Chapter 9 Significance Testing

Exercises for Section 9.2

In all that follows, the hypotheses relate to values for true (or population) means or proportions. The evidence we have about the truth or otherwise of those hypotheses comes from what is happening in the sample data.

1. Here μ is the population or true mean volume. **(a)** $H_0 : \mu = 750$. **(b)** $H_1 : \mu < 750$. **(c)** We would check whether the sample mean volume, $\overline{x}$, from the 40 bottles tested is too much smaller than 750 for the difference to be explained simply in terms of sampling variation.

2. **(a)** $H_0 : \mu_{white} - \mu_{blue} = 0$. **(b)** $H_1 : \mu_{white} - \mu_{blue} > 0$. **(c)** We would check whether the sample mean blood pressure from the white-collar sample, $\overline{x}_{white}$, is sufficiently much larger than the sample mean from the blue-collar sample, $\overline{x}_{blue}$, that the difference could not be explained simply in terms of sampling variation.

3. **(a)** $H_0 : \mu_{French} - \mu_{math} = 0$. **(b)** $H_1 : \mu_{French} - \mu_{math} > 0$. **(c)** We would check whether the sample mean comprehension mark from the French class, $\overline{x}_{French}$, was sufficiently larger than the sample mean from the Additional Mathematics class, $\overline{x}_{math}$, that the difference could not be explained simply in terms of sampling variation.

4. **(a)** $H_0 : \mu = 10$. **(b)** $H_1 : \mu \neq 10$. **(c)** We would check whether the sample mean diameter, $\overline{x}$, from the 50 ball-bearings tested is too far from 10 (in either direction) for the difference to be explained simply in terms of sampling variation.

5. **(a)** $H_0 : p = 0.5$. **(b)** $H_1 : p \neq 0.5$. **(c)** We would check whether the sample proportion of the 50 ball-bearings tested with a diameter greater than the target, $\widehat{p}$, is too far from 0.5 (in either direction) for the difference to be explained simply in terms of sampling variation.

6. **(a)** $H_0 : p = 0.5$. **(b)** $H_1 : p \neq 0.5$. **(c)** We would check whether the sample proportion of heads, $\widehat{p}$, in the 1000 coin tosses is too far from 0.5 (in either direction) for the difference to be explained simply in terms of sampling variation.

7. $H_0 : p_{giveaway} - p_{none} = 0$. **(b)** $H_1 : p_{giveaway} - p_{none} < 0$, expecting people attracted by free gifts to be less loyal. (Could also argue for a "$\neq$" alternative.) **(c)** We would check whether the sample proportion of the giveaway sample who were still doing business with the bank 5 years later, $\widehat{p}_{giveaway}$, is sufficiently much smaller than the corresponding proportion for the no-giveaway sample, $\widehat{p}_{none}$, that the difference could not be explained simply in terms of sampling variation.

8. **(a)** $H_0 : \mu = 0$. **(b)** $H_1 : \mu \neq 0$. **(c)** We would check whether the sample mean net-earnings per person, $\overline{x}$, for the 1000 customers studied is sufficiently far from zero that the difference could not be explained simply in terms of sampling variation.

Exercises for Section 9.3

1. Let p be the true proportion sucking their left thumbs in the womb.

(a) The research hypothesis is that birth-stress "pushes infants towards left-handedness," and thus there should be fewer "left handers" before birth than there are after birth. Let p be the true proportion of babies who are "left-handed" before birth. We thus want to test the sceptical $H_0 : p = 0.1$ (before birth is the same as after) versus $H_1 : p < 0.1$ (from the research hypothesis).

We have a sample of $n = 224$ babies of which a sample proportion $\widehat{p} = 12/224 = 0.05357$ suck their left thumbs. Now $\text{se}(\widehat{p}) = \sqrt{\frac{0.05357 \times 0.94643}{224}} = 0.01504$ and the t-test statistic is $t_0 = \frac{0.05357 - 0.1}{0.015045} = -3.086$. This tells us that the sample mean from the data is more than 3 standard errors below the value of 0.1 hypothesized for the true mean. The (1-tailed) P-value is $\text{pr}(Z \leq -3.086) = 0.001$. There is very strong evidence against H_0 in favor of H_1, or in terms of p, there is very strong evidence that fewer than 10% of babies suck their left thumbs.

[Warning: the 10% rule gives n to be at least 960, which is not true, so large-sample theory is a little suspect.]

(b) The study premise is that the thumb-sucking behavior of fetuses relates to left and right handedness after birth (apart from some switching due to such things as "birth stress"). We also assume that Belfast left-handedness rates are 10% or more. Our analysis relates to a population from which these babies can be considered a random sample.

2. Let p be the true probability of a person dying in the month before her or his birthday. The research hypothesis is that this probability p should be lower than for other months because of the postponing effect. We will assume that, if such an effect did not exist, the month before the birthday would be just like a randomly chosen month and so the probability of dying in that month would be 1 chance in 12. In these terms, our research hypothesis says that $p < \frac{1}{12}$.

(a) We wish to test the sceptical $H_0 : p = \frac{1}{12}$ (a month like any other) versus $H_1 : p < \frac{1}{12}$ (from the research hypothesis).

We have a sample of $n = 348$ individuals for which the sample proportion dying in the month before the birthday is $\widehat{p} = \frac{16}{348} = .04598$. Now $\text{se}(\widehat{p}) = \sqrt{\frac{0.04598 \times 0.95402}{348}} = 0.011227$ from which we obtain $t_0 = \frac{0.04598 - 0.08333}{0.01123} = -3.327$. This tells us that the sample proportion from the data is more than 3.3 standard errors below the value of $\frac{1}{12}$ hypothesized for the true probability. The (1-tailed) P-value is thus $\text{pr}(Z \leq -3.327) = 0.0004$.
There is very strong evidence against H_0 in favor of H_1, or more concretely, there is very strong evidence in favor of the postponing-death theory.

[Warning: the 10% rule gives n to be at least 960, which is not the case, so large-sample theory is a little suspect.]

(b) These were all "Notable Americans." To generalize we would have to assume that "ordinary" people have the same survival behavior as "notable" people as far as postponing death goes. We assume some sort of uniformity of the birth and death rates throughout the year. For example, if most births were in the summer and most deaths in the winter for reasons which had nothing to do with "postponing" death, our estimate of $\widehat{p}$ would be small.

3. Let μ be the true mean nicotine content. We will test $H_0 : \mu = 18$ versus $H_1 : \mu > 18$ (as the prior claim is one sided). We have a sample of $n = 12$ cigarettes for which the sample mean nicotine content is $\overline{x} = 19.1$ with a standard deviation of $s = 1.9$. Now $\text{se}(\overline{x}) = \frac{s}{\sqrt{n}} = \frac{1.9}{\sqrt{12}} = 0.54848$. The t-test statistic is thus $t_0 = \frac{19.1-18}{0.54848} = 2.0055$. This tells us that the sample mean from the data is more than 2 standard errors above the value of 18 hypothesized for the true mean.

Using $T \sim \text{Student}(df = n-1 = 11)$, the (1-tailed) P-value is $\text{pr}(T \geq 2.00555) = 0.035$. There is some evidence against H_0 in favor of H_1, or more concretely, we do have some evidence that the claim is false.

4. Let p_S be the true proportion of smoking mothers with infants getting colic and p_{NS} be the true proportion of non-smoking mothers with infants getting colic. There is not enough information given for us to determine whether the investigators suspected some particular effect of smoking or whether they just thought they noticed something. So we will test the sceptical $H_0 : p_S - p_{NS} = 0$ (smoking makes no difference) versus the 2-sided alternative $H_1 : p_S - p_{NS} \neq 0$. Of a sample of $n_S = 200$ babies of smoking mothers, a sample proportion $\widehat{p}_S = 0.4$ had colic compared with a proportion $\widehat{p}_{NS} = 0.2$ among $n_{NS} = 400$ babies of nonsmoking mothers. We are comparing proportions from independent samples (situation (a) in Fig. 8.5.1), so $\text{se}(\widehat{p}_S - \widehat{p}_{NS}) = \sqrt{\frac{0.4\times0.6}{200} + \frac{0.2\times0.8}{400}} = 0.04$. The test statistic is thus $t_0 = \frac{(0.4-0.2)-0}{0.04} = 5$. This tells us that our estimated difference in proportions from the data is more than 5 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 5) = 0.0000$. There is very strong evidence against H_0. There is very strong evidence that a true difference exists, or more concretely, very strong evidence that smoking mothers are more likely to have colicky babies. (We deduce the direction of the effect from the sample estimates. Later we will state as a rule never to perform a test without also constructing a confidence interval from which we can read off the likely the size of the difference.)

5. Let p_{ES} be the proportion knowing that Christ was resurrected on Easter Sunday and p_{GF} be the proportion knowing that Christ was crucified on Good Friday. We will test $H_0 : p_{ES} - p_{GF} = 0$ (no difference) versus the 2-sided alternative $H_1 : p_{ES} - p_{GF} \neq 0$, as we have no prior reason to expect a difference in one direction of the other.

In our sample of size $n = 1101$ people, the corresponding sample proportions were $\widehat{p}_{ES} = 0.66$ and $\widehat{p}_{GF} = 0.61$, thus suggesting that more people know what Easter Sunday commemorates. This is a situation (c) comparison in Fig. 8.5.1 so $\text{se}(\widehat{p}_{ES} - \widehat{p}_{GF}) = \sqrt{\frac{0.34+0.39-(0.66-0.61)^2}{1101}} = 0.02571$. Our test statistic is thus $t_0 = \frac{(0.66-0.61)-0}{0.02571} = 1.945$. This tells us that our estimated difference in proportions from the data is nearly 2 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 1.945) = 0.052$. We do have some evidence against H_0. We do have some evidence that a real difference exists, or more concretely, that more people know what Easter Sunday commemorates. (The direction of the difference is deduced from the sample estimates.)

6. Let p_B be the probability of accepting if claimed beneficial and p_{NB} be the probability of accepting if claimed not beneficial. We will test the sceptical hypothesis $H_0 : p_B - p_{NB} = 0$ (whether the paper "found" that social-work intervention was beneficial or not makes no difference to the probability of acceptance) versus the 2-sided alternative $H_1 : p_B - p_{NB} \neq 0$ on the grounds that the story did not contain enough information for us to know what Epstein hypothesized before starting the study. [We strongly suspect that his research hypothesis was that articles claiming intervention was beneficial would be *more* likely to be accepted. If this was the case, the alternative hypothesis should be $H_1 : p_B - p_{NB} > 0$.]

