Thomas W. Yee

## \*Figures from "Vector Generalized Linear and Additive Models: With an Implementation in **R**"

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## Preface

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Thomas Yee Auckland, New Zealand November 2015

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Some, imagining they can best commend themselves to the Eternal by means of statues, are eagerly desirous of them, as if they were certain to obtain more reward from brazen figures unendowed with sense, than from the consciousness of duties honourably and uprightly performed. —Ammianus Marcellinus

## Contents

| 1 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 1  |
|---|---|----|
| 2 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 7  |
| 3 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 29 |
| 4 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 33 |
| 5 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 41 |
| 6 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 49 |
| 7 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 61 |
| 8 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 67 |
| 9 | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 | 71 |

| 10  | Figures from Vector Generalized Linear and Additive Models:With an Implementation in R© T. W. Yee, 201573       |
|-----|---|
| 11  | Figures from Vector Generalized Linear and Additive Models:With an Implementation in R© T. W. Yee, 201579       |
| 12  | Figures from Vector Generalized Linear and Additive Models:With an Implementation in R© T. W. Yee, 201585       |
| 13  | Figures from Vector Generalized Linear and Additive Models:With an Implementation in R© T. W. Yee, 201589       |
| 14  | Figures from Vector Generalized Linear and Additive Models:With an Implementation in R© T. W. Yee, 201593       |
| 15  | Figures from Vector Generalized Linear and Additive Models:With an Implementation in R© T. W. Yee, 201597       |
| 16  | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 |
| 17  | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 |
| Α   | Figures from Vector Generalized Linear and Additive Models:<br>With an Implementation in R<br>© T. W. Yee, 2015 |
| Rei | ferences  |

Chapter 1
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Fig. 1.1 Rank-1 constrained quadratic ordination (CQO). (a) The mean abundance is  $\mu_s(\nu) = E(Y_s|\nu)$  for  $s = 1, \ldots, S$  species, and  $\nu = c^T x_2$  is a latent variable. (b) Zooming in on the subinterval [A, B] in (a). This is approximately linearly on the  $\eta$  scale, meaning a RR-VGLM would be suitable.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 1.2 Flowchart for different classes of models. Legend: LM = linear model, V = vector, G = generalized, A = additive, O = ordination, Q = quadratic, U = unconstrained, RCIM = row-column interaction model. See also Table 1.1. Apart from the LM, the models of the bottom half are more to be viewed as computational building blocks.



Fig. 1.3 Paths of the estimated LASSO coefficients in a LM fit. The response log(los) is regressed against the variables admit (black), age75 (red), procedure (green) and sex (blue) and intercept, in the data frame azpro from COUNT. The first plot has log  $\lambda$  as its *x*-axis, whereas second plot has the quantity  $\sum_{k=2}^{p} |\beta_k|$  in (1.43). The upper numbers are the number of variables in the model. Package glmnet is used here.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 1.4 Some functional forms derived from the S formula language based on elementary functions and operators. The formula heads each plot, followed by footnotes. The number of parameters in the regression is given. An intercept term is assumed in all—and the blue  $\bullet$  point indicates its value at the location  $x = x_0$ . Plots (k)–(l) are contour images with various colours denoting different fitted values (blue for the intercept). The value  $x_0 = x_0 = 0.4$  here, and variables  $x, x_2$  and  $x_3$  are defined on the unit interval. Function bs() resides in the splines package.

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**Fig. 2.1** Properties of some common link functions  $g_j$  suitable for a probability. (a)  $g_j(p)$ ; (b)  $g'_j(p)$ ; (c)  $g''_j(p)$ ; (d)  $g_j^{-1}(p)$ . The legend in (a) is common for all plots. The calls to (a)-(c) are of the form link.function(p, deriv = d) for d = 0, 1 and 2 (Table 1.2).



Fig. 2.2 Wald (orange) and likelihood ratio test (blue) statistics plotted against  $p_2$ , for: (a)  $p_1 = 0.5$ , (b)  $p_1 = 0.25$  (vertical dashed lines). Actually, the Wald statistic here is the square of the usual Wald statistic, and  $p_2 = 0.01(0.01)0.99$  is discrete. The data follows Hauck and Donner (1977).

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Fig. 2.3 Completely separable data (blue circles). Adding the two orange hollow points results in quasi-completely separable data. The logistic regression estimate of the slope will tend to infinity in both cases.



