95$) distribution hold.

**(b)** Using the Binomial($n = 100, p = 0.95$) distribution, $\text{pr}(91 \leq X \leq 98) = \text{pr}(X \leq 98) - \text{pr}(X \leq 90) = 0.9347$. [Recall that with discrete distributions, whether or not end points are included in an interval is critically important.]

**(c)** Using the Binomial($n = 1000, p = 0.95$) distribution, pr($935 \le X \le 965$) = pr($X \le 965$) − pr($X \le 934$) = 0.9758. [Note that with $M = 1000$ we have an even larger probability of falling into a substantially narrower interval than we did in (a) with $M = 100$.]

***(d)** The margin of error is $1.96 \times \sqrt{\frac{0.95\times 0.5}{n}}$. If we want $1.96 \times \sqrt{\frac{0.95\times 0.5}{n}} \le 0.01$, we need to take (solving for $n$) $n \ge \left(\frac{1.96}{0.01}\right)^2 \times 0.95 \times 0.5 = 1824.76$. If we wanted to run a simulation at this level of precision we would use $M = 2000$.

**21. (a)** A histogram of 500 observations from a Chi-square distribution with 4 degrees of freedom is shown below.




**(b)** 100 samples of size 9 were generated. For each sample a 95% confidence interval was obtained using the formula $\overline{x} \pm t\,\text{se}(\overline{x})$. Here, $df = n - 1 = 8$ and $t = 2.306$. The one hundred 95% confidence intervals were:

```
(1.5800846, 7.845032); (1.9556367, 6.000979); (1.6196485, 5.745535);
   ......     .....      .....     .....      .....     .....
(2.0805119, 5.502391); (1.8812885, 4.266043); (1.2595665, 3.630293).
```

The proportion of our intervals that contained the true mean (4) was $\frac{88}{100} = 0.88$.

**(c)** We next generated 100 samples of size 25. Here, $df = n - 1 = 24$ and the $t$ multiplier is $t = 2.0639$. In this case the proportion of our intervals that contained the true mean (4) was $\frac{93}{100} = 0.93$.

**(d)** When 1000 samples of size 9 and 1000 samples of size 25 were generated, the proportion of intervals that contained the true mean (4) was $\frac{903}{1000} = 0.903$ and $\frac{946}{1000} = 0.946$ respectively. For a discussion of the issues involved here, see page 411 of the book.

**(e)** It is working very well by the time $n = 25$ and it is not too terrible even at $n = 9$.