Of the $n_B = 70$ articles claiming benefit, a proportion $\widehat{p}_B = 0.53$, were accepted, whereas of $n_{NB} = 70$ claiming no benefit only a proportion $\widehat{p}_{NB} = 0.14$ were accepted. We are comparing proportions from two independent samples so $\text{se}(\widehat{p}_B - \widehat{p}_{NB}) = \sqrt{\frac{0.53\times 0.47}{70} + \frac{0.14\times 0.86}{70}} = 0.07265$. Our test statistic is thus $t_0 = \frac{0.53-0.14}{0.07265} = 5.368$. This tells us that our estimated difference in proportions from the data is more than 5 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 5.368) = 0.0000$. There is very strong evidence against H_0. There is very strong evidence that a true difference exists, or more concretely, that journals are more likely to accept articles claiming intervention is beneficial. (The direction of the effect is deduced from the data estimates.)

[Warning: The 10% rule require n_B to be at least 11 and n_{NB} to be at least 243, so large-sample theory is a little suspect.]

We are assuming that the 70 journals to get the "beneficial" version were selected at random and the journals made decisions independently, e.g., we do not have the situation where different journals are using the same referees to determine their decisions.

7. Let μ_{HS} be the true mean length in hedge-sparrow nests and μ_{GW} be the true mean length in garden-warbler nests.
We will test the sceptical null hypothesis $H_0 : \mu_{HS} - \mu_{GW} = 0$ (type of nest makes no difference) versus the 2-sided alternative $H_1 : \mu_{HS} - \mu_{GW} \neq 0$ (as there is no information about a direction of difference from a prior research hypothesis).

The $n_{HS} = 58$ eggs in hedge sparrow nests had a sample mean length of $\overline{x}_{HS} = 22.6$ and standard deviation of $s_{HS} = 0.8759$ compared with $\overline{x}_{GW} = 21.9$ and $s_{GW} = 0.7860$ for the $n_{GW} = 91$ eggs in garden warbler nests. Now, $\text{se}(\overline{x}_{HS} - \overline{x}_{GW}) = \sqrt{\frac{0.8759^2}{58} + \frac{0.7860^2}{91}} = 0.14148$ so that $t_0 = \frac{(22.6-21.9)-0}{0.14148} = 4.948$. This tells us that our estimated difference in means from the data is nearly 5 standard errors from zero. Using $T \sim$ Student with $df = \min(n_{HS} - 1, n_{GW} - 1) = 57$, the (2-tailed) P-value is $2 \times \text{pr}(T \geq 4.948) = 0.0000$. There is very strong evidence that a true difference in mean lengths exists, or more concretely, that eggs in hedge-sparrow nests tend to be larger. (The direction of the difference is deduced from the sample estimates.)

We cannot immediately conclude that the type of nest causes the observed differences in size as this is observational data. There may be other mechanisms such as bigger birds tending to select hedge-sparrow nests, or differences (e.g., in food supplies) between habitats containing mainly hedge sparrows or mainly garden warblers.

Review Exercises 9

Throughout the following Review Exercises we continue to abbreviate "confidence interval" to "CI." In choosing the alternative hypothesis for testing we have used the conservative 2-sided alternative unless it is clear that there was a research hypothesis that should determine the null. In many cases the researchers probably did have a research hypothesis and we have a strong suspicion about what that hypothesis would have been. In these cases, we have discussed the consequences of the use of our "suspected" research hypothesis.

1. (a) Let p_T and p_C be the respective true probabilities that a person will return a completed questionnaire with or without telephone contact. We will test the sceptical null hypothesis $H_0 : p_T - p_C = 0$ (phone calls make no difference) versus the 2-sided alternative $H_1 : p_T - p_C \neq 0$. [If the very plausible proposition that "a followup telephone call would increase the likelihood of a response" was the research hypothesis, then we should test versus $H_1 : p_T - p_C > 0$.]

Of the sample of $n_T = 239$ people followed up by telephone a proportion $\widehat{p}_T = \frac{134}{239} = 0.5607$ responded, versus a proportion $\widehat{p}_C = \frac{186}{836} = 0.2225$ of the $n_C = 836$ people in the control group.

Since we are comparing proportions from independent samples, $\text{se}(\widehat{p}_T - \widehat{p}_C) = \sqrt{\frac{0.56067\times0.43933}{239} + \frac{0.22249\times0.77751}{836}} = 0.03518$. Our test statistic is $t_0 = \frac{0.56067-0.22249}{0.03518} = 9.613$. This tells us that our estimated difference in proportions from the data is more than 9 standard errors from zero! The P-value is 0 to many more than 4 decimal places whether we do it 1- or 2-tailed. There is very strong evidence that a true difference exists, or more concretely, that phone calls increase the response rate.

(b) A 95% CI for $p_T - p_C$ is [0.27,0.41]. With 95% confidence, calls increase the percentage responding by between 27 and 41 percentage points.

(c) Even though there is substantially less nonresponse in the treatment group, it is still quite high so nonresponse bias would still be a worry. If only people they contacted by phone were sent questionnaires, this could add further bias.

3. (a) Let p_{none} be the true probability that a regular purchaser (no incentive) will buy again and p_{coup} be the true probability that a purchaser using a coupon will buy again. We will test $H_0 : p_{none} - p_{coup} = 0$ (no change) versus the 2-sided alternative $H_1 : p_{none} - p_{coup} \neq 0$. [If the very plausible proposition that "people buying using a coupon would be less loyal" was the research hypothesis, then we should test versus $H_1 : p_{none} - p_{coup} > 0$. In this problem, the change has no effect on the conclusions reached.]

From the data we get sample estimates $\widehat{p}_{none} = 0.87$ and $\widehat{p}_{coup} = 0.49$ from samples of size $n_{none} = 23,794$ and $n_{coup} = 671$ respectively.

We are comparing proportions from independent samples so $\text{se}(\widehat{p}_{none} - \widehat{p}_{coup}) = \sqrt{\frac{0.87\times0.13}{23794} + \frac{0.49\times0.51}{671}} = 0.019421$. From this we obtain test statistic $t_0 = \frac{(0.87-0.49)-0}{0.019421} = 19.566$. This tells us that our estimated difference in proportions from the data is more than 19 standard errors from zero! The P-value is vanishingly small whether we perform the test 1- or 2-tailed. It is clear that there is a true difference, or more concretely, there is very strong evidence that brand loyalty is lower when customers are attracted by inducements.

(b) A 95% CI for $p_{none} - p_{coup}$ is [0.34, 0.42]. With 95% confidence, brand loyalty is lower by between 34 and 42 percentage points when a coupon offer is involved.

(c) Among new customers, the reduction in brand loyalty will probably be higher as new customers may have only switched to the brand during the coupon special.

(d) The real issue here is whether such a promotion attracts sufficient new profits to be cost effective. (It does not matter if only a low proportion of the customers who switched during the promotion stayed with the brand.) To test this, it would be better to look at sales trends before and after the promotion and analyze these to see if there has been any significant jump in sales. (Why might you expect to see a short term drop in sales immediately after a promotion?)

5. (a) Let p_{pay} be the true proportion who would cooperate if the payment is made and p_{con} be the true proportion who would cooperate under control conditions (no payment). We will test $H_0 : p_{pay} - p_{con} = 0$ (payments make no difference to the probability of cooperation) versus the 2-sided alternative $H_1 : p_{pay} - p_{con} \neq 0$. [If the very plausible proposition that "payment would increase the probability of cooperation" was the research hypothesis, then we should test versus $H_1 : p_{pay} - p_{con} > 0$. The resulting P-value would be half the size of the one quoted below. For this problem, this would result in somewhat stronger evidence for the existence of the effect.]

The sample proportions from the data were $\widehat{p}_{pay} = 0.793$ and $\widehat{p}_{con} = 0.672$ from samples of size $n_{pay} = 111$ and $n_{con} = 116$ respectively.
We are comparing proportions from independent samples so $\text{se}(\widehat{p}_{pay} - \widehat{p}_{con}) = \sqrt{\frac{0.793\times 0.207}{111} + \frac{0.672\times 0.328}{116}} = 0.058129$. Our test statistic is $t_0 = \frac{(0.793-0.672)-0}{0.058129} = 2.0816$. This tells us that our estimated difference in proportions from the data is more than 2 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 2.082) = 0.037$. There is some evidence that a true difference exists, or more concretely, that payments increase cooperation rates.

(b) The 95% CI for $p_{pay} - p_{con}$ is [0.007, 0.235]. In this environment, with 95% confidence, a \$5 payment increases the percentage cooperating by somewhere between 0.7 percentage points (almost no increase) and 24 percentage points.

(c) Paying participants reduces the number of people you will be able to afford to survey. So one of the tradeoffs is response rate versus sample size.

7. Let p_{before} represent the true proportion of those opening counts before the promotion whose accounts were still open 6 months later. Let p_{during} be the corresponding true proportion for accounts opened during the promotion.

(a) We will test $H_0 : p_{before} - p_{during} = 0$ (no difference in loyalty) versus the 2-sided alternative $H_1 : p_{before} - p_{during} \neq 0$. [If the very plausible proposition that "people opening accounts during the promotion should be less loyal" was the research hypothesis, then we should test versus $H_1 : p_{before} - p_{during} > 0$ resulting in a P-value half the size of the one presented below. In this problem, this would have no real effect on the conclusions reached.]

Our data gives sample proportions of $\widehat{p}_{before} = \frac{178}{200} = 0.89$ and $\widehat{p}_{during} = \frac{158}{200} =$

0.79 from samples of size $n_{before} = 200$ and $n_{during} = 200$, respectively.
Since we are comparing proportions from separate samples (situation (a) in Fig.8.5.1), $\text{se}(\widehat{p}_{before} - \widehat{p}_{during}) = \sqrt{\frac{0.89\times 0.11}{200} + \frac{0.79\times 0.21}{200}} = 0.036318$. From this we obtain $t_0 = \frac{(0.89-0.79)-0}{0.036318} = 2.7535$. This tells us that our estimated difference in proportions from the data is more than 2.75 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 2.75) = 0.006$. There is strong evidence against H_0, or more concretely, strong evidence that the induced customers are less loyal.