Fig. 2.4 Polynomials of degree 1–4 fitted to two data sets. (a) mcycles from MASS. (b) cars from datasets.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 2.5 (a) Proportion of fish caught that are rainbow trout from Lake Otamangakau (lakeO) caught by an angler who frequented the spot. The variable Year is year-1900. (b) Smoothing the same data with a cubic regression spline (truncated power basis) with one knot located at the year 1980. A boundary effect on the RHS is evident.



Fig. 2.6 Smoothing some data with a regression spline (B-spline). Each segment of the spline is coloured differently. The term is effectively bs(x, knots = c(1, 3.08, 6.03)). The true function is the sine function (dashed) and n = 50.



Fig. 2.7 Truncated power series basis for cubic splines (2.39) The black dashed lines are  $1, x, x^2, x^3$ . The coloured solid lines are  $(x - \xi_k)^3_+$  for knots  $\xi_1 = 1, \xi_2 = 2, \xi_3 = 3, \xi_4 = 4$  and  $\xi_5 = 5$ .



Fig. 2.8 B-splines of order 1-4 ((a)–(d)), where the interior knots are denoted by vertical lines. The boundary knots are at 0 and 11. The basis functions have been plotted from left to right.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 2.9 Smoothing some data with cubic regression splines with knots of varying multiplicities. The knots are at x = 2 and 4.



Fig. 2.10 (a)–(d) Linear combinations of B-splines of degrees 0-3 fitted to some scatter plot data; the formula is similar to (2.55) The knots are equally-spaced on the unit interval.



Fig. 2.11 (a) Cubic smoothing spline fitted to the proportion of fish caught that are rainbow trout from lake0. The *x*-axis is year. The smoother has 1 nonlinear degrees of freedom. (b)–(d) Derivatives of the smooth of orders 1–3. In contrast, Fig. 2.15 fits a local linear regression to these data.

Fig. 2.12 O-splines: number of knots K selected from n unique  $x_i$ , for smooth.spline() is the top function. The lower function is (2.56)for vsmooth.spline(). Both axes are on a logarithmic scale. The top function intersects with the dashed lines at (50, 50), (200, 100), (800, 140) and (3200, 200); logarithmic interpolation is used for other n values.





Fig. 2.13 Local linear regression with n = 40 points. The kernel weights have been divided by 10 for scaling purposes. The vertical line is at the target point  $x_0 = 0.7$  so that three curves/lines intersect at  $(x_0, \hat{f}(x_0))$ . The shaded region is effectively the window for computing  $\hat{f}(x_0)$ .



Fig. 2.14 Kernel functions from Table 2.2. All but one has compact support.

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Fig. 2.15 (a) Local linear regression fitted to the proportion of fish caught that are rainbow trout from lake0. The smoother has a bandwidth of h = 2.5 and uses the Gaussian kernel function. (b)  $\hat{f}'(x)$ . In contrast, Fig. 2.11 fits a cubic smoothing spline to these data.

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Fig. 2.16 Asymptotic equivalent kernel for  $\hat{f}'(x)$  from (2.82) The 101  $x_i$  are equally-spaced on [0, 1] (as shown by the rugplot). The  $x_0$  values are 0.05, 0.25, 0.5 (vertical dashed lines), and the bandwidth is 0.2. The kernel function  $K = \phi(\cdot)$ .

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Fig. 2.17 (a) Eigenvalues of the smoother matrix of a cubic smoothing spline. Here, n = 20 and the  $x_i$  are equidistant on [0,1]. (b) The same on a logarithmic scale. The two unit eigenvalues correspond to constant and linear functions. The corresponding eigenvectors are plotted in Fig. 2.18.

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Fig. 2.18 Successive eigenvectors corresponding to the eigenvalues of Fig. 2.17.



**Fig. 2.19** Equivalent kernel of a cubic spline,  $\kappa(u)$  (Eq. (2.92))



Fig. 2.20 Fitted values from some logistic regression models applied to chinese.nz. The response is the proportion of New Zealand Chinese who are female. The terms are year, poly(year, 2), bs(year, 4). Area sizes of the points are proportional to the number of people.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 2.21 Four residual types for the regression spline fit of Fig. 2.20. The fitted values are plotted on the x-axis.

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Fig. 3.1 An extension of the fitted model fit.travel from Sect. 14.2.1, where the cost variable is smoothed with regression splines. The variable gcost stands for 'generalized cost'.