The 95% CI for $p_{before} - p_{during}$ is [0.029, 0.17]. With 95% confidence, the true percentage of those accounts opened during the promotion that "remain loyal" is lower by somewhere between 3 and 17 percentage points than that for accounts opened before the promotion.

(b) The actual number of accounts retained and the value of the accounts to the bank. The cost of the promotion.

9. **(a)** Let p_{smoke} be true proportion of smokers who then have a stroke and p_{nonsm} be the corresponding true proportion for nonsmokers. We will test $H_0 : p_{smoke} - p_{nonsm} = 0$ (smoking makes no difference) versus the 2-sided alternative $H_1 : p_{smoke} - p_{nonsm} \neq 0$ (as there is no description of a research hypothesis suggesting a particular direction).
Our data gives sample proportions of $\widehat{p}_{smoke} = \frac{171}{3435} = 0.049782$ and $\widehat{p}_{nonsm} = \frac{117}{4437} = 0.026369$ from samples of size $n_{smoke} = 3435$ and $n_{nonsm} = 4437$, respectively. We are comparing proportions from separate samples (situation (a) in Fig 8.5.1) so $\text{se}(\widehat{p}_{smoke} - \widehat{p}_{nonsm}) = \sqrt{\frac{0.049782\times 0.950218}{3435} + \frac{0.026369\times 0.973631}{4437}} = 0.0044224$. Thus our test statistic is $t_0 = \frac{(0.049782-0.026369)-0}{0.0044224} = 5.294$. This tells us that our estimated difference in proportions from the data is more than 5 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 5.294) = 0.0000$. There is very strong evidence against H_0, i.e., very strong evidence of a true difference (smokers are more likely to have strokes).

(b) The 95% CI for the true difference, $p_{smoke} - p_{nonsm}$, is [0.015, 0.032]. With 95% confidence, the true percentage of smokers having strokes is higher by between 1.5 and 3.2 percentage points than the percentage for nonsmokers. Put another way, the risk is increased by somewhere between 1.5 and 3.2 chances in 100.

11. **(a)** Let μ_{morn} and μ_{aft} be the respective population mean pH levels for morning and afternoon patients. We will test $H_0 : \mu_{morn} - \mu_{aft} = 0$ (no difference between morning and afternoon) versus the 2-sided alternative $H_1 : \mu_{morn} - \mu_{aft} \neq 0$ (as there is no description of a research hypothesis suggesting a particular direction).

Summary statistics from the data are $\overline{x}_{morn} = 3.94$ and $s_{morn} = 2.51$ from a sample of size $n_{morn} = 50$, and $\overline{x}_{aft} = 2.93$ and $s_{aft} = 2.39$ from a sample of size $n_{aft} = 49$. Since we are dealing with separate or independent samples, $\text{se}(\overline{x}_{morn} - \overline{x}_{aft}) = \sqrt{\frac{2.51^2}{50} + \frac{2.39^2}{49}} = 0.49252$. The resulting t-test statistic is $t_0 = \frac{(3.94-2.93)-0}{0.49252} = 2.051$. This tells us that our estimated difference in means from the data is more than 2 standard errors from zero. Using Student's t distribution with $df = \min(n_{A.no} - 1, n_{A.sim} - 1) = 48$, the (2-tailed) P-value is $2 \times \text{pr}(T \geq$

2.051) = 0.046. We do have some evidence against H_0, i.e., evidence that a true difference exists (lower pH on average for morning patients).

The 95% CI for the true difference, $\mu_{morn} - \mu_{aft}$, is [0.02, 2.00]. With 95% confidence, the true mean pH level for morning patients is bigger than it is for afternoon patients by somewhere between 0.02 and 2.0 units.

(b) let p_{aft} and p_{morn} be the respective population proportions of morning and afternoon patients with a pH level below 2.5. We will test $H_0 : p_{aft} - p_{morn} = 0$ versus $H_1 : p_{aft} - p_{morn} \neq 0$.
Our data gives sample proportions of $\widehat{p}_{aft} = \frac{31}{49} = 0.63265$ and $\widehat{p}_{morn} = \frac{21}{50} = 0.42$ from samples of size $n_{aft} = 200$ and $n_{morn} = 200$ respectively. Since we are working with proportions from independent samples (situation (a) in Fig. 8.5.1), $\text{se}(\widehat{p}_{aft} - \widehat{p}_{morn}) = \sqrt{\frac{0.63265\times 0.36735}{49} + \frac{0.42\times 0.58}{50}} = 0.098056$. The resulting test statistic is $t_0 = \frac{(0.63265-0.42)-0}{0.098056} = 2.169$. This tells us that our estimated difference in proportions from the data is more than 2 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 2.169) = 0.03$. We do have some evidence against H_0, i.e., evidence that a true difference exists (more afternoon patients have a pH below 2.3).
The 95% CI for the true difference, $p_{aft} - p_{morn}$, is [0.02,0.40]. With 95% confidence, the true percentage of afternoon patients with a pH below 2.5 is larger than the corresponding percentage for morning patients by somewhere between 2 and 40 percentage points.

(c) It opens the possibility of biases. One would need to be assured that the allocation to morning or afternoon lists could not depend in any way on the metabolism of the patient.

13. **(a)** There are substantial proportions of reoffenders in both groups.

(b) We expect bias against the classification system as not paroling the "worst" prisoners should lower the reoffending rate in the high risk group and make the rates in the 2 groups more similar.

(c) Longer followup times would lead to higher proportions reoffending in both groups.

15. Let p_{Asian} be the true proportion of Asians voting Republican in 1998 and p_{Hispan} be the corresponding proportion for Hispanics. We will test $H_0 : p_{Asian} - p_{Hispan} = 0$ versus $H_1 : p_{Asian} - p_{Hispan} \neq 0$ (as there is no description of a research hypothesis suggesting a particular direction).
From our data we have sample proportions $\widehat{p}_{Asian} = 0.42$ and $\widehat{p}_{Hispan} = 0.35$ from samples of size $n_{Asian} = 100$ and $n_{Hispan} = 500$. For independent proportions, $\text{se}(\widehat{p}_{Asian} - \widehat{p}_{Hispan}) = \sqrt{\frac{0.42\times 0.58}{100} + \frac{0.35\times 0.65}{500}} = 0.05376802$.
$t_0 = \frac{(0.42-0.35)-0}{0.05376802} = 1.3019$. This tells us that our estimated difference in proportions from the data is only 1.3 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 1.3019) = 0.193$. We have no evidence of a real difference.
The 95% CI for the true difference, $p_{Asian} - p_{Hispan}$, is [−0.035, 0.18]. With 95% confidence, the percent-Republican vote for Asian Americans could be anywhere between

3.5 percentage points lower than it is for Hispanic Americans and 18 percentage points higher.

17. (a) Let p_{sinpar} be the true proportion of single parents who are stressed by relationships with parents and let p_{alone} be the corresponding proportion for those living alone. We will test $H_0 : p_{sinpar} - p_{alone} = 0$ versus $H_1 : p_{sinpar} - p_{alone} \neq 0$.
From the data we have the sample proportions $\widehat{p}_{sinpar} = 0.129$ from a sample of size $n_{sinpar} = 575$ and $\widehat{p}_{alone} = 0.103$ from a sample of size $n_{alone} = 875$. As we are comparing proportions from two independent samples (situation (a) in Fig. 8.5.1), we have $\text{se}(\widehat{p}_{sinpar} - \widehat{p}_{alone}) = \sqrt{\frac{0.129\times0.871}{575} + \frac{0.103\times0.897}{875}} = 0.017349$.
The resulting test statistic is $t_0 = \frac{(0.129-0.103)-0}{0.017349} = 1.4986$. This tells us that the estimated difference in proportions from our data is about 1.5 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 1.4986) = 0.13$. We have no evidence that a true difference exists.

The 95% CI for the true difference, $p_{sinpar} - p_{alone}$, is $[-0.008, 0.060]$. With 95% confidence, the true percentage stressed by relationships with parents for single parents is somewhere between being about 1 percentage point smaller than for those living alone and being 6 percentage points larger.

(b) Here we are only looking at those living as single parents. Let p_{smoke} be the true proportion of them who smoke and let $p_{unhealthy}$ be the true proportion with unhealthy eating practices. We will test $H_0 : p_{smoke} - p_{unhealthy} = 0$ versus $H_1 : p_{smoke} - p_{unhealthy} \neq 0$.
We have data on a sample of size $n = 496$ for which the sample proportions are $\widehat{p}_{smoke} = 0.541$ and $\widehat{p}_{unhealthy} = 0.432$. We are performing a situation (c) comparison from Fig. 8.5.1 so $\text{se}(\widehat{p}_{smoke} - \widehat{p}_{unhealthy}) = \sqrt{\frac{0.541+0.432-(0.541-0.432)^2}{575}} = 0.040884$. The resulting test statistic is $t_0 = \frac{(0.541-0.432)-0}{0.040884} = 2.6661$. This tells us that the estimated difference in proportions from our data is more than 2.6 standard errors from zero. The (2-tailed) P-value is $2 \times \text{pr}(Z \geq 2.6661) = 0.008$. We have strong evidence against H_0, i.e., we have strong evidence that a true difference exists (more likely to smoke than have unhealthy eating practices).

The 95% CI for the true difference, $p_{smoke} - p_{unhealthy}$, is [0.03 , 0.19]. With 95% confidence the true percentage who smoke is higher than the percentage who would report unhealthy eating practices by somewhere between 3 and 19 percentage points.

(c) Here we are only looking at those living with a partner and child(ren). Let p_{underw} be the true proportion of them falling into the underweight category and let p_{overw} be the true proportion falling into the overweight category. We will test $H_0 : p_{underw} - p_{overw} = 0$ versus $H_1 : p_{underw} - p_{overw} \neq 0$.
We have data on a sample of size $n = 915$ for which the sample proportions are $\widehat{p}_{underw} = 0.253$ and $\widehat{p}_{overw} = 0.216$. We are performing a situation (b) comparison from Fig. 8.5.1, so $\text{se}(\widehat{p}_{underw} - \widehat{p}_{overw}) = \sqrt{\frac{0.253+0.216-(0.253-0.216)^2}{915}} = 0.022607$. The resulting test statistic is $t_0 = \frac{(0.253-0.216)-0}{0.022607} = 1.6367$. This tells us that the estimated difference in proportions from our data is about 1.6 standard errors from zero. The (2-tailed) P-value is (2-tailed) P-value $= 2 \times \text{pr}(Z \geq 1.6367) = 0.10$. We have only weak evidence of a true difference.