Fig. 3.2 Log-likelihood  $\ell$  as a function of (i)  $\mu$  and (ii)  $\eta = \text{logit } \mu$ , for the V1 data frame. The model is a logistic regression with 3 or more hits defining 'success'. The vertical dashed lines are at the MLE  $\hat{\mu}$ . The dashed curves are the quadratic approximation to  $\ell$  at  $\hat{\mu}$ .

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Fig. 3.3 Hat values from a proportional odds model fitted to the severity of pneumoconiosis data set in coalminers, called **pneumo**.

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Fig. 4.1 (a) Scatter plot of diastolic ( $^{\circ}$ ) and systolic ( $\times$ ) blood pressures (mm Hg) versus age, for a random sample of 100 European-type females from xs.nz. (b) A vector smoothing spline fit overlaid on the same. Each component function has 2 effective nonlinear degrees of freedom (ENDF).

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Fig. 4.2 (a) Data used from (4.6). The black curves are the true functions; (b)–(f) Equivalent kernels based on (4.6) for various values of  $\rho$ . The row of the influence matrix corresponds to  $\hat{f}_1(x=0)$ , and is midway between the boundaries.

## Fig. 4.3 O-

smoothing spline (blue curve) fitted to scatter plot data  $\bullet$ , which is equal to a least squares fit (black line) plus the sum of individual B-spline basis functions; cf. Fig. 2.8. It corresponds to the decomposition (4.12). The rugplot denotes the position of the  $x_i$ .



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Fig. 4.4 A bivariate odds ratio model fitted as a VGAM to a subset of female Europeans from xs.nz, with household cat and dog ownership as the responses. The plots are the  $\hat{f}_{(j)2}(x_2)$ .

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 4.5 Joint probabilities from fit.cd1.



Fig. 4.6 The first three plots are the fitted component functions overlaid; the models are fit1.cd are fit3.cd. The fourth plot is the estimated log odds ratios of fit3.cd.

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Fig. 5.1 (a) RR-VGLM-binomial model applied to several disease responses. The x-axis is  $\hat{\nu}$ , a linear combination of 11 binary psychological variables. The y-axis is disease prevalence. (b) The same on a log scale.

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Fig. 5.2 Monthly average prices of Grain series, January 1961–October 1972, in data frame grain.us.

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Fig. 5.3 Canonical variables of the Grain Price series, January 1961–October 1972.

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Fig. 5.4 Mosaic plot of alcoff; the area sizes are proportional to the counts.

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Fig. 5.5 Hourly and effective daily effects of a rank-0 Goodman's RC model fitted to alcoff. This is output from plot(grc0.alcoff).



Fig. 5.6 Rasch fixed effects model to exam1, Eq. (5.29). Only a few people and items are labelled.

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Fig. 5.7 Quasi-variances computed for the ships data in MASS. The fitted model is a quasi-Poisson-GLM fitted to the ship data (McCullagh & Nelder, 1989) with respect to ship types (A–E) on the damage rate on a log scale. (a) Confidence intervals for contrasts with type A ships based on conventional standard errors; (b) Comparison intervals based on quasi-variances; (c) 5% LSD intervals (arrows) based on quasi-variances overlaid on (b). For (a)–(c), the formulas are  $\hat{\beta}_i \pm 2 \operatorname{SE}(\hat{\beta}_i)$ ,  $\hat{\beta}_i \pm 2 \sqrt{q_i}$  and  $\hat{\beta}_i \pm 2(0.025) \sqrt{q_i/2}$ , respectively.

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Fig. 6.1 Four hypothetical models for community ecology, along a gradient or latent variable. Plot (c) corresponds to a species packing model.

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Fig. 6.2 Ordination of 11 species from the hspider data frame; a Poisson model with unequal tolerances, called plut.hs.2. (a) lvplot() output. (b) persp() output, which is a perspective plot and a 'continuous' version of (a).



Fig. 6.3 Ordination of 12 species from the hspider data frame; a Poisson model with equal tolerances, called plet.hs. (a) lvplot() output. (b) persp() output, which is a perspective plot and a 'continuous' version of (a).



Fig. 6.4 Perspective plot of a rank-2 CQO fitted to the hspider data frame, called p2et.hs. All 12 species are fitted, using an equal-tolerances Poisson model, but only the response surfaces of the dominant species may be seen. See also Fig. 6.6 for an ordination diagram of the above.