The 95% CI for the true difference, $p_{underw} - p_{overw}$, is $[-0.007,\ 0.081]$. With 95% confidence, the true percentage who are underweight is somewhere between being 0.7 percentage points smaller than the percentage who are overweight and being 8 percentage points larger.

19. (a) If people were just guessing, the chances of identifying the one of the three slices that was different would be one in three.

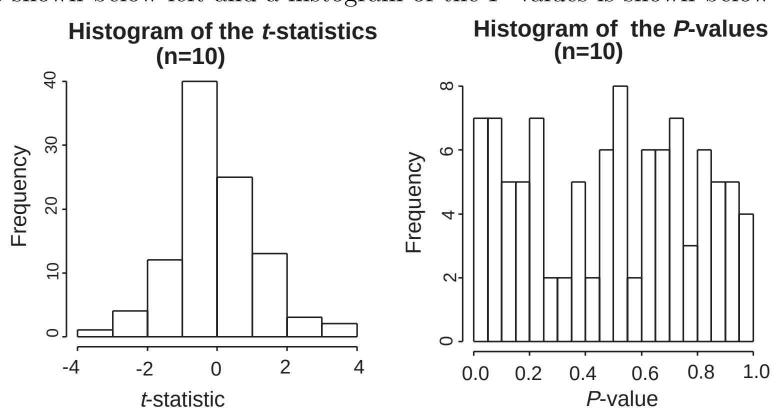
(b) We test $H_0 : p = \frac{1}{3}$ versus $H_1 : p > \frac{1}{3}$ (there is some ability to discriminate). We have $\widehat{p} = \frac{16}{27} = 0.5925926$, $\text{se}(\widehat{p}) = \sqrt{\frac{0.5925926 \times 0.4074074}{27}} = 0.094561$, and $t_0 = \frac{0.59259 - 0.33333}{0.094561} = 2.7417$. The sample proportion of correct identifications is over 2.7 standard errors above $\frac{1}{3}$. The (1-tailed) P-value is $\text{pr}(Z \geq 2.7417) = 0.003$. We have strong evidence against H_0, i.e., strong evidence that the true proportion of correct identifications is greater than "just guessing".
(Warning: The sample size is too small for this large sample theory.)

(c) P-value tells us that we have strong evidence that the identification rate is better than 1/3. The magazine has got it wrong.

(d) Possible differences in appearance can be catered for by using blindfolds. There is the possibility of learning over the 3 attempts so we could have more people and make only one identification each. Other ideas?

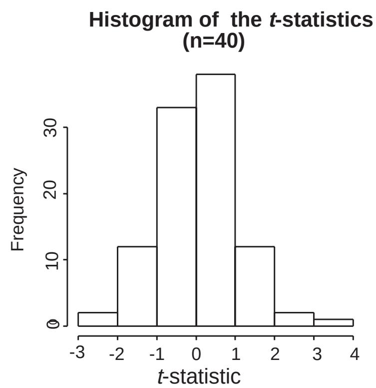
(e) If you use $H_0 : p = 1/2$ the result is not significant.

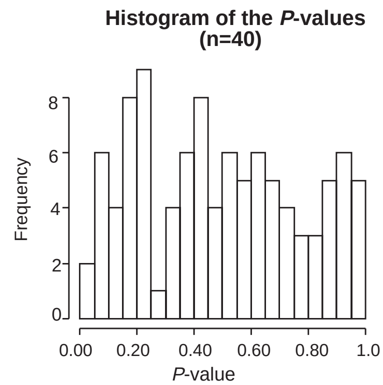
21. (a) One hundred samples, each of size $n = 10$, were generated under circumstances in which the null hypothesis was true. For each sample the t-statistic and the P-value for testing $H_0 : \mu = 5.517$ were obtained. A histogram of the t_0 values is shown below left and a histogram of the P-values is shown below right.



Our histogram of t_0-values is centered at about 0 (with a reasonably symmetric bell shape). When H_0 is true, P-values less than or equal to 0.05 occur 5% of the time over the long run. The proportion of our 100 P-values that was less than 0.05 was $\frac{7}{100} = 0.07$ or 7%. Your results will be somewhat different.

(b) We repeated (a) using 100 samples each of size $n = 40$. A histogram of the t_0 values is shown below left and a histogram of the P-values is shown below right.



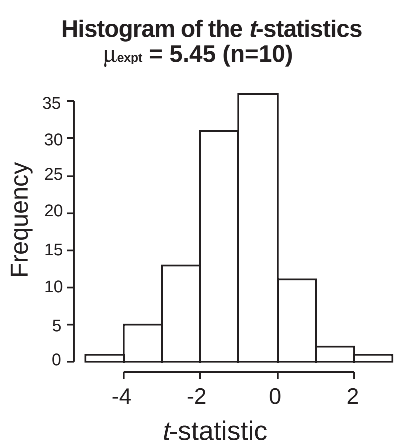


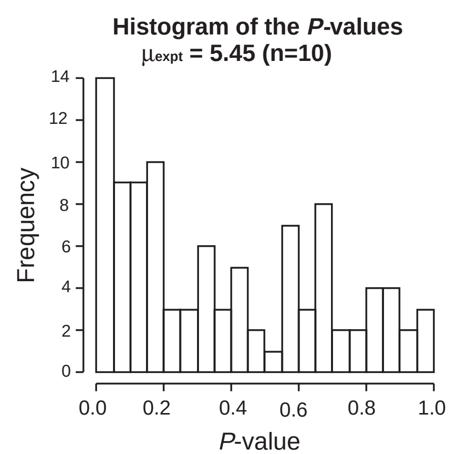
Our histogram of t_0 values is centered at about 0, bell shaped and looks somewhat right skewed. (We might have expected it would be more symmetric – see Note 1 to follow.) The proportion of our 100 P-values less than or equal to 0.05 was $\frac{2}{100} = 0.02$. Your results will be somewhat different.

Notes: We make the folowing points about (a) and (b).

1. The reason our histograms are not necessarily symmetric like Student's t-distribution (your one might be) is that we are only using 100 values and there is quite a bit of variation, from histogram to histogram, in histograms of 100 values. (Some are given at the end of this set of answers for comparative purposes.) If we had used t_0 values from hundreds of thousands of samples, our histogram would look like a t distribution.
2. It can be shown that when H_0 is true, the P-value is equally likely to fall anywhere between 0 and 1 (technically they have a Uniform(0,1) distribution) with 5% of them falling below 0.05 in the long run. Our histograms in (a) and (b) do look like histograms of samples of size 100 from the Uniform distribution. Some are given at the end of this set of answers for comparative purposes.

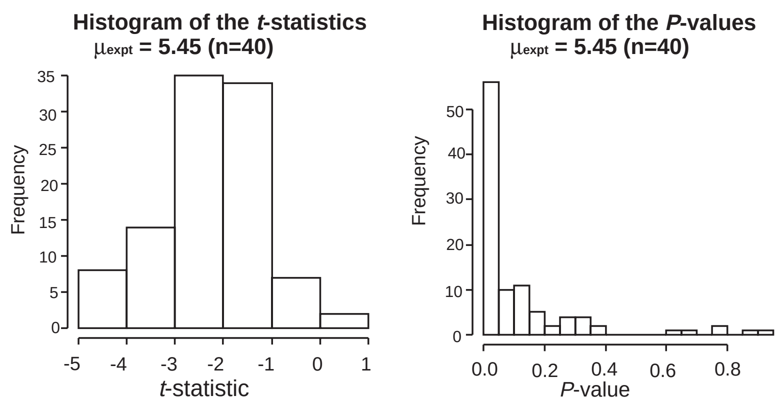
(c) *Samples of size 10*: 100 samples of size $n = 10$ with $\mu_{\text{expt}} = 5.45$ were generated (i.e., H_0 is false in that the experiment is slightly biased).





For each sample the t-statistic and the P-value for testing $H_0 : \mu = 5.517$ were obtained. A histogram of our 100 t_0 values is shown above left and a histogram of our P-values is shown above right. We see that the distribution of t_0 values is no longer centered at 0, but is now centered at approximately -1. The distribution of P-values is no longer uniform in shape but is now negatively skewed and beginning to stack up against the left hand side of the plot. The proportion of our P-values that were less than or equal to 0.05 is now bigger at $\frac{14}{100} = 0.14$ (cf. 0.05) but still fairly small.

Samples of size 40: We repeated the above experiment using samples of size $n = 40$ under exactly the same conditions. Histograms of the t_0 values (below left) and the P-value (below right) for each sample for testing $H_0 : \mu = 5.517$ follow.

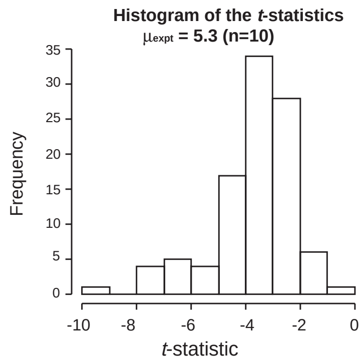


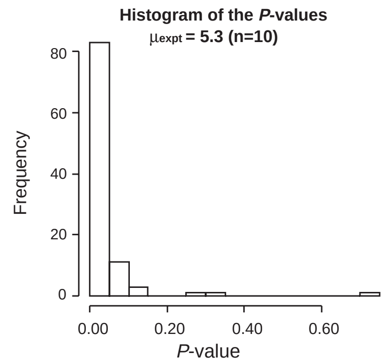
We see that the distribution of t_0 values is now centered at approximately -2.5. Note also how the histogram has become very skewed and stacked up against the left-hand side of the plot. The proportion of the P-values less than or equal to 0.05 is much bigger at $\frac{56}{100} = 0.56$ or nearly 60%.

The intended lesson is that it is easier to detect departures from a null hypothesis with larger samples.

(d) We now shift the true value of μ even further away from the hypothesized value.

Samples of size 10: Histograms of our 100 t_0 values and the P-values from the 100 samples are given below. We should compare these plots with our other plots for $n = 10$. The distribution of t_0 values has moved further to the left (now centered around approximately -3.5), the P-value histogram is stacked more strongly against the left-hand side and the proportion of our P-values less than or equal to 0.05 is $\frac{83}{100} = 0.83$.

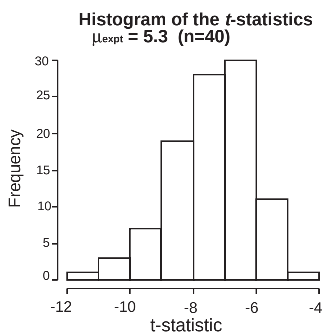


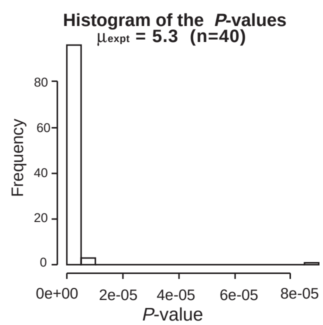


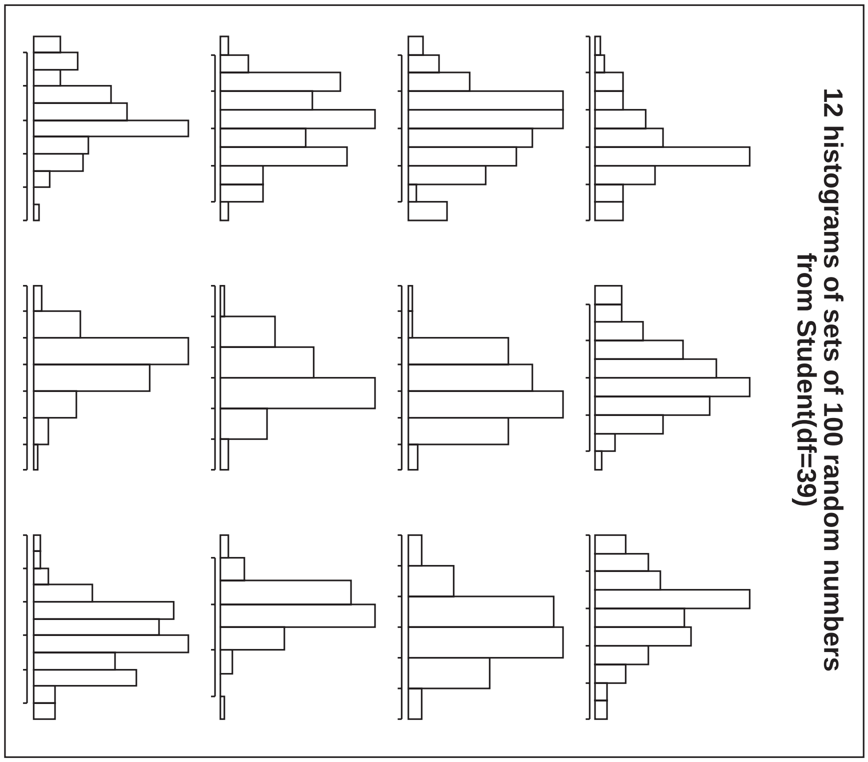
Samples of size 40:

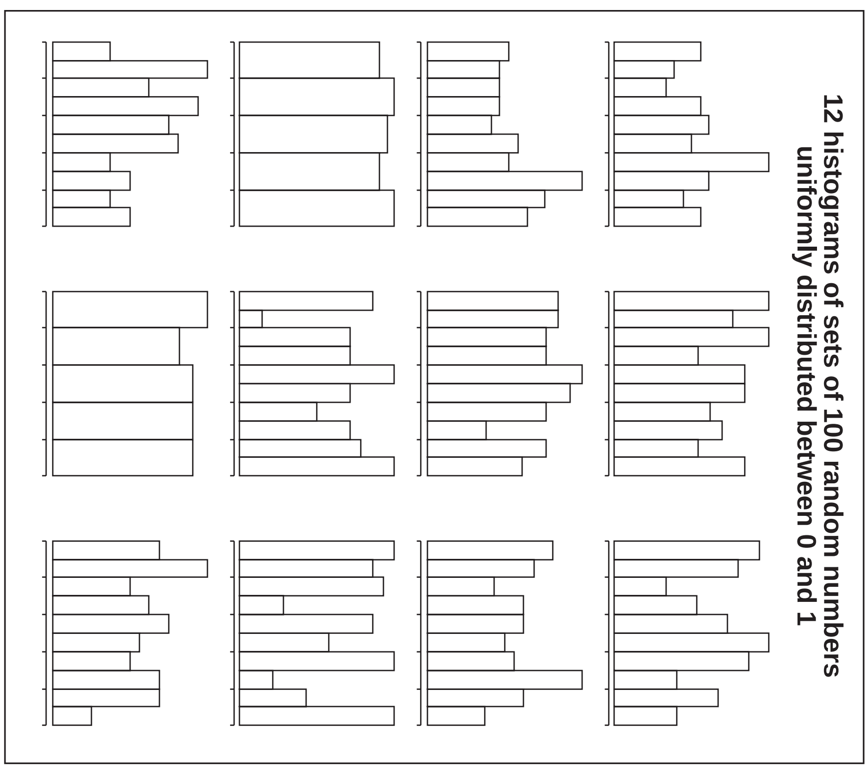
We should compare these plots (given below) with our other plots for $n = 40$. The distribution of t_0 values has moved further to the left (now centered around approximately -7), the P-value histogram is stacked more strongly against the left-hand side and *all* of our P-values were less than or equal to 0.05.

The intended lesson is that it is easier to detect larger departures from a null hypothesis than it is to detect smaller ones. It is also easier with larger samples.






12 histograms of sets of 100 random numbers
from Student(df=39)

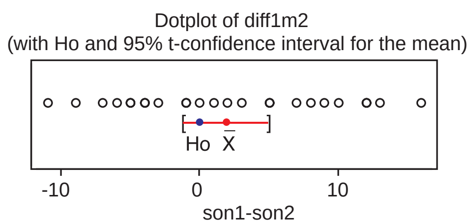

12 histograms of sets of 100 random numbers
uniformly distributed between 0 and 1

Chapter 10 Data on a Continuous Variable

All answers in this chapter were computed using Minitab.

Exercises for Section 10.1.2

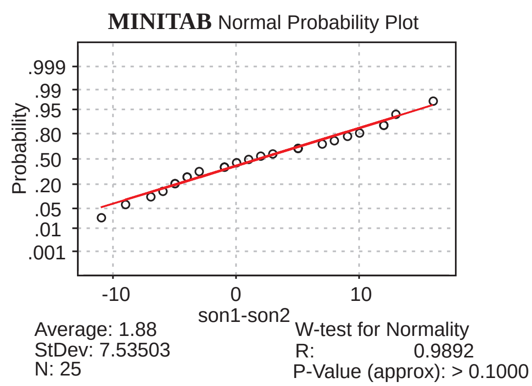
1. (a) We plot the differences (son1-son2). The following dot plot or stem-and-leaf plot do not show up any unusual points, though the data tends to be fairly uniformly spread. However, the Normal probability plot is close to a straight line and the W-test shows no evidence of non-Normality (P-value > 0.1).

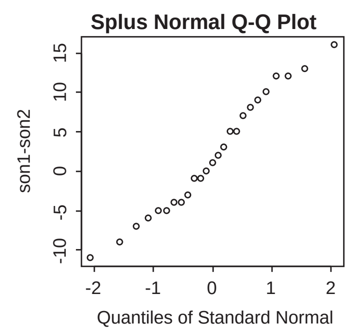


Character Stem-and-Leaf Display

```
Stem-and-leaf of diff1m2   N  = 25
Leaf Unit = 1.0


   1    -1 1
   6    -0 97655
  11    -0 44311
  (4)    0 0123
  10     0 55789
   5     1 0223
   1     1 6
```





Note: We have included another Normal probability plot (from Splus). Here the data axis is the vertical axis and the Normal distribution axis is the horizontal axis. This is the reverse of the Minitab plot. We have done this to illustrate that there are differences between packages in the way they orient their Normal probability plots. Apart from the choice and labelling of axes they are, however, the same type of plot.]

(b) Let μ_{diff} be the population mean of the differences. We wish to test $H_0 : \mu_{diff} = 0$ versus $H_1 : \mu_{diff} \neq 0$. Using a paired-comparison t-test, $t_0 = 1.25$ and P-value $= 0.22$, i.e., no evidence against H_0. There is no evidence of a difference between the head lengths. Assuming Normality, a 95% t-confidence interval for μ_{diff} is $[-1.23, 4.99]$, so at this level of confidence, the true mean difference could be anywhere between -1.23 (1st sons smaller) and 4.99 (1st sons larger). This information is depicted on the dot plot above.

(c) How were the families selected? How were the measurements taken? Was a standard procedure strictly followed?

2. People would vary in how they administered the procedure. The size of any systematic difference between the two sets of calipers will vary with how the head measurement is taken and from what part of the head it is taken. As the cardboard calipers wear, they will tend to give bigger measurements.

Exercises for Section 10.1.3

1. Let $\tilde{\mu}_{diff}$ be the population median of the differences. We wish to test $H_0 : \tilde{\mu}_{diff} = 0$ versus $H_1 : \tilde{\mu}_{diff} \neq 0$. Using a sign test we have 13 plus signs, 11 minus signs, and 1 zero. Intuitively such a result is not significant. (Think about tossing a fair coin.) In fact, P-value = 0.84. There is no evidence of a difference, i.e., no evidence of a difference in head length. A sign 95% confidence interval for the true median difference $\tilde{\mu}_{diff}$ is $[-3.80, 6.60]$.

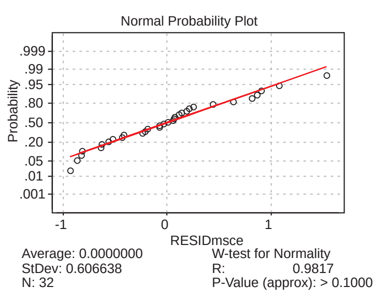
2. Let $\tilde{\mu}$ be the median score. We wish to test $H_0 : \tilde{\mu} = 28$ versus $H_1 : \tilde{\mu} \neq 28$. Using the sign test, we have 10 plus signs, and 4 minus signs with P-value = 0.18. We have no evidence against H_0, i.e., no evidence that cyclozocine is an effective treatment. A sign 95% confidence interval for $\tilde{\mu}$ is $[27, 51]$ so that with 95% confidence, the true median is somewhere between 27 and 51. Note that the interval contains the hypothesized value of 28.