Fig. 6.5 The effect of the argument isd.latvar on the site scores. All response curves have unit tolerances (because  $\mathbf{T}_s = \mathbf{I}_R \forall s$ ), and the optimums are located the same relative distance from each other. The site scores are uniformly distributed over the latent variable space, and have been scaled to have a standard deviation isd.latvar. The tick marks are at the same values.



Fig. 6.6 Ordination diagrams of 12 species from the hspider data frame; the Poisson model p2et.hs with equal tolerances. A convex hull surrounds the site scores. In (a), the circles indicate the abundance of each species at 95% of its maximum abundance. In (b), the arrows display the contribution of each environmental variable towards each of the ordination axes.



Fig. 6.7 Trajectory plot of three hunting spiders species. A rank-1 Poisson CQO is fitted to these. Site numbers have been placed on each curve.



Fig. 6.8 Two calibrated sites from a rank-1 equal-tolerances Poisson QRR-VGLM fitted to the hunting spiders data without those 2 sites. The thick vertical lines are the CQO sites scores  $\hat{\nu}_i$ , and the thinner vertical lines are the calibrated sites scores  $\tilde{\nu}_i$ . Each *i* has the same colour and line type.

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Fig. 6.9 Perspective plot of an unequaltolerances Poisson CQO model fitted to the trap0 trout data. Legend: "B" = brown trout, "R" = rainbow trout, "F" = female, "M" = male. The trap was located at the Te Whaiau Trap, hence the "TW". The BFTW response curve has unit tolerance.



Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 6.10 (a)–(b) compares the UQO solution with the truth for a rank-1 Poisson simulated data set. (a) Estimated optimums  $\hat{u}_j$  versus  $u_j$ . (b) Estimated site scores  $\hat{\nu}_i$  versus  $\nu_i$ . (c) CQO fitted to the original data. (d) CQO fitted to the scaled UQO site scores. In (a)–(b) the dashed orange line is a simple linear regression through the points.

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Fig. 7.1 Estimated (uncentred) component functions of plcao.hs, a rank-1 Poisson RR-VGAM fitted to the hspider data. The x-axis is  $\hat{\nu}$ .

Fig. 7.2 Perspective plot of a rank-1 Poisson RR-VGAM fitted to the trap0 data. The latent variable of this CAO, which is predominantly the day of the year, has unit variance. The responses are combinations of male and female rainbow and brown trout, all captured at the Te Whaiau Trap of Lake Otamangakau. See also Fig. 6.9.





Fig. 7.3 Component functions (uncentred) of a rank-1 binomial RR-VGAM fitted to a subset of the xs.nz data. The latent variable of this CAO is a linear combination of 11 binary psychological variables.



Fig. 7.4 Top row: component functions (centred) of a rank-1 binomial RR-VGAM fitted to a subset of the xs.nz data, cf. Fig. 7.3. The latent variable of this CAO is a linear combination of 11 binary psychological variables. Bottom row: first derivatives of the respective fitted functions.

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Fig. 8.2 Fitted logistic regression VGAM (the response is nofriend) to European-type people in xs.nz with (a) noxmean = FALSE, (b) noxmean = TRUE. In (a), the point A, which has 0 as its SE, has been explicitly added to the plot, whereas it has been omitted in (b). The x-coordinate of A is at the mean of the variable sex01. The y-coordinate is 0 because component functions are centred.

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Fig. 9.1 Fitted curves for three binary regression models fitted to completely-separable data, with n = 20. A grey vertical line at  $x_2 = 0$  is plotted. If there were an additional two points at (0,0) and (0,1) then the data would have quasi-complete separation.

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Fig. 10.1 (a)–(b) Scatter plots of the ethanol data stratified by C and E respectively. In (a) C has been split into low, medium and high subgroups. In (b) E has been split into three similar subgroups—see Fig. 10.2 for an expansion. Plots (c)–(d) are the estimated (centred) component functions of the VCM (10.5) where both the intercept and slope are a smoothing spline of the variable E. The dashed lines are pointwise  $\pm 2$  SEs about the  $\hat{\beta}_j(E)$ . The points about the curves are the working residuals.



Fig. 10.2 Expansion of Fig. 10.1(b) with a least squares line added to each subset.