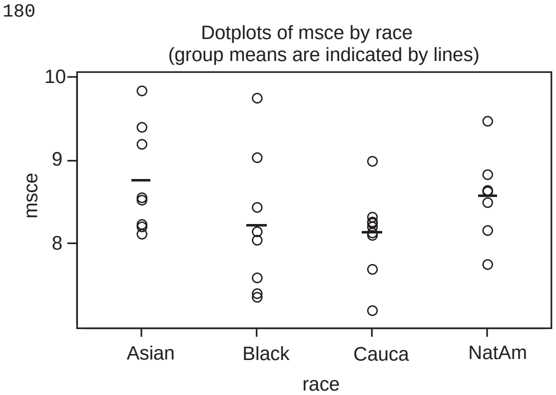
Exercises for Section 10.3

1.

```
Analysis of Variance for msce
Source   DF      SS      MS     F     P
race      3   2.136   0.712  1.75 0.180
Error    28  11.408   0.407
Total    31  13.544

Level       N      Mean     StDev
Asian       8    8.7587    0.6397
Black       8    8.2237    0.8389
Cauca       9    8.1311    0.4851
NatAm       7    8.5729    0.5377
```

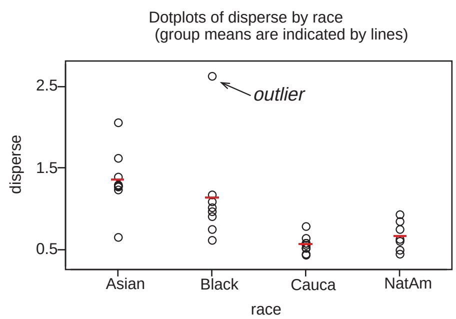


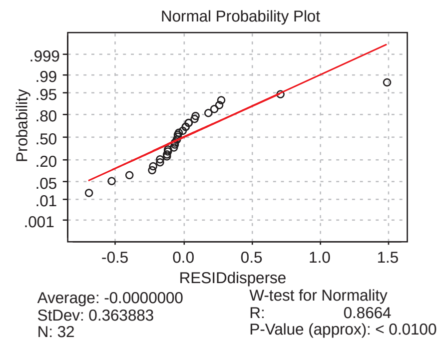


For the MSCE data we test H_0: population means all equal versus H_1: population means not all equal. Using an F-test, $f_0 = 1.75$ and P-value = 018 (see the printout above). There is no evidence against H_0, that is, no evidence of racial differences. The dot plots indicate that the four samples have acceptably similar spreads (the standard deviations range from 0.49 to 0.84). The (combined) Normal probability plot of the residuals is closely linear (apart from displaced end points, which is not atypical of Normal plots; see Fig. 10.1.3). The W-test has P-value > 0.1 providing no evidence of non-Normality.

2.

```
Analysis of Variance for disperse
Source     DF        SS        MS        F        P
race        3     3.473     1.158     7.90    0.001
Error      28     4.105     0.147
Total      31     7.578
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  ---+---------+---------+---------+---
Asian       8    1.3563    0.3933                    (------*------)
Black       8    1.1463    0.6259               (------*------)
Cauca       9    0.5667    0.1063   (-----*------)
NatAm       7    0.6743    0.1774    (-------*------)
                                   ---+---------+---------+---------+---
Pooled StDev =   0.3829              0.40      0.80      1.20      1.60
```





For the DISPERSION data we wish to test H_0: population means all equal versus H_1: population means not all equal. Using an F-test, $f_0 = 7.90$ and P-value $= 0.001$. There is very strong evidence of racial differences. Looking at the 95% confidence intervals for the four individual means in the computer printout above, we see that the Asian confidence interval does not overlap with the Caucasian or Native American confidence intervals, and the Black confidence interval does not overlap with the Caucasian. We will not go any further with this analysis because of the presence of the outlier labelled in the dot plot, and worries about differences in spreads between the groups.

3. The dot plot above shows a high outlier at 2.63. The numerator of the F-test measures how far apart the sample means are. Removing the outlier will reduce the mean of the Black group. This will move three of the means closer together, thus reducing the numerator. However, removing the outlier will substantially reduce the internal variation of the Black data thus reducing the denominator. Since means are less sensitive than standard deviations to outliers, the F-ratio might be expected to increase, though it is hard to tell.

4. We have the following computer printout when the outlier is removed.

```
Analysis of Variance for disperse
Source     DF        SS        MS        F        P
race        3    3.0148    1.0049    17.08    0.000
Error      27    1.5887    0.0588
Total      30    4.6035
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  ---------+---------+---------+-------
Asian       8    1.3563    0.3933                        (----*----)
Black       7    0.9343    0.1943             (-----*----)
Cauca       9    0.5667    0.1063  (----*----)
NatAm       7    0.6743    0.1774     (----*-----)
                                   ---------+---------+---------+-------
Pooled StDev =   0.2426                 0.70      1.05      1.40
```

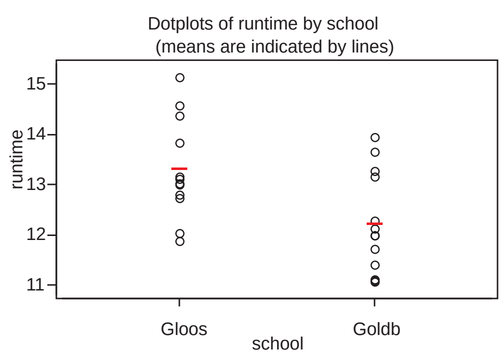
We see that $f_0 = 17.08$ and P-value $= 0.000$, again indicating very strong evidence of racial differences. However, the individual 95% confidence intervals for the Asian and Black groups no longer overlap, so that the Asian group is clearly different from the other three. The value of f_0 has increased, as suggested in 3.

5. The spread for the Asian group is much greater than that for the other three, which are quite similar. The standard deviation for the Asian group is 0.3933 and that for the Caucasion group is 0.1063, a ratio of nearly 4. The F-test and confidence intervals for differences between the means may be of doubtful validity.

[In fact a Levene test for differences in spread was nonsignificant indicating that the apparent differences in spread could have arisen just through sampling variation.]

Review Exercises 10

1. (a)



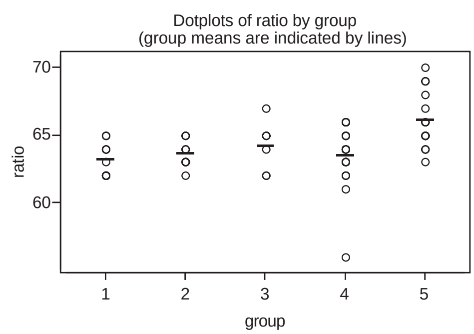
From the dot plots, running times seem longer on average at Glooscap. The spreads look similar. Let μ_{Gloo} and μ_{Cold} be the respective mean running times for Glooscap and Coldbrock. We wish to test $H_0 : \mu_{Gloo} - \mu_{Cold} = 0$ versus $H_0 : \mu_{Gloo} - \mu_{Cold} \neq 0$. Using a Welch two-sample t-test we have P-value $= 0.012$. There is reasonably strong evidence of a difference between the two schools. A 95% confidence interval for the difference in the means is $[0.26, 1.91]$, that is, a difference in true mean running times of between about 0.3 and 1.9 seconds.

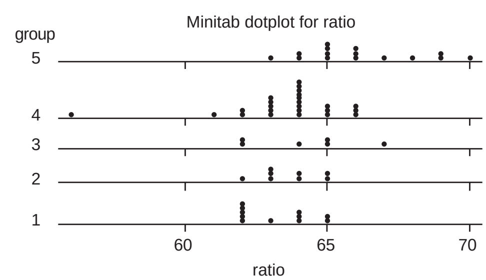
The dot plot looks reasonable, the individual Normal probability plots (not shown) look reasonably linear and both groups give P-values > 0.1 on a W-test for Normality. The Normal theory methods appear to be applicable.

***(b)** Let $\tilde{\mu}_{Gloo}$ and $\tilde{\mu}_{Cold}$ be the respective median running times. We wish to test $H_0 : \tilde{\mu}_{Gloo} - \tilde{\mu}_{Cold} = 0$ versus $H_0 : \tilde{\mu}_{Gloo} - \tilde{\mu}_{Cold} \neq 0$. The Mann-Whitney (Wilcoxon) test gives P-value $= 0.036$, which provides some evidence of a school difference. An approximate 95% confidence interval for the difference in true (or population) medians is $[0.16, 1.92]$.

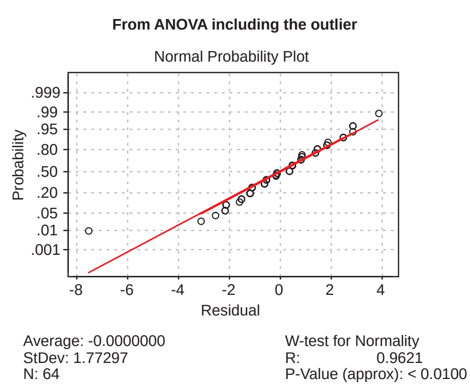
(c) The problem here is that we have an observational study, not an experiment, so that we cannot prove causality, namely, that the coach makes a difference. For example, the better runners might go to Glooscap. (How would you prove that coaching makes a difference?)

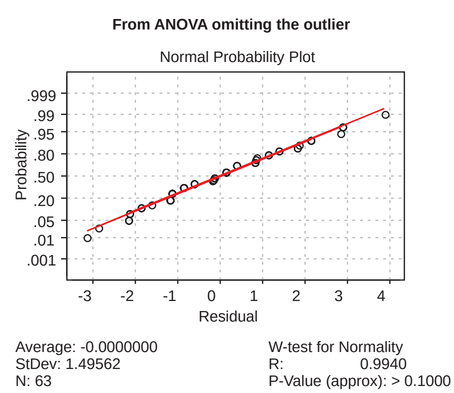
3. (a) We have included two sets of dot plots from Minitab. The left-hand set comes from the analysis of variance program and makes no adjustment for overprinting. The right-hand set comes from Minitab's specialist dot plot program and uses stacking to avoid overprinting. This data is clearly heavily rounded and overprinting is a real problem here. We see that 56 in group 4 is an outlier. Also group 5 has a larger mean and a larger spread than the other groups.





(b) We wish to test H_0 : population group means all equal versus H_1 : population group means not all equal. Using the F-test we have $f_0 = 5.99$ and P-value $= 0.000$. There is very strong evidence of a difference in the group means. The outlier shows up very clearly in the Normal probability plot of the residuals (and is the cause of the significant P-value for the W-test).





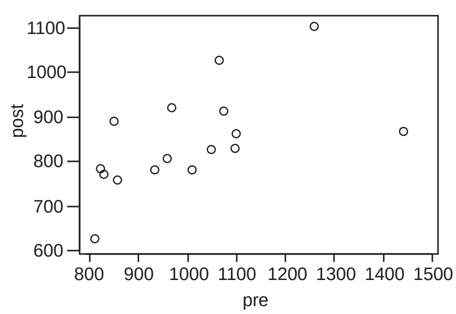
Without the outlier we have the following output.

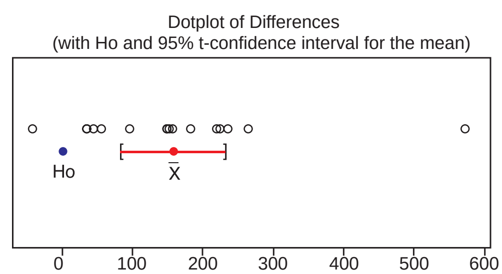
```
Analysis of Variance for ratio
Source     DF        SS        MS        F        P
setup       4     72.17     18.04     7.55    0.000
Error      58    138.69      2.39
Total      62    210.86
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  ------+---------+---------+---------+
1          11    63.182     1.250   (-----*-----)
2           8    63.625     1.061     (------*------)
3           6    64.167     1.941       (--------*-------)
4          23    63.870     1.290         (----*---)
5          15    66.133     2.066                        (----*----)
                                   ------+---------+---------+---------
```

We see that $f_0 = 7.55$ with P-value $= 0.000$. The conclusion that real differences exist between the true means is unchanged. If we leave out the outlier, the 95% confidence interval for the mean of group 5 does not overlap with the other four confidence intervals. The combined Normal probability plot of the residuals (above) is reasonable, and the maximum ratio of two standard deviations is (just) less than 2.

(c) Without the outlier, Fisher's pairwise comparisons are:
$1-2$: $[-1.88, 0.10]$, $1-3$: $[-2.56, 0.59]$, $1-4$: $[-1.82, 0.45]$, and $1-5$: $[-4.18, -1.72]$; $2-3$: $[-2.21, 1.13]$, $2-4$: $[-1.52, 1.03]$, and $2-5$: $[-3.86, -1.15]$; $3-4$: $[-1.12, 1.72]$, $3-5$: $[-3.46, -0.47]$, and $4-5$: $[-3.29, -1.24]$.
The group 5 mean is clearly different from the other 4 means. The intervals for differences between the other means contain zero so we cannot demonstrate the existence of real differences. As the confidence intervals show, however, we also cannot rule out the possibility of quite large differences in either direction.

5. (a)



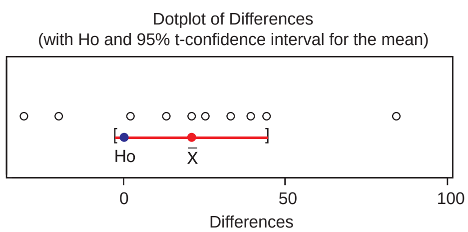


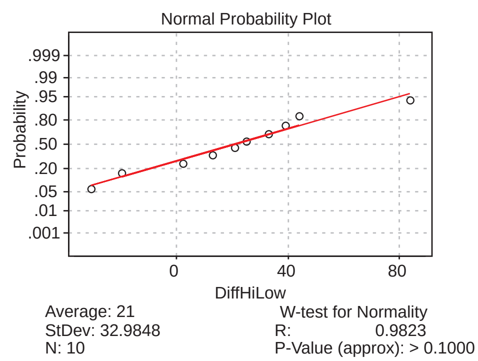
Using a scatter plot, we see that the poststerilization-factor-V level tends to get larger as the presterilization-factor-V level gets larger. There is a definite upward trend.

(b) We use the paired comparison method. If *diff* = *pre* − *post* we wish to test $H_0 : \mu_{diff} = 0$ versus $H_1 : \mu_{diff} \neq 0$. We use a two-sided test as there is no suggestion that there was research hypothesis that predicted a direction of difference. Using a t-test, $t_0 = 4.50$ with P-value $= 0.000$. There is very strong evidence that sterilization makes a difference. A 95% confidence interval for the true mean difference, μ_{diff}, is $[82.9, 232.2]$. Since *diff* = *pre* − *post* gives the reduction in factor V with sterilization, we can say with 95% confidence that sterilization decreases factor V levels by somewhere between 83 and 230 units, on average. At least that would be our conclusion if we were happy with the way the data looked.

(c) A dot plot of the differences (above) indicates an outlier. It also shows up very clearly in the Normal probability plot (not shown) which has a P-value of approximately 0.01. After removing the outlier (donor number 16) the Normal probability plot and W-test become satisfactory (not shown). Retesting without the outlier gives us $t_0 = 5.62$ with P-value $= 0.000$, so that there is no change in our conclusion about the existence of a difference. However, the confidence interval for the true difference is now $[80.4, 179.6]$, which is a lot shorter. We can say with 95% confidence that sterilization decreases factor V levels by somewhere between 80 and 180 units.

7. (a) We use the method of paired comparisons, as we have measurements on the same brand. Let $\textit{diff} = \textit{high} - \textit{low}$. We wish to test $H_0 : \mu_{diff} = 0$ versus $H_1 : \mu_{diff} > 0$. Using a one-sample t-test, $t_0 = 2.01$ with a (one-sided) P-value of 0.037. There is some evidence against H_0, i.e., or some evidence that high-recall commercials do tend to have higher activity indices.

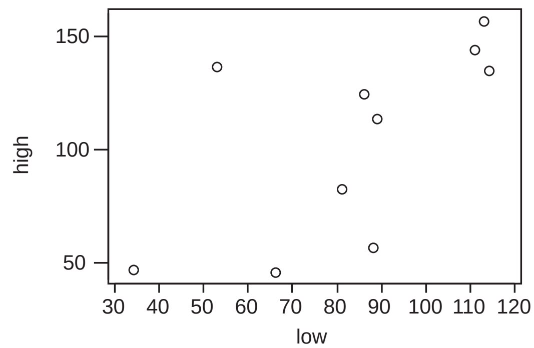




The dot plot, Normal probability plot and the W-test indicate that Normality is a reasonable assumption.

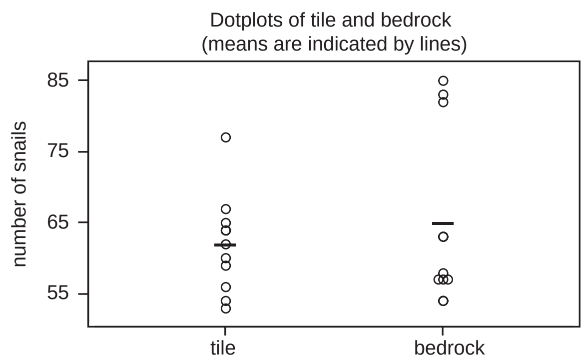
(b) No, as all that is established is that a difference in mean activity levels between high and low-recall commercials exists. This does not establish that the relationship between activity and recall is very close. We note that two brands actually had negative differences.

(c) The following scatter plot shows that there is a weak relationship (upwards trend) which seems to be almost nonexistent for the seven observations closest to the origin.



(d) Through randomization one can try and eliminate any systematic bias due to the order in which the ads are seen, e.g., effects due to experimental subjects becoming more tired or inattentive over time.

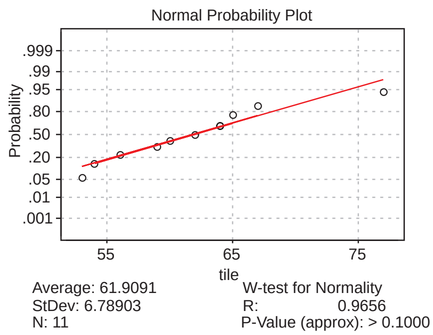
9. (a)

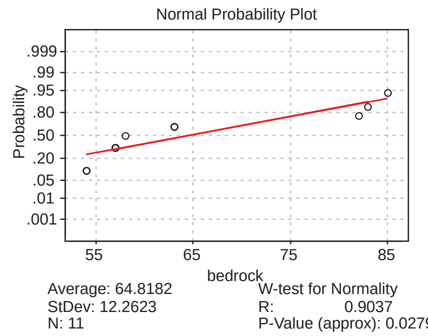


The dot plot suggests that average numbers of snails on bedrock might possibly be greater than on tile.

Using a Welsh two-sample t-test to test $H_0 : \mu_{tiles} - \mu_{bedr} = 0$ versus $H_1 : \mu_{tiles} - \mu_{bedr} \neq 0$, we obtain $t_0 = -0.69$ and P-value $= 0.5$ providing no evidence of a true difference.

From the dot plot, there is a hint of an outlier in the tile sample. However, the following Normal probability plot and W-test (below left) are supportive of the Normality assumption. The bedrock sample looks a little strange in the dot plot (we have done some staggering to cope with overprinting). There are 8 points below the sample mean, a large gap and then 3 larger observations. We see under the Normal probability plot (below right) that W-test has P-value $= 0.03$ indicating significant departures from Normality.