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Fig. 10.3 Tobit model fitted to some simulated data. This mimics the type of problem motivating Tobin (1958), viz. spending is non-negative, and is linear beyond a certain income. Values of zero are plotted with a different colour and symbol for clarity. (a) The purple dashed line is a naïve fit that treats all values as if they were 'real'. The estimate and the truth are similar. (b) The 3 types of fitted values currently distinguished by the argument type.fitted.

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Fig. 10.4 Smooths of a SUR model applied to the gew data. The fitted model is called fit.rs.

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Fig. 11.1 Acceptance-rejection method for generating random variates. The solid blue points are accepted; the hollow orange points are rejected. (a) The density g = dunif() is used to generate Beta(2, 4) random variates. The vertical line at y = 0.25 denotes the position of the mode, thus defining C = f(0.25). (b) A Kumaraswamy(3, 4) density (f; blue) is overshadowed by a scaled Beta(3, 2) density  $(C \cdot g(y); \text{ purple dashed})$ ; the scaling constant is C = 1.5.

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Fig. 11.2 Contour plot of qnbinom(0.995, size = size, mu = mu), where  $\mu$  and k are on a log-scale. Regions of the  $(\mu, k)$ -space with a value less than 1000, say, might have the EIM computed by the exact method (11.5). Some of the contour levels appear jagged due to the discrete nature of qnbinom().



Fig. 11.3 Profile log-likelihood  $\ell(a_{21})$  for rrnb.azpro;  $\ell(\hat{a}_{21})$  is the highest point. The MLE and likelihood-ratio confidence limits are the orange horizontal lines. The Wald confidence limits are the grey vertical lines. The point at  $a_{21} = 0$  is  $\ell$  for NB-2 (intercept-only for k). The point at  $a_{21} = 1$  is  $\ell$  for NB-1.



Fig. 11.4 Standard deviation,  $SD(Y_i^*)$ , in the beta-binomial distribution, from (11.13) with  $N_i = 10$ . (a) As a function of  $\mu$ , for  $\rho = 0.33$  (blue),  $\rho = 0.67$  (green) and  $\rho = 0$  (orange dashed). (b) As a function of  $\rho$ , where  $\mu_i = 0.5$ .

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**Fig. 12.1** Log-likelihood  $\ell(a)$  as a function of the location parameter, for a random sample of size 10 from a standard Cauchy distribution. The dashed vertical line denotes  $\hat{a}$ , and the purple  $\times$  denotes the data.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 12.2 QQ-plots of 2 simulated data sets. A Birnbaum-Saunders distribution is fitted to both. (a)–(b) Data from a Birnbaum-Saunders distribution; (c)–(d) data from a Fréchet distribution. Plots (a) and (c) are based on the q-type function, and both axes are on a log-scale. Plots (b) and (d) use the p-type function. A x = y line appears in all plots.

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Fig. 12.3 The 2-parameter gamma distribution (12.10). (a)  $SE(\hat{b})$  are the solid curves, for various values of the scale parameter *b* given in the legend of (b). The purple dashed line is  $SE(\hat{s})$ . (b) The same with both axes on a log-scale. (c) The densities with shape parameters  $s = e^0$  (solid lines) and  $e^1$  (dashed). (d) The same as (c) with one axis on a log-scale.

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Fig. 13.1 250 random vectors generated from a standard bivariate normal distribution, with various values of  $\rho$ .

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Fig. 13.2 Fitted (centred) component functions of a VGAM  $N_2$  fitted to diastolic and systolic blood pressures data, versus age. The data set are 5649 male Europeans from xs.nz. From left to right, those for  $\hat{\mu}_1$  and  $\hat{\mu}_2$ , those for  $\log \hat{\sigma}_1$  and  $\log \hat{\sigma}_2$ , those for  $\log((1 + \hat{\rho})/(1 - \hat{\rho}))$ .

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 13.3 500 random vectors generated from two copulas. (a) Bivariate Gaussian copula with  $\alpha = \rho = 0.8$ . (b) Bivariate Frank copula with  $\alpha = 50$ .

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Fig. 14.1 Biplot of a RR-MLM fitted to marital status in xs.nz, with the latent variables being linear combinations of 11 binary psychological variables. The x- and y-axes are  $\hat{\nu}_1$  and  $\hat{\nu}_2$ , respectively. The groups are: 1 single, 2 married/partnered (baseline), 3 divorced/separated, 4 widowed.





Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 14.3 Nonparametric nonparallel cumulative logit model fitted to the **pneumo** data: (a) the two centred functions are overlaid on to one plot; (b) the fitted values as a function of let with sample proportions plotted proportional to their counts; (c) reversed cumulative probabilities  $\hat{P}(Y \ge j)$  for j = 2, 3. (d) Proportional odds model: the marginal effects for variable let for each of the 3 levels.

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Fig. 15.1 (a) The Box-Cox transformation  $(y^{\lambda} - 1)/\lambda$  for various values of  $\lambda$ . (b) The Yeo-Johnson transformation  $\psi(\lambda, y)$ .

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.2 Hexagonal binning plot of BMI versus age for European-type women in xs.nz.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.3 Plot of the VGAM component functions of w0.LMSO, a lms.bcn() fit of BMI versus age for European-type women in the xs.nz data frame. These are (centred)  $\hat{\lambda}(x)$ ,  $\hat{\mu}(x)$ , log  $\hat{\sigma}(x)$ .

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.4 (a) Quantile and (b) density plots of w0.LMS. In (b) the estimated densities are for 20 and 30 year olds, and the vertical lines denote an approximate healthy range.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.5 Output of qtplot() when there are non-primary variables. The two groups are European and Pacific island women. The fitted 50-percentiles differ by a constant.

Vector Generalized Linear and Additive Models: With an Implementation in R


Fig. 15.6 Loss functions for: (a) quantile regression with  $\tau = 0.5$  ( $L_1$  regression) and  $\tau = 0.9$ ; (b) expectile regression with  $\omega = 0.5$  (least squares) and  $\omega = 0.9$ . Note: (a) is also known as the asymmetric absolute loss function or pinball loss function.



Fig. 15.7 (a) Scatter plot of simulated Poisson counts (15.19). The points have been jittered slightly. (b) Fitted  $\mathcal{L}^*(\xi = \exp\{\beta^*_{(s)1} + f^*_{(s)2}(x_{i2})\}, \sigma = 1, \tau = (\frac{1}{4}, \frac{3}{4})^T)$  model to the data. The smooth curves are the fitted  $\tau = 0.25$  and 0.75 quantiles from a vector smoothing spline fit. The step functions are the output from **qpois()** at the corresponding  $\tau$  values. The actual sample proportions lying below the curves are 35.2 and 79.6 percent. See also Fig. 15.8(b).

Vector Generalized Linear and Additive Models: With an Implementation in R



**Fig. 15.8** (a) Fitted functions  $\hat{f}_{(s)2}(x_{i2})$  in (15.20) overlaid, with pointwise  $\pm 2$  SE bands. (b) Same as Fig. 15.7(b) except the quantiles (purple curves) are constrained to be parallel on the  $\eta$ -scale (log scale here). The actual sample proportion lying below the curves are 36.4 and 78.4 percent.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.9 (a) Quantile plot from the onion method applied to some simulated Poisson data. The quantiles are positive, nonparallel and noncrossing. Here,  $\tau$  has values 0.2(0.1)0.9. (b) A zoom-in view of the LHS of (a).

Then Fig. 15.9(a) was produced by

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.10 (a)–(c) Density plots of expectile derived g (orange solid lines; (15.30)) for the original f of standard normal, standard uniform and standard exponential distributions (blue dashed lines). (d) Illustration of the interpretation of expectiles in terms of centres of balance, the hollow triangles at positions  $c_1$  and  $c_2$ . Here, the vertical dashed line is at the 0.1-expectile, the solid triangle at  $\mu(\omega = 0.1)$ , which means that (15.33) is satisfied with  $\omega = 0.1$ .



**Fig. 15.11** 'Quantile' plot from amlnormal() applied to women.eth0: 5, 25, 50, 75, 95 'percentile' curves. Each regression curve is a regression spline with 3 degrees of freedom (1 = linear fit).

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 15.12 Quantile plot from an amlpoisson() fit using simulated data. The dashed step functions are the qpois() quantiles calibrated with the true mean function and the proportion under the expectile curves. The response has been jitted to aid clarity.



Fig. 15.13 (a) Melbourne maximum temperature data, called melbmaxtemp, in °C. (b) Onion method using amlnormal() expectiles applied to the data with an assortment of  $w_j$  values. The sample proportions below the curves are 1, 4.8, 20.2, 54.9, 65.2, 74.8, 82.7, 88.2, 91.7, 94.8 percent.

Vector Generalized Linear and Additive Models: With an Implementation in R

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**Fig. 16.1** GEV densities for values  $\mu = 0$ ,  $\sigma = 1$ , and  $\xi = -\frac{1}{3}$ , 0,  $\frac{1}{3}$  (Weibull-, Gumbel- and Fréchet-types respectively). The orange curve is the CDF, the dashed purple segments divide the density into areas of  $\frac{1}{10}$ . The bottom RHS plot has the densities overlaid.

Vector Generalized Linear and Additive Models: With an Implementation in R



**Fig. 16.2** GPD densities for values  $\mu = 0$ ,  $\sigma = 1$ , and  $\xi = -\frac{1}{3}$ , 0,  $\frac{1}{3}$  (beta-, exponential- and Pareto-types, respectively). The orange curve is the CDF, the dashed purple segments divide the density into areas of  $\frac{1}{10}$ . The bottom RHS plot has the densities overlaid.



Fig. 16.3 Gumbel plot of the two highest annual sea levels of the Venice data (venice).

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 16.4 Intercept-only GEV model fitted to the portpirie annual maximum sea levels data. (a) Scatter plot, and the dashed horizontal lines are the resulting 95% and 99% quantiles. (b) Probability plot. (c) Quantile plot. (d) Density plot. (e)–(f) Return level plots, with slight changes in the x-axis labelling.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 16.5 VGAM fitted to the Venice sea level data (fit1).

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 16.6 (a) Quantile plot of the Venice sea level data (fit2). (b) Venice data overlaid with the fitted values of fit3. This model underfits.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 16.7 (a) The fitted 99 percentile of fit2 and the 99 percentile data values (4th highest sea level each year). (b) Fitted median predicted value (MPV) of the Venice sea level data from fit2. The points are the highest sea levels for each year.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 16.8 The rain daily rainfall data. (a) Scatter plot. (b) Mean excess plot.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 16.9 Intercept-only GPD model fitted to the rain daily rainfall data. (a) Scatter plot, with the solid black horizontal line denoting the threshold at 30. The dashed horizontal lines are the resulting 90% and 99% quantiles. (b) Probability plot. (c) Quantile plot. (d) Density plot.

Chapter 17
Figures from Vector Generalized Linear and Additive Models: With an Implementation in R
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Fig. 17.1 Decision tree diagram for (a) zero-alteration versus (b) zero-inflation. They depict (17.4) and (17.7) respectively. Here,  $Y^*$  corresponds to a parent distribution such as the binomial or Poisson, and Y is the response of interest. The probabilities  $\omega$  and  $\phi$  dictate the decisions.





Fig. 17.2 Probability functions of a (a) zero-inflated Poisson with  $\phi = 0.2$ , (b) zero-deflated Poisson with  $\omega = -0.035$ . Both are compared to their parent distribution which is a Poisson( $\mu = 3$ ) in orange.



Fig. 17.3 Estimated component functions for the deermice VGAM.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. 17.4 Capture probability estimates with approximate  $\pm 2$ pointwise SEs, versus wing length with (blue) and without (orange) fat content present fitting a  $\mathcal{M}_h$ -VGAM, using the **prinia** data.

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Fig. A.1 The first few Newton-like iterations for a Poisson regression fitted to the V1 data set. The solid orange curve is  $\ell(\theta)$  with  $\theta = \mu$ . The initial value is  $\theta^{(1)} = 0.2$ . Each iteration  $\theta^{(a)}$  corresponds to the maximum of the quadratic (dashed curves) from the previous iteration.

Vector Generalized Linear and Additive Models: With an Implementation in R



Fig. A.2 Negative binomial NB( $\mu$ , k) distribution fitted to the machinists data set. The *y*-axis is  $\ell$ . Let  $\theta = k$  and  $\theta^* = \log k$ . (a)  $\ell(\theta)$  is the solid blue curve. (b)  $\ell(\theta^*)$  is the solid blue curve. Note: for  $H_0: \theta = \theta_0$  (where  $\theta_0 = \frac{1}{3}$ ), the likelihood-ratio test, score test and Wald test statistics are based on quantities highlighted with respect to  $\ell$ . In particular, the score statistic is based on the tangent  $\ell'(\theta_0)$ .

Vector Generalized Linear and Additive Models: With an Implementation in R

## References

Yee, T. W. (2015). Vector Generalized Linear and Additive Models: With an Implementation in R. New York, USA: Springer.