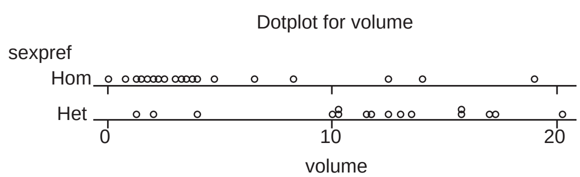




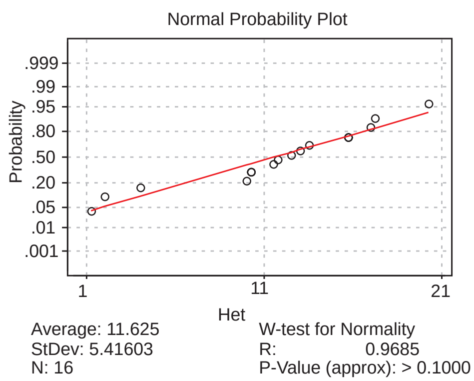
(b) A Mann-Whitney (Wilcoxon) test has P-value $= 1.000$! The reason for this strange result is that it uses a t_0-statistic that takes the value of zero; this has as one-sided P-value of 0.5, which is doubled. An approximate 95% confidence interval for the difference in the medians is $[-10, 7]$. There is clearly no evidence of a difference. However, we need to be careful about the bedrock sample. The Mann-Whitney test is strictly a test to see if two independent samples come from the same distribution, and, although we don't have significance, the two samples are very different looking.

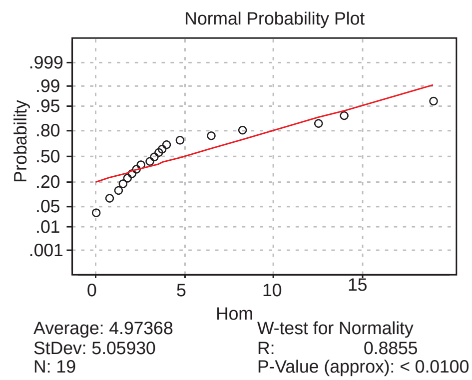
(c) You would need to randomize the placing of the tiles and the selection of bedrock samples to avoid any systematic bias.

11. **(a)** From the following dot plots, it appears that average INAH-3 volume is larger for heterosexuals than homosexuals. (The question remaining to be answered in (b) is whether the shift we are seeing might just be due to sampling variation.) We see that the heterosexual data appear slightly skewed while the homosexual data are more strongly skewed, but in the opposite direction.



The Normal probability plot and the W-test for the heterosexual data provide no evidence against the Normality assumption (below left), while the plot for the homosexual data emphasizes the skewness and the W-test shows significant non-Normality.

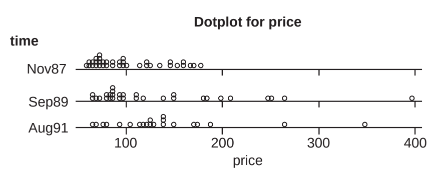


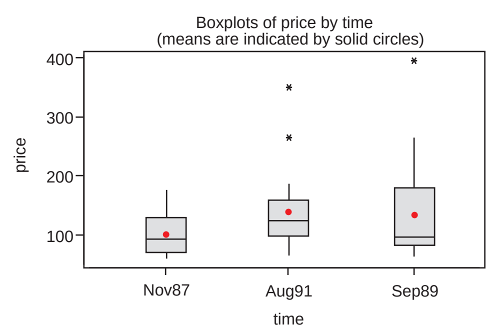


(b) We use a two-sample t-test to test $H_0 : \mu_{het} - \mu_{hom} = 0$ versus $H_1 : \mu_{het} - \mu_{hom} \neq 0$. The Welch test gives $t_0 = 3.73$ with P-value $= 0.0008$, giving very strong evidence for a difference. A 95% confidence interval for $\mu_{het} - \mu_{hom}$ is given by $[3.0, 10.3]$.

(c) This is an observational study, so we cannot prove causality. The samples are not random, as a very high percentage (about 38%) of the heterosexual men died of AIDS.

13. **(a)** Dot plots and box plots follow.





The means and standard deviations are: $\overline{x}_{87} = 101.59$, $s_{87} = 36.11$; $\overline{x}_{89} = 134.37$, $s_{89} = 76.89$; and $\overline{x}_{91} = 139.33$, $s_{91} = 66.19$. There is a substantial increase in the sample mean from November 1987 to September 1989 and almost no difference between September 1989 and August 1991. There is also a substantial increase in the spread after 1987. From the dot plots we see that this is in part due a few more expensive homes in 1989 and 1991. The box plots show similar trends, though the differences don't appear to be so obvious because of the compressed vertical scale.

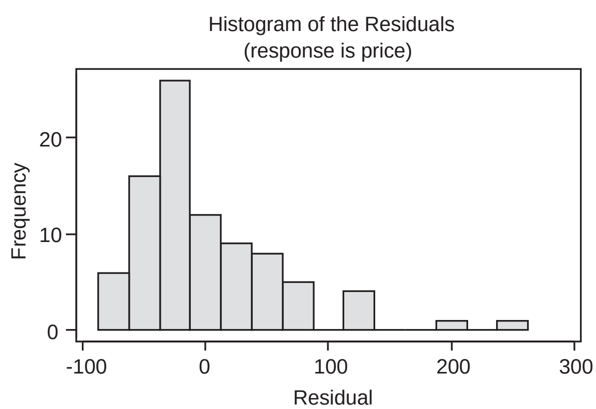
(b) We wish to test H_0: three means equal versus H_1: three means not all equal. The printout for the F-test follows. We see that $f_0 = 3.65$ with P-value $= 0.030$, yielding some evidence against H_0. The individual 95% confidence intervals overlap so we cannot immediately conclude that 1987 is different.

```
One-way Analysis of Variance

Analysis of Variance for price
Source     DF        SS        MS        F        P
time        2     26299     13149     3.65    0.030
Error      85    306033      3600
Total      87    332331
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  --------+---------+---------+--------
Aug91      21    139.33     66.19              (----------*---------)
Nov87      37    101.59     36.11   (-------*------)
Sep89      30    134.37     76.89              (--------*-------)
                                   --------+---------+---------+--------
Pooled StDev =    60.00                   100       125       150
```

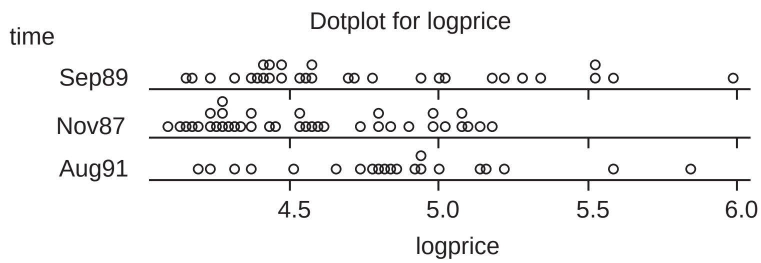
(c) Using Fisher's pairwise comparisons, we have $91-87:[5.1,70.3]$, $89-87:[3.5,62.1]$, $91-89:[-29.0,38.9]$. The intervals are quite wide, indicating a fairly large degree of uncertainty about the differences between the true means. For example, with 95% confidence, the true 1987 mean was smaller than that for 1989 by somewhere between \$3500 and \$62,000.

(d) We see that $s_{89} > 2s_{87}$. There are outliers present, and the 1989 data are clearly skewed. The histogram of the residuals is skewed.

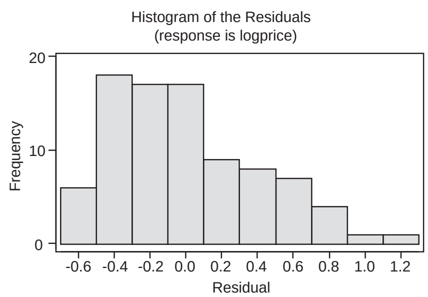


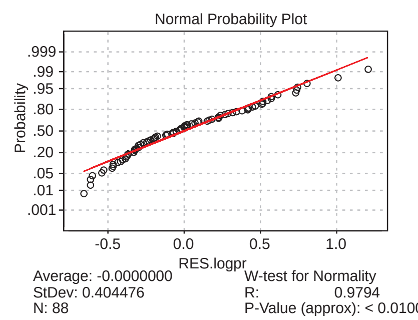
The above analysis is therefore suspect.

(e) Except for possible outliers, the dot plots indicate that some of the skewness seems to have been removed, and the spreads are now more similar.



(f) Using the F-test with the logarithmic data, we get $f_0 = 4.00$ and P-value $= 0.022$, so that our conclusion is unchanged. The data are still skewed, as seen from the histogram of the residuals (below left) and the slight curvature in their Normal probability plot (below right).





We find that the standard deviations are now similar. We again conclude that there is a significant increase in house prices from 1987 to 1989, and no evidence of a change from 1989 to 1991.

(g) An increase in a mean house price does not imply that all individual house prices go up; some will go down as well. The top end of the market may tend to rise or fall while the bottom end stays fairly static. Furthermore, any increase in the average may be due to just a few expensive houses being sold. These comments would apply to all houses. We would need to look at houses sold more than once or, if there are few in this category, compare houses with similar valuations.

15. **(a)** Select a random sample of 10 out of 20, and assign them to the standard treatment.

(b) You could use a paired-comparison method based on the differences.

(c) No, as we have two independent samples.

(d) You can again use a paired-comparison method, though there are more complicated methods of analyzing design III.

(e) To allow for any carry-over effect or changes over time.

(f) Design III. Any carry-over effect will be balanced out: half of the subjects will get treatment 1 first and the other half treatment 2 first. This is in contrast to design II, where you may not get 10 subjects with each ordering.

17. (a) (i) One sample. Confidence interval (as we are not told what "effective" means).

(ii) The percentages are approximately Normal with equal standard deviations.

(iii) No placebo is used for a comparison. Also, some patients will have more headaches than others so that the (Binomial) percentages will have different standard deviations. Generalizability: How similar are the people under study to those the treatment will be marketed to?

(b) (i) (We would need random assignment of plots to A or B, i.e., a completely randomized design.) Two independent samples. Confidence interval.

(ii) The data set for each method is Normally distributed and the sets are independent.

(iii) Variability in the fertility, for example, of the plots, which may become confounded with the method difference. Generalizability: How similar is the land in the experiment to that potatoes will ultimately be grown on?

(c) (i) More than two independent samples. Confidence intervals.

(ii) Assume that the numbers trapped for each color are Normally distributed and that the four standard deviations are all equal. Also, assume that the four samples are independent. (We need to have some randomized method, such as a randomized block design, for allocating the color to each board.)

(iii) There may be a variation in the numbers of beetles in different parts of the field.

(d) (i) Paired data. Hypothesis test.

(ii) Differences Normally distributed with the same standard deviation.

(iii) There may be a carry-over learning effect. The order of using the thread needs to be randomized so that half the students use the right-hand thread first and the other half use the left-hand thread